MATHEMATICS -I EM

FIRST YEAR

Intermediate Vocational
Bridge Course

NAME: ___________________________________

ROLL No.___________________________________

Question Bank

1.FUNCTIONS

1. If A={0 , $\frac{\lambda}{6}$, $\frac{\lambda}{4}$, $\frac{\lambda}{3}$, $\frac{\lambda}{2}$} and f:A→B f(x)=$\cos x$ then find B.
2. If A={-2,-1,0,1,2} and f:A→B f(x)=x^2 +x+1 then find B. **(model)**
3. If f={(1,2), (2,-3),(3,-1)} then find **(Mar19)** (i) 2f (ii)f^2 (iii)f+2 (iv)$\sqrt{f}$
4. If f=(4,5)(5,6)(6,-4) and g=(4,-4)(6,5)(8,5) then find (i) f+g (ii)f-g (iii)2f+4g (iv) f+4 (v) fg (vi)$\frac{f}{g}$ (vii)$\sqrt{f}$ (viii)|f| (ix) f^2 (x) f^3
5. If f(x)=2x-1 and g(x)=x^2 then find (i) (3f-2g)(x) (ii) (fg)(x) (iii)$\frac{\sqrt{f}}{g}$(x) (iv) (f+g+2)(x)
6. If f(x) = $\frac{1-x^2}{1+x^2}$ then show that f($\tan\theta$)=$\cos 2\theta$ **mar20**
7. If f(x)=$\log\left|\frac{1+x}{1-x}\right|$ then show that f($\frac{2x}{1+x^2}$) =2f(x)
8. If f(x) =4x-1; g(x)=x^2+2 then find (i) (gof)(x) (ii) go(fof)(0) (iii) (gof)$\frac{a+1}{4}$ (iv) (fof)(x)
9. f(x) =2, g(x) =x^2, h(x) =2x then find (fogoh)(x)
10. If f(x) = ax+b then find f^{-1}(x)
11. If f(x) =5^xthen find f^{-1}(x)
12. If f(x) =2x-3, g(x)=x^3+5 then find $(fog)^{-1}$(x).
13. If f(x) = $\frac{x+1}{x-1}$ then find (fof)(x)
14. If f(x)=$\frac{1}{x}$; g(x)=$\sqrt{x}$ then find (gof)(x) and g$\sqrt{f}$ (x)
15. If A={1,2,3,4} and f:A→R and f(x) = $\frac{x^2-x+1}{x+1}$ then find the range of f

2. MATHEMATICAL INDUCTION

1. Prove that 1+2+3+…..+n=$\frac{n(n+1)}{2}$ using mathematical induction.
2. Prove that $1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$ using mathematical induction.
3. Prove that $1^3+2^3+3^3+\ldots+n^3=\frac{n^2(n+1)^2}{4}$ using mathematical induction.
4. Prove that 1+3+5+….+2n-1=n^2 using Mathematical induction.
5. Prove that a+(a+d)+(a+2d)+….+a+(n-1)d =$\frac{n}{2}$[2a+(n-1)d] using mathematical induction.
6. Prove that a +ar +ar^2+….+ar^{n-1}=$\frac{a(r^n-1)}{r-1}$ using mathematical induction.
7. Prove that 3+3^2+3^3….3^n=$\frac{3}{2}$(3^n-1) using Mathematical induction.
8. Prove that 1.2.3+2.3.4+3.4.5+…n terms =$\frac{n(n+1)(n+2)(n+3)}{4}$ using mathematical induction.
9. Prove that $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}$+…. n terms=$\frac{n}{2n+1}$ using Mathematical induction.
10. Prove that $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}$+…. n terms=$\frac{n}{3n+1}$ using Mathematical induction.
11. Prove that 2+7+12+….(5n-3) =$\frac{n(5n-1)}{2}$ using Mathematical induction.
12. Prove that $4^3+8^3+12^3+\ldots+$n terms =$16n^2(n+1)^2$ using Mathematical induction.
13. Prove that $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$ using Mathematical induction.
14. Prove that 2+3.2+4.2^2….n terms=n2^n using Mathematical induction.
15. Prove that 2.3+3.4+4.5+…+ n terms=$\frac{n(n^2+6n+11)}{3}$ using Mathematical induction.

3. MATRICES

1. If A=$\begin{bmatrix}3 & 9 & 0\\1 & 8 & -2\end{bmatrix}$ and B=$\begin{bmatrix}4 & 0 & 2\\7 & 1 & 4\end{bmatrix}$ then find A+B
2. If A=$\begin{bmatrix}1 & 2 & 3\\3 & 2 & 1\end{bmatrix}$ and B=$\begin{bmatrix}3 & 2 & 1\\1 & 2 & 3\end{bmatrix}$ then find 3B-2A
3. If A=$\begin{bmatrix}0 & 1 & 2\\2 & 3 & 4\\4 & 5 & 6\end{bmatrix}$, B=$\begin{bmatrix}1 & -2 & 0\\0 & 1 & -1\\-1 & 0 & 3\end{bmatrix}$then find A-B and 4B-3A.
4. If A=$\begin{bmatrix}2 & 3 & 1\\6 & -1 & 5\end{bmatrix}$, B=$\begin{bmatrix}1 & 2 & -1\\0 & -1 & 3\end{bmatrix}$ and A+B-X=[0] Then find the matrix X.
5. Find the trace of the matrix $\begin{bmatrix}1 & 3 & -5\\2 & -1 & 5\\1 & 0 & 1\end{bmatrix}$
6. If A=$\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ and B=$\begin{bmatrix}0 & 4\\-1 & 2\end{bmatrix}$ then find AB and BA
7. If A=$\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ then find A^2
8. If A=$\begin{bmatrix}2 & 4\\-1 & K\end{bmatrix}$ and A^2=[0] then find K.
9. If A=$\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$ then show that A^2-4A-5I=[0]
10. If A=$\begin{bmatrix}i & 0\\0 & -i\end{bmatrix}$ then show that A^2=-I
11. If A=$\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}$ then find A+A^T
12. If A=$\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}$ then find A.A^T
13. If A=$\begin{bmatrix}-2 & 1\\5 & 0\\-1 & 4\end{bmatrix}$; B=$\begin{bmatrix}-2 & 3 & 1\\4 & 0 & 2\end{bmatrix}$ then find 2A+B^T
14. If A=$\begin{bmatrix}2 & -4\\-5 & 3\end{bmatrix}$ then find A.A^T
15. If A=$\begin{bmatrix}-1 & 2 & 3\\2 & 5 & 6\\3 & x & 7\end{bmatrix}$ is a symmetric matrix, find the values of x.
16. A=$\begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}$ then show that A.A^T=I
17. If A=$\begin{bmatrix}1 & 4 & 7\\2 & 5 & 8\end{bmatrix}$ then verify that $(A+B)^T=A^T+B^T$
18. If A=$\begin{bmatrix}1 & 5 & 3\\2 & 4 & 0\\3 & -1 & -5\end{bmatrix}$; B=$\begin{bmatrix}2 & -1 & 0\\0 & -2 & 5\\1 & 2 & 0\end{bmatrix}$ then find 3A-4B^T
19. If A=$\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$; B=$\begin{bmatrix}0 & 4\\-1 & 2\end{bmatrix}$ then find BA^T
20. If A=$\begin{bmatrix}0 & 4\\-1 & 2\end{bmatrix}$ then find AA^T

21. Find the determinant A=$\begin{bmatrix} 2 & 1 \\ 41 & -5 \end{bmatrix}$

22. Find the determinant A=$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

23. Find the determinant of A=$\begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix}$

24. Find the determinant of A=$\begin{bmatrix} 2 & -1 & 4 \\ 0 & -2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$

25. Find the determinant of A=$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

26. Find the determinant of A=$\begin{bmatrix} 2 & -1 & 4 \\ 4 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

27. Find the determinant of $\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & 4 \\ -3 & 7 & 6 \end{bmatrix}$

28. Find the determinant of $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 2 \\ 4 & 5 & 6 \end{bmatrix}$

29. Find the determinant of $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$

30.Find the determinant of $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

31.Find the determinant of $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

32. Find the determinant of $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}$ where $1,\omega,\omega^2$ are cube roots of unity.

33. Find x of $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{vmatrix}$=45

34. Find $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \\ -4 & -2 & 5 \end{vmatrix}$

35. Prove that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$=0

36. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}=2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

37. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix}$ = $a^3+b^3+c^3-3abc$

38. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ $=(a+b+c)^3$

39. Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}=2(a+b+c)^3$

40. Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$=(a-b)(b-c)(c-a)

41. Prove that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$=(a-b)(b-c)(c-a)(ab + bc + ca)

42. Show that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ = abc(a-b)(b-c)(c-a)

43. Show that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ = 0

44. Show that $\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}=\begin{vmatrix} a & b & c \\ x & y & z \\ yz & zx & xy \end{vmatrix}$ without expanding the matrix

45. If A=$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then find A^{-1}

46. If A=$\begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$ then find Adj (A)

47. If A=$\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ then show that Adj A=$3A^T$

48. If A=$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then find $A^3=A^{-1}$

49. If A=$\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ find $(A^T)^{-1}$

50. Solve the following system of equation using matrix invariant.
x-y+3z=5; 4x+2y+z=0; -x+3y+z=5

51. Solve the following system of equation using matrix invariant.
2x-y+3z=8; -x+2y+z=4; 3x+y-4z=0

52. Solve the following system of equation using matrix invariant.
3x+4y+5z=18; 2x-y+8z=13; 5x-2y+7z=20

53. Solve the following system of equation using Cramer's Rule.
x-y+3z=5; 4x+2y-z=0; -x+3y+z=5

54. Solve the following system of equation using Cramer's Rule.
2x-y+3z=9; x+y+z=6; x-y+z=2

55.Solve the following system of equation using Cramer's Rule.
2x-y+3z=8; -x+2y+z=4; 3x+y-4z=0

4&5 VECTOR ALGEBRA

1. Let $\bar{a}=\bar{i}+2\bar{j}+3\bar{k}$ and $\bar{b}=3\bar{i}+\bar{j}$ find the unit vector in the direction of $\bar{a}+\bar{b}$

2. If the vectors $-3\bar{i}+4\bar{j}+\lambda\bar{k}$ and $\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear then find λ and μ.

3. If the points whose position vectors are $3\bar{i}-2\bar{j}-\bar{k}$, $2\bar{i}+3\bar{j}-4\bar{k}$, $-\bar{i}+\bar{j}+2\bar{k}$ and $4\bar{i}+5\bar{j}+\lambda\bar{k}$ are coplanar then show that $\lambda=\frac{-146}{47}$.

4. If OA=$\bar{i}+\bar{j}+\bar{k}$, AB=$3\bar{i}-2\bar{j}+\bar{k}$, BC=$\bar{i}+2\bar{j}-2\bar{k}$ and CD=$2\bar{i}+\bar{j}+3\bar{k}$. Then find the vector OD

5. Let a=$2\bar{i}+4\bar{j}-5\bar{k}$, b=$\bar{i}+\bar{j}+\bar{k}$ and c = $\bar{j}+2\bar{k}$. Find the unit vector in the opposite direction of a+b+c.

6. OABC is a parallelogram, if $\overline{OA}=\bar{a}$ and $\overline{OC}=\bar{c}$. Find the vector equation of the side BC.

7. Find the vector equation of the plane passing through the points $\bar{i}-2\bar{j}+5\bar{k}$,$-5\bar{j}-\bar{k}and-3\bar{i}+5\bar{j}$.

8. If $\bar{a}=6\bar{i}+2\bar{j}+3\bar{k}$ and $\bar{b}=2\bar{i}-9\bar{j}+6\bar{k}$, then find the angle between the vectors $\bar{a}$ and $\bar{b}$.

9. If|a|=11, |b|=23 and |a − b|=30. Then find the angle between the vectors $\bar{a}$ and $\bar{b}$also find |a + b|.

10. If the vectors $\lambda\bar{i}-3\bar{j}+5\bar{k}$ and $2\lambda\bar{i}-\lambda\bar{j}-\bar{k}$ are perpendicular to each other, find λ.
11.If $\bar{a}=2\bar{i}-\bar{j}+3\bar{k}$ and $\bar{b}=\bar{i}-3\bar{j}-5\bar{k}$. Find the vector $\bar{c}$, such that $\bar{a}$, $\bar{b}$ and $\bar{c}$ form the sides of a triangle.
12. If $|a|=2$, $|b|=3$ and $|c|=4$ and each of $\bar{a}$, $\bar{b}$ and $\bar{c}$ are perpendicular to the sum of the other two vectors then find the magnitude of a+b+c.
13. Find the area of the parallelogram for which the vector . $\bar{a}=2\bar{i}-3\bar{j}$ and $\bar{b}=3\bar{i}-\bar{k}$ are adjacent sides.
14. If $4\bar{i}+\frac{2P}{3}\bar{j}+P\bar{k}$ is parallel to the vector $\bar{i}+2\bar{j}+3\bar{k}$. Find the value of P.
15. If $|a|=13$, $|b|=5$ and a.b=60. Then find $|\bar{a}X\bar{b}|$
16.If $a=7\bar{i}-2\bar{j}+3\bar{k}$, $b=2\bar{i}+8\bar{k}$ and $c=\bar{i}+\bar{j}+\bar{k}$, then compute $\bar{a}x\bar{b}$, $\bar{a}x\bar{c}$ and $\bar{a}x(\bar{b}+\bar{c})$. Verify whether the cross product is distributive over vector addition.
17. If $a=3\bar{i}-\bar{j}+2\bar{k}$, $b=-\bar{i}+3\bar{j}+2\bar{k}$, $c=4\bar{i}+5\bar{j}-2\bar{k}$ and $d=\bar{i}+3\bar{j}+5\bar{k}$. Then compute the following.
(i) $(\bar{a}x\bar{b})$ X$(\bar{c}x\bar{d})$ (ii) $(\bar{a}x\bar{b}).\bar{c}-(\bar{a}x\bar{d}).\bar{b}$
18. If the vectors $a=2\bar{i}-\bar{j}+\bar{k}$, $b=\bar{i}+2\bar{j}-3\bar{k}$ and $c=3\bar{i}+P\bar{j}+5\bar{k}$ are coplanar, then find P.
19. Find the equation of the plane passing through the points A(2,3,-1), B(4,5,2) and C(3,6,5)
20. Find the shortest distance between the skew lines $r=(6\bar{i}+2\bar{j}+2\bar{k})+t(\bar{i}-2\bar{j}+2\bar{k})$ and $r=(-4\bar{i}-\bar{k})+s(3\bar{i}-2\bar{j}-2\bar{k})$
21. Simplify the following
(i) $(\bar{i}-2\bar{j}+3\bar{k})X(2\bar{i}+\bar{j}-\bar{k}).(\bar{j}+\bar{k})$
(ii)$(2\bar{i}-3\bar{j}+\bar{k}).(\bar{i}-\bar{j}+2\bar{k})X(2\bar{i}+\bar{j}+\bar{k})$
22. Find λ in order that the four points A(3,2,1), B(4,λ,5), C(4,2,-2) and D(6,5,-1) be coplanar.
23. Find the volume of the tetrahedron having the edges $\bar{i}+\bar{j}+\bar{k}$, $\bar{i}-\bar{j}$ and $\bar{i}+2\bar{j}+\bar{k}$
24.Compute $[\bar{i}-\bar{j} \quad \bar{i}-\bar{k} \quad \bar{k}-\bar{i}]$
25. If $\bar{a}=(1,-2,1)$; $\bar{b}=(2,1,1)$ and $\bar{c}=(1,2,-1)$ then find $|\bar{a}\ x(\bar{b}\ x\ \bar{c})|$ and $|(\bar{a}\ x\ \bar{b})x\bar{c}|$
26. If $\bar{a}=2\bar{i}+2\bar{j}-3\bar{k}$, $\bar{b}=3\bar{i}-\bar{j}+2\bar{k}$, then find the angle between $(2\bar{a}+\bar{b})$ and $(\bar{a}+2\bar{b})$.
27. Simplify the following
(a) $(\bar{i}-2\bar{j}+3\bar{k})X(2\bar{i}+\bar{j}-\bar{k})x(\bar{j}+\bar{k})$
(b)$(2\bar{i}-3\bar{j}+\bar{k}).(\bar{i}-\bar{j}+2\bar{k})x(2\bar{i}+\bar{j}+\bar{k})$
28.If $\bar{a}$, $\bar{b}$, $\bar{c}$ are non coplanar vectors, then find the value of $\frac{(a+2b-c)[(a-b)x(a-b-c)]}{[a \quad b \quad c]}$

6. TRIGONOMETRIC RATIOS AND FUNCTIONS

1. Find the value of $\cos 225°-\sin 225°+\tan 495°-\cot 495°$
2. Find the value of $\sin^2\frac{\pi}{10}+\sin^2\frac{4\pi}{10}+\sin^2\frac{6\pi}{10}+\sin^2\frac{9\pi}{10}$
3. Find the value of $\cos^2 45°+\cos^2 135°+\cos^2 225°+\cos^2 315°$
4. Find the value of $\sin^2\frac{2\pi}{3}+\cos^2\frac{5\pi}{6}-\tan^2\frac{3\pi}{4}$
5. If tan 20°=P, then prove that $\frac{\tan 610°+\tan 700°}{\tan 560°-\tan 470°}=-\frac{1-P^2}{1+P^2}$
6. Show that $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}=1$
7. If tan 20°=λ, then prove that $\frac{\tan 160°-\tan 110°}{1+\tan 160°\tan 110°}=\frac{1-\lambda^2}{2\lambda}$
8. Prove that $(\sin\theta+\cos\theta)^2+(\cos\theta+\sec\theta)^2-(\tan^2\theta+\cot^2\theta)=7$
9. Prove that $\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta+\cos\theta)^2}=\frac{1-\cos\theta}{1+\cos\theta}$
10. If $\frac{2\sin\theta}{1+\cos\theta+\sin\theta}=x$ then prove that $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}=x$
11. Show that $\cos^4\alpha+2\cos^2\alpha(1-\frac{1}{\sec^2\alpha})=1-\sin^4\alpha$
12. Prove that
$2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$
13. Prove that
$(\tan\theta+\cot\theta)^2=\sec^2\theta+\mathrm{cosec}^2\theta=\sec^2\theta\ .\ \mathrm{cosec}^2\theta$
14. If $\tan^2\theta=1-e^2$ then show that $\sec\theta+\tan^3\theta\ \mathrm{cosec}\ \theta=(2-e^2)^{\frac{3}{2}}$
15. Prove that
$3(\sin x-\cos x)^4+6(\sin x+\cos x)^2+4(\sin^6 x+\cos^6 x)=13$
16. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$
17. If 3sin θ+4cos θ=5 then find the value of 4sin θ-3cos θ.
18. If 3sin A+5cos A =5 then show that 5sin A-3cos A =$\pm$3
19. If a cosθ -b sinθ= C then show that a sinθ + b cosθ = $\pm\sqrt{a^2+b^2-c^2}$
20. If cos θ+sin θ =$\sqrt{2}$ cos θ, then prove that cos θ-sin θ=$\sqrt{2}$ sin θ.
21. If x = $a\cos^3\theta$; y = $b\sin^3\theta$ then eliminate θ.
22. Prove that
$\sin 780°\sin 480°+\cos 240°\cos 300°=\frac{1}{2}$
23. Find the value of
$\sin 330°\cos 120°+\cos 210°\sin 300°$

COMPOUND ANGLES

1. Find the value of sin 75°, cos 75°, tan 75°
2. Prove that $\cos 100°\cos 40°+\sin 100°\sin 40°=\frac{1}{2}$
3. Prove that tan 75°+cot 75°=4
4. Prove that $\cos 100°\cos 40°+\sin 100°\sin 40°=\frac{1}{2}$
5. Show that cos 42°+cos 78°+cos 162°=0
6. If sin(θ+α) =cos(θ+α) then find tan θ in term of tan α
7. Find the value of $\sin^2 82\frac{1}{2}^{\circ}-\sin^2 22\frac{1}{2}^{\circ}$
8. Find the value of $\cos^2 112\frac{1}{2}^{\circ}-\sin^2 52\frac{1}{2}^{\circ}$
9. Find the value of
$\tan 20°+\tan 40°+\sqrt{3}\tan 20°\tan 40°$

10. Find the value of tan 56°-tan 11°-tan 56° tan 11°

11. If $\sin\alpha=\frac{1}{\sqrt{10}}$; $\sin\beta=\frac{1}{\sqrt{5}}$ and α and β are acute, then show that $\alpha+\beta=\frac{\pi}{4}$.

12. If $\sin(A+B)=\frac{24}{25}$, $\tan A=\frac{3}{4}A$, A+B are acute then find the value of $\cos B$.

13. If A+B = 45°, then prove that $(1+\tan A)(1+\tan B)=2$

14. If A+B = 225°, then prove that $\frac{\cot A+\cot B}{(1+\cot A)(1+\cot B)}=2$

15. If $A-B=\frac{3\pi}{4}$, then show that $(1-\tan A)(1+\tan B)=2$

16. If $A+B+C=\frac{\pi}{2}$, then prove that $\cot A+\cot B+\cot C=\cot A\cot B\cot C$

17. If $A+B+C=\frac{\pi}{2}$, then prove that $\tan A\tan B+\tan B\tan C+\tan C\tan A=1$

18. If A+B+C = 180°, then prove that $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

19. If A+B+C = 180°, then prove that $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$

20. Find the expansion of
(i) $(A+B-C)$ (ii) $\cos(A-B-C)$.

21. If $\sin(A+B)=\frac{24}{25}$ and $\cos(A-B)=\frac{4}{5}$ where $0<A<B<\frac{\pi}{4}$, then find tan 2A.

MULTIPLE SUB MULTIPLE ANGLES

1.Prove that $\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}=\tan\frac{\theta}{2}$

2. Prove that $\frac{\sin 4\theta}{\sin\theta}=8\cos^3\theta-4\cos\theta$

3. Prove that $\cos^6 A+\sin^6 A=1-\frac{3}{4}\sin^2 2A$

4. Prove that $\frac{\sin 3\theta}{1+2\cos 2\theta}=\sin\theta$ and hence find the value of sin 15°

5. Find the value of $\sin^2 42°-\sin^2 12°$.

6. If $\tan\frac{A}{2}=\frac{5}{6}$ and $\tan\frac{B}{2}=\frac{20}{37}$. Then show that $\tan\frac{C}{2}=\frac{2}{5}$

7. Prove that $\frac{\sin 2\theta}{1+\cos 2\theta}=\tan\theta$

8. Prove that $\frac{1-\cos 2\theta}{\sin 2\theta}=\tan\theta$. Simplify $\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta}$.

9. Prove that $\frac{\cos 3A+\sin 3A}{\cos A-\sin A}=1+\sin 2A$

10. If $\frac{\sin\alpha}{a}=\frac{\cos\alpha}{b}$ then prove that $a\sin 2\alpha+b\cos 2\alpha=b$

11. Prove that $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}=4$

12. Prove that $\frac{\sin 2A}{1-\cos 2A}\frac{1-\cos A}{\cos A}=\tan\frac{A}{2}$

13. Prove that $\frac{\cos^3\theta-\cos 3\theta}{\cos\theta}+\frac{\sin^3\theta+\sin 3\theta}{\sin\theta}=3$

14. Prove that $\sin A\sin(60°+A)\sin(60°-A)=\frac{1}{4}\sin 3A$

15. Prove that $\cos A\cos(60°+A)\cos(60°-A)=\frac{1}{4}\cos 3A$

16. Prove that $\tan A\tan(60°+A)\tan(60°-A)=\tan 3A$

17. Prove that
$(1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1+\cos\frac{7\pi}{10})(1+\cos\frac{9\pi}{10})=\frac{1}{16}$

18. Prove that
$\cos^2\frac{\pi}{10}+\cos^2\frac{2\pi}{5}+\cos^2\frac{3\pi}{5}+\cos^2\frac{9\pi}{10}=2$

19. Prove that $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}=\frac{1}{8}$

20. Prove that $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}=\frac{1}{32}$

6. TRANSFORMATIONS

1. Prove that $\sin 34°+\cos 64°-\cos 4°=0$

2. Prove that $\cos 55°+\cos 65°+\cos 175°=0$

3. Prove that $\cos 35°+\cos 85°+\cos 155°=0$

4. Prove that $\frac{\sin 70°-\cos 40°}{\cos 50°-\sin 20°}=\frac{1}{\sqrt{3}}$

5. Prove that $4(\sin 24°+\cos 6°)=\sqrt{15}+\sqrt{13}$

6. Prove that $\cos^2 76°+\cos^2 16°-\cos 76°\cos 16°=\frac{3}{4}$

7. Prove that $\sin 10°+\sin 20°+\sin 40°+\sin 50°=\sin 70°+\sin 80°$

8. Prove that $\sin 50°-\sin 70°+\sin 10°=0$

9. Prove that $\cos 48°\cos 12°=\frac{3+\sqrt{5}}{8}$

10. Prove that $\sin 78°+\cos 132°=\frac{\sqrt{5}-1}{4}$

11. Prove that $\cos^2\theta+\cos^2(\frac{2\pi}{3}+\theta)+\cos^2(\frac{2\pi}{3}-\theta)=\frac{3}{2}$

12. Prove that
$\sin^2(\alpha-45°)+\sin^2(\alpha+15°)-\sin^2(\alpha-15°)=\frac{1}{2}$

13. Prove that $\frac{\sin(n+1)\alpha-\sin(n-1)\alpha}{\cos(n+1)\alpha+2\cos n\alpha+\cos(n-1)\alpha}=\tan\frac{\alpha}{2}$

14. If $x+y=\frac{2\pi}{3}$ and $\sin x+\sin y=\frac{3}{2}$ then find x and y.

15. If $\cos x+\cos y=\frac{4}{5}$, $\cos x-\cos y=\frac{2}{7}$. Then show that $14\tan\frac{x-y}{2}+5\cot\frac{x+y}{2}=0$

16. If $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{a+b}{a-b}$ then prove that $a\tan\beta=b\tan\alpha$.

17. If $m\sin B=n\sin(2A+B)$ then show that $(m+n)\tan A=(m-n)\tan(A+B)$

18. If $\tan(A+B)=\lambda\tan(A-B)$ then show that $(\lambda+1)\sin 2B=(\lambda-1)\sin 2A$.

19. If A+B+C = 180° then prove that $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

20. If A+B+C = 180° then prove that $\sin 2A-\sin 2B+\sin 2C=4\cos A\sin B\cos C$

21. If A+B+C = 180° then prove that $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$

22. If A+B+C = 90° then prove that $\cos 2A+\cos 2B+\cos 2C=1+4\sin A\sin B\sin C$

23. In Δle ABC prove that
$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

24. If A+B+C = 0° then prove that

sin 2A +sin 2B+sin 2C = -4sin A sin B sin C

25. If A+B+C = 270° then prove that
cos 2A +cos 2B+cos 2C =1-4sin A sin B sin C

26. If A+B+C=2S. Then prove that
$\cos(S-A)+\cos(S-B)+\cos(S-C)$
$=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

7. TRIGONOMETRIC EQUATION

1. Solve tan θ+3cot θ = 5sec θ
2. Solve $2\cos^2\theta - \sqrt{3}\sin\theta+1= 0$
3. Solve $4\cos^2\theta + \sqrt{3} = 2(\sqrt{3}+1)\cos\theta$
4. Solve $7\sin^2\theta +3\cos^2\theta = 4$
5. Solve $\cot^2\theta -(1+\sqrt{3})\cot\theta+\sqrt{3} = 0$
6. Solve $1+\sin^2\theta =3\sin\theta\cos\theta$
7. Solve sin 5θ+sin θ = sin 3θ
8. Solve cos 8θ+cos 2θ = cos 5θ
9. Solve $\cos\theta.\cos 2\theta.\cos 3\theta = \frac{1}{4}$
10. Solve $\sqrt{3}\cos\theta+\sin\theta = \sqrt{2}$
11. Solve $\sqrt{3}\sin\theta-\cos\theta = \sqrt{2}$
12. Solve $\tan\theta + \sec\theta = \sqrt{3}$
13. Solve $1+|\cos x| +|\cos^2 x| +......\infty$
14. Solve 4sin x. sin 2x. sin 4x = sin 3x
15. Solve 3cosec θ = 4sin θ
16. If $a\cos 2\theta+b\sin 2\theta =c$. Then prove that
$\tan\theta_1+\tan\theta_2 =\frac{2b}{c+a}$ ‘$\tan\theta_1.\tan\theta_2 = \frac{c-a}{c+a}$
17. Solve $\sin^{-2}\theta - \cos\theta = \frac{1}{4}$

8. HYPERBOLIC FUNCTIONS

1. Prove that
$\sinh(\alpha+\beta) = \sinh\alpha\cosh\beta +\cosh\alpha\sinh\beta$
2. Prove that
$\cosh(\alpha+\beta) = \cosh\alpha\cosh\beta +\sinh\alpha\sinh\beta$
3. Prove that $\tanh(\alpha+\beta) = \frac{\tanh\alpha+\tanh\beta}{1+\tanh\alpha\tanh\beta}$
4. Prove that
$\sinh(\alpha-\beta) = \sinh\alpha\cosh\beta -\cosh\alpha\sinh\beta$
5. Prove that
$\cosh(\alpha-\beta) = \cosh\alpha\cosh\beta - \sinh\alpha\sinh\beta$
6. Prove that sinh 2x = 2sinh x cosh x
7. Prove that $\cosh 2x =\cosh^2 x + \sinh^2 x$
8. Prove that $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$
9. Prove that $\sinh 3x = 3\sinh x+4\sinh^3 x$
10. Prove that $\cosh 3x = 4\cosh^3 x -3\cosh x$
11. Prove that $\tanh 3x = \frac{3\tanh x+\tanh^3 x}{1+3\tanh^2 x}$
12. Prove that $\frac{\tanh x}{\operatorname{sech} x-1}+\frac{\tanh x}{\operatorname{sech} x+1} = -2\operatorname{cosech} x$
13. Prove that
$[\cosh x+\sinh x]^n = \cosh nx + \sinh nx$
14. If $\cosh x=\frac{5}{2}$, The prove that
$\cosh 2x = \frac{23}{2}$; $\sinh 2x = \frac{5\sqrt{21}}{2}$
15. If $u = \log_e(\tan(\frac{\pi}{4}+\frac{\theta}{2}))$ then prove that cosh u = sec θ
16. If $\sinh x=\frac{3}{4}$ then find cosh 2x and sinh 2x.

9. LIMITS AND CONTINUITY

Compute the following limits

1. $\lim_{x\to 3}\frac{x^2-8x+15}{x^2-9}$
2. $\lim_{x\to 0^+}\frac{|x|}{x}$; $\lim_{x\to 0^-}\frac{|x|}{x}$
3. $\lim_{x\to 2^+}([x]+x)$; $\lim_{x\to 2^-}([x]+x)$
4. $\lim_{x\to 0}\frac{\tan x}{x}$
5. $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$
6. $\lim_{x\to 0}[\frac{e^x-1}{\sqrt{1+x}-1}]$
7. $\lim_{x\to 0}[\frac{a^x-1}{b^x-1}]$ (a>0,b>0, b≠1)
8. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$
9. $\lim_{x\to 0}[\frac{e^{3x}-1}{x}]$
10. $\lim_{x\to 0}[\frac{e^x-\sin x-1}{x}]$
11. $\lim_{x\to 0}[\frac{e^{3+x}-e^3}{x}]$
12. $\lim_{x\to 0}[\frac{e^{\sin x}-1}{x}]$
13. $\lim_{x\to 0}[\frac{3^x-1}{\sqrt{1+x}-1}]$
14. $\lim_{x\to a}\frac{\sin(x-a)tan^2(x-a)}{(x^2-a^2)^2}$
15. $\lim_{x\to 0}\frac{\cos ax-\cos bx}{x^2}$
16. $\lim_{x\to a}\frac{x\sin a-a\sin x}{x-a}$
17. $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$
18. $\lim_{x\to 0}\frac{1-\cos 2mx}{sin^2 nx}$
19. $\lim_{x\to\infty}\frac{x^2+5x+2}{2x^2-5x+1}$
20. $\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$
21. $\lim_{x\to\infty}(\sqrt{x+1}-\sqrt{x})$
22. $\lim_{x\to\infty}(\sqrt{x^2+1}-x)$
23. Evaluate $\lim_{x\to 0}[\frac{\sin(a+bx)-\sin(a-bx)}{x}]$
24. Evaluate $\log_{x\to 2}\frac{2x^2-7x-4}{(2x-1)(\sqrt{x}-2)}$
25. $\lim_{x\to\infty}\frac{2x^2-x+3}{x^2-2x+5}$
26. $\lim_{x\to\infty}\frac{11x^3-3x+4}{13x^3-5x^2-7}$
27. $\lim_{x\to\infty}\frac{3x^2-4x+5}{2x^3+3x-7}$

CONTINUITY

1. If f defined by $f(x) = \begin{cases}\frac{\sin 2x}{x}, & \text{if } x\neq 0\\ 1, & \text{if } x=0\end{cases}$
continued at 0
2. If f given by $f(x) = \begin{cases}K^2x-K & \text{if } x\geq 0\\ 2, & \text{if } x<1\end{cases}$

is a continuous function on R, then find the value of K.

3. Show that $f(x) = \begin{cases} \frac{\cos ax - \cos bx}{x^2}, & x \neq 0 \\ \frac{1}{2}(b^2 - a^2), & x = 0 \end{cases}$ where a and b are real constants is continuous at 'a'.

4. Find real constants a,b. so that the function f given by $f(x) = \begin{cases} \sin x, & x \leq 0 \\ x^2 + a, \text{if } 0 < x < 1 \\ bx + 3, \text{if } 1 \leq x \leq 3 \\ -3, & x > 0 \end{cases}$ is continuous on R.

10.DIFFERENTIATION

1. Find the derivative of $\sin(\log x)$ (x>0)
2. Find the derivative of $(x^3 + 6x^2 + 12x - 13)^{100}$
3. Find the derivative of $\sin^{-1}\sqrt{x}$
4. Find the derivative of $\log(\cosh 2x)$
5. Find the derivative of $(\cot^{-1} x^3)^2$
6. Find the derivative of $\log(\sec x + \tan x)$
7. Find the derivative of $e^{\sin^{-1} x}$
8. Find the derivative of $\sin^{-1}(3x - 4x^3)$
9. Find the derivative of $\cos^{-1}(4x^3 - 3x)$
10. Find the derivative of $\tan^{-1}(\frac{2x}{1-x^2})$
11. Find the derivative of $\tan^{-1}(\frac{a-x}{1+ax})$
12. If y = $\tan^{-1}(\frac{2x}{1-x^2})$ +$\tan^{-1}(\frac{3x-x^3}{1-3x^2})$ - $\tan^{-1}(\frac{4x-4x^3}{1-6x^2+x^4})$ then show that $\frac{dy}{dx} = \frac{1}{1+x^2}$
13. If y=$\tan^{-1}[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}]$ for 0< $|x|$ <1 find $\frac{dy}{dx}$.
14. Find the derivative of $\sin^{-1}(\frac{b+a\sin x}{a+b\sin x})$ (a$> 0, b > 0$)
15. Find the derivative of $\cos^{-1}(\frac{b+a\cos x}{a+b\cos x})$ (a$> 0, b > 0$)
16. Find the derivative of $\tan(2x)$ from first principle.
17. Find the derivative of $x \sin x$ from first principle.
18. Find the derivative of x^2+2 from definition method.
19. Find $\frac{d}{dx}[\frac{\cos x}{\cos x+\sin x}]$
20. Find the derivative of a^x using first principles.
21. Find the derivative of $\cos 2x$ using first principles

11. APPLICATION OF DIFFERENTIATION

1. If the increase in the side of a square is 2%. Then the approximate percentage of increase in its area.
2. Find dy and Δy of y=f(x) = x^2+x at x=10 when Δx=0.1
3. Find Δy and dy for the functions y = e^x+x when x=5 and Δx=0.02
4. Find the equations of the tangent and the normal to the curve y = $5x^4$ at the point(1,5).
5. Find the slope of the tangent to the curve y=x^3-x+1 at the point whose x coordinate is 2.
6. Find the slope of the tangent to the curve y=$3x^4$-4x at x=4.
7. Find the lengths of sub-tangent and sub-normal at a point on the curve y = b$\sin\frac{x}{a}$.
8. Find the lengths of normal and sub-normal of a point on the curve y = $\frac{a}{2}(e^{\frac{x}{a}} + e^{\frac{-x}{a}})$.
9. Show that the curves y^2 =4(x+1) and y^2 =36(9-x) intersect orthogonally.
10. Show that the curves $6x^2$-5x+2y=0 and $4x^2$+$8y^2$=3 touch each other at($\frac{1}{2}$, $\frac{1}{2}$).
11. If the tangent at any point on the curve $x^{2/3}$ +$2^{2/3}$ =$a^{2/3}$ intersects the coordinate axes in A and B, then show that length AB is constant.
12. Show that the curves x^2+y^2 =2 and $3x^2$+y^2 =4x have a common tangent at the point (1,1).
13. Find the equation of tangent and normal to the curve y = x^3+$4x^2$ at (-1,3)
14. Show that the length of the sub normal at any point on the curve y^2 =4ax is a constant.
15. Show that the length of the sub tangent at any point on the curve y^2 =4ax is a constant.
16. A particle is moving in a straight line, so that after t seconds its distance is from a fined point on the line is given by s=f(t)=8t+t^3 find
    i) The velocity at time t =2sec
    ii) The initial velocity
    iii) Acceleration at t=2sec.
17. A particle moving along a straight line has the relation S=t^3+2t+3 connecting the distance"s' described by the particle in time t. Find the velocity and acceleration of the particle at t=4 seconds.
18. The distance – time formula for the motion of a particle along a straight line is S = t^3-9t^2+24t-18. Find when and where the velocity is zero.
19. Find the equation of tangent and normal to the curve of y=x^4-6x^3+13x^2-10x+5 at (0,5).

12. LOCUS

SHORT ANSWER QUESTIONS

1. Find the equation of locus of a point P, if the distance of P from A(3,0) is twice the distance of P from B(-3,0)
2. Find the equation of a point which is at a distance from A(4,-3).
3. Find the equation of locus of a point which equidistant from the points A(-3,2) and B(0,4).

4. Find the equation of locus of a point P, such that the distance of P from the origin is twice the distance of P from A(1,2).
5. Find the equation of locus of a point P, the square of whose distance from the origin is 4 times its y-coordinate.
6. Find the equation of locus of a point, such that $PA^2+PB^2=2c^2$ where A=(a,0), B(-a,0) and $0< |a| < |c|$

Essay type questions

7. Find the equation of locus P, if the line segment joining(2,3) and (-1,5) subtends a right angle at P.
8. Find the equation of the locus of P,if A=(4,0); B=(-4,0) and $|PA - PB|$=4
9. Find the equation of the locus of P, if A =(2,3), B=(2,-3) and $|PA + PB|$=8
10. A(5,3) and B(3,-2) are two fixed points. Find the equation of the locus of P. So that the area of triangle is 9.
11. If the distance from P to the points (2,3) and (2,-3) are in the ratio 2:3, then find the equation of the locus of P.
12. A(1,2), B(2,-3) and C(-2,3) are three points, a point P moves such that $PA^2+PB^2=2PC^2$. Show that the equation of the locus of P is 7x-7y+4=0.

13. TRANSFORMATION OF AXES

Short answer questions

1. When the origin is shifted to (-2,3) by transformation of axes let us find the co-ordinate of (1,2) w.r.t new axes.
2. When the origin is shifted to (2,3) by translation of axes, the co-ordinates of a point P are changed as (4,-3). Find the co-ordinates of P in the original system.
3. Find the point to which the origin is to be shifted. So that the point(3,0) may change to (2,-3).
4. Find the point to which the origin is to be shifted so as to remove the first degree terms from the equation $4x^2+9y^2$- 8x+36y+4=0
5. When the axes are rotated through an angle 30°. Find the new coordinates of (0,5),(-2,4) and (0,0).
6. When the axes are rotated through an angle 60°. Find the original co-ordinates of (3,4),(-7,2) and (2,0).
7. Find the angle through which the axes are to be rotated so as to remove the xy term in the equation x^2 + 4xy - y^2- 2x +2y – 6 = 0.

Essay type questions

8. When the origin is shifted to the point(2,3), the transformed equation of a curve is x^2 + 3xy - $2y^2$+ 17x -7y – 11 = 0. Find the original equation of the curve.
9. When the origin is shifted to (-1,2) by the translation of axes, find the transformed equation to $x^2 + y^2$ + 2x - 4y +1 = 0.
10. When the axes are rotated through an angle 45°. Find the original equation of the curve $17x^2$ -16xy + $17y^2$ = 225
11. When the axes are rotated through an angle $\frac{\pi}{4}$. Find the transformed equation $3x^2$ +10xy + $3y^2$ = 9

14. STRAIGHT LINES

Short answer questions.

1.Find the equation of straight line joining through the point(2,3) and making non-zero intercept on the co-ordinate axes whose sum is zero.
2. Find the value of x, if the slope of the line passing through (2,5) and (x,3) is 2.
3. Find the value of y if the line joining the points (3,y),(2,7) is parallel to the line joining the points (-1,4) (0,6).
4. Find the equation of straight line which makes an angle of $\frac{\pi}{4}$ with x-axis and passing through the points (0,0)
6. Find the sum of the squares of the intercepts of the line 4x-3y=12 on the coordinate axes.
7. Find the equation of straight line which makes an angle of $\alpha = 150°$with x-axis and passing through (1,2).
8.Transform the straight line 4x – 3y + 12 = 0 into
a) slope – intercept form b) Intercept form
c) normal form
9. Find the ratios in which i) x-axis and ii) y-axis divide the line segment AB joining A(2,-3) & B(3,-6)
10. Find the value of K if the lines 2x-3y+K=0, 3x-4y+13=0 and 8x-11y+33=0 are concurrent.
11. Find the angle between straight line y=4-2x; y=3x+7
12. Find the length of perpendicular drawn from the point (-2,-3) to the straight line 5x-2y+4=0.
13. Find the distance between parallel lines 3x-4y = 12 and 3x-4y = 7
14. Find the value of p if the straight lines 3x+7y-1=0 and 7x-py+3=0 are mutually perpendicular.
15. Find the foot of the perpendicular drawn from (4, 1) upon the straight line $3x - 4y + 12 = 0$.
16. Find the image the point(1,2) is the straight line 3x+4y-1=0.
17. Find the foot of the perpendicular drawn from the point(3,0) on to the line 4x+12y-41=0
18. If the straight lines ax +by +c=0, bx +cy +a=0, cx + ay +b=0 are concurrent, then prove that $a^3+b^3+c^3$=3abc

19. Find the equation of the line which passes through(0,0) and the point of intersection of the lines x+y+1=0 and 2x-y+5=0.
20. Show that the distance of the point(6,-2) from the line 4x+3y=12 is half the distance of the point(3,4) from the line (4x-3y=12).
21.Transform the equation of the line x+y+2=0 into i)slope-intercept form ii) intercept form iii) normal form

15. PAIR OF STRAIGHT LINES

Essay type questions
1.Find the acute angle between the pair of lines represented x^2-7xy+12y^2=0
2. Find the centroid and area of a triangle formed by the lines 2y^2- xy - 6x^2 = 0; x+y+4 = 0
3. Find the equation of pair of lines intersecting at (2,-1) and perpendicular to the pair of line 6x^2-13xy-5y^2=0.
4. Find the equation of pair of lines intersecting at (2,-1) and perpendicular to the pair of line 6x^2-13xy-5y^2=0.
5. Find the combined equation of pair of bisectors of the angle between the pair of straight lines represented by 6x^2-11xy+3y^2=0
6. Show that the equation 2x^2-13xy-7y^2+x+23y-6=0 represents a pair of straight lines and also find the angle between and the coordinates of the point of intersection of lines.
7. Show that the equation 8x^2 - 24xy + 18y^2- 6x + 9y - 5 = 0 represents a pair of parallel lines and find the distance between them.
8. Show that the lines joining the origin to the points of inter section of curve x^2 - xy + y^2+ 3x + 3y - 2 = 0 and the straight line x-y-$\sqrt{2}$ =0 are normally perpendicular.
9. Find the values of K. If the lines joining the origin to the points of intersection of the curve 2x^2 -2 xy + 3y^2+ 2x - y - 1 = 0 and the lines x+2y+K are mutually perpendicular.
10. Find the angle between the lines joining the origin to the points of intersection of the curve 7x^2 -4 xy + 8y^2+ 2x - 4y - 8 = 0 with the straight line 3x-y=2.
11. Find the condition for the lines joining the origin to the points of intersection of the circle x^2+y^2=a^2 and the line lx+my=1 to coincide.

16. THREE DIMENSIONAL COORDINATES

Short Answer questions.
1.Find x if the distance between (5,-1,7) and (x,5,1) is 9 units.
2. Show that the points (2,3,5) (-1,5,-1) and (4,-3,2) form a straight angled isosceles triangle.
3. Show that the points (1,2,3) (2,3,1) and (3,1,2) form an equilateral triangle.
4. Show that the points (1,2,3) (7,0,1) and (-2,3,4) are collinear.
5. Find the coordinated of vertex C of ΔABC, if its centroid is origin and the vertices A,B are (1,1,1) and (-2,4,1) respectively.
6. If (3,2,-1)(4,1,1) and (6,2,5) are three vertices and (4,2,2) is a centroid of tetrahedron . find the fourth vertex.
7. Find the distance between the midpoint of line segment AB and the point(3,-1,2) where A=(6,3,-4) and B=(-2,-1,2)
8. The three consecutive vertices of a parallelogram are given as (2,4,-1)(3,6,-1)(4,5,1). Find the fourth vertex.

17. DIRECTION COSINES AND DIRECTION RATIOS

Short Answer questions
1.If the line makes angles $\alpha, \beta, \gamma$with the +ve directions of x,y,z axes. What is the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$
2. What are the direction cosine of the line joining the points(-4,1,7) and (2,-3,2)
3. If (6,10,10)(1,0,-5)(6,-10,1) are thee vertices of a triangle. Find the direction ratios of its sides. Also show that it is a right angle triangle.
4. Find the ratio in which the XZ-plane divides the line joining A(-2,3,4) and B(1,2,3)
5. Show that the lines PQ and RS are parallel, if P=(2,3,4); Q=(4,7,8) R=(-1,-2,1) S=91,2,5)
Essay questions
6. Find the direction cosines two lines which are connected by its relation l+m+n=0 and mn-2nl-2lm=0
7. Find the direction cosines of two lines which are connected by the relation l -5m+3n=0 and 7l^2+5m^2-3n^2=0
8. Find the angle between the lines where direction cosines are given by the equations 3l+m+5n=0 and 6mn-2nl+5lm=0
9. Find the angle between the lines where direction cosines satisfy equations l+m+n=0 ; l^2+m^2-n^2=0

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Question Bank Answers

1.FUNCTIONS

1. If A={0 , $\frac{\lambda}{6}$, $\frac{\lambda}{4}$, $\frac{\lambda}{3}$, $\frac{\lambda}{2}$} and f:A→B f(x)=cos x then find B.

Sol: f(x)=cos x

$f(\frac{\pi}{6})=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$f(\frac{\pi}{4})=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$f(\frac{\pi}{3})=\cos\frac{\pi}{3}=\frac{1}{2}$

$f(\frac{\pi}{2})=\cos\frac{\pi}{2}=0$

∴B=f(A)={1, $\frac{\sqrt{3}}{2}$, $\frac{1}{\sqrt{2}}$, $\frac{1}{2}$,0}

2. If A={-2,-1,0,1,2} and f:A→B f(x)=x^2 +x+1 then find B.

Sol: Given f:A→B defined by f(x)= x^2 +x+1

f(-2) : $(-2)^2$ +(-2)+1=4-2+1=3

f(-1) : $(-1)^2$ +(-1)+1=1-1+1=1

f(0) : $(0)^2$ +(0)+1=0+0+1=1

f(1) : $(1)^2$ +(1)+1=1+1+1=3

f(2) : $(2)^2$ +(2)+1=4+2+1=7

∴B=f(A)={3,1,1,3,7}={3,1,7}

3. If f={(1,2), (2,-3),(3,-1)} then find (i) 2f (ii)f^2 (iii)f+2 (iv)$\sqrt{f}$

Sol: f={(1,2), (2,-3),(3,-1)}

⇒f(1)=2, f(2)=-3, f(3)=-1

(i) 2f

(2f)(x)=2f(x)

2f(1)=2f(1)=2(2)=4

2f(2)=2f(2)=2(-3)=-6

2f(3)=2f(3)=2(-1)=-2

∴2f={(1,4),(2,-6),(3,-2)}

(ii) f^2

$f^2(x)=(f(x))^2$

$f^2(1)=(f(1))^2=(2)^2=4$

$f^2(2)=(f(2))^2=(-3)^2=9$

$f^2(x)=(f(3))^2=(-1)^2=1$

∴ f^2={(1,4), (2,9), (3,1)}

(iii) 2+f

(2+f)(x) = f(x)+2

(2+f)(1) = f(1)+2 = 2+2=4

(2+f)(2) = f(2)+2 = -3+2=-1

(2+f)(3) = f(3)+2 = -1+2=1

∴2+f= {(1,4), (2,-1),(3,1)

(iv)$\sqrt{f}$

$\sqrt{f(x)}=\sqrt{f(x)}$

$\sqrt{f(1)}=\sqrt{2}$

$\sqrt{f(2)}=\sqrt{-3}$ not defined

$\sqrt{f(3)}=\sqrt{-1}$ not defined

∴ $\sqrt{f}$ ={(1,$\sqrt{2}$)

4. If f=(4,5)(5,6)(6,-4) and g=(4,-4)(6,5)(8,5) then find (i) f+g (ii)f-g (iii)2f+4g (iv) f+4 (v) fg (vi)$\frac{f}{g}$ (vii)$\sqrt{f}$ (viii)|f| (ix) f^2 (x) f^3

Sol: Given f=(4,5)(5,6)(6,-4) and g=(4,-4)(6,5)(8,5)

(i).f+g={(4,5-4),(6,-4+5)}={(4,1),(6,1)}

(ii) f-g={(4,5-(-4)(6,-4-5)}={(4,9),(6,-9)}

(iii) 2f+4g

2f=(4,2x5),(5,2x6),(6,(2x-4))=(4,10)(5,12)(6,-8)

4g=(4,4x(-4)(6,4x5)(8,4x5)=(4,-16)(6,20)(8,20)

∴2f+4g={4,(10+(-16),6,-8+20)={(4,-6)(6,12)}

(iv) f=4

f+4=f+4=(4,5+4)(5,6+4)(6,-4+4)=(4,9)(5,10)(6,0)

(v) fg

fg=f.g={(4,5(-4)),(6,-4(5))}={(4,-20),(6,-20)}

(vi) $\frac{f}{g}$

$\frac{f}{g}$ = {(4,$\frac{-5}{4}$),(6,$\frac{-4}{5}$)}

(vii) $\sqrt{f}$

$\sqrt{f}$={(4,$\sqrt{5}$),(5,$\sqrt{6}$)}

(viii) |f|={(4,5),(5,6),(6,4)}

(ix) f^2={(4,5^2),(5, 6^2)(6, $(-4)^2$)}={(4,25),(5,36),(6,16)}

(x) f^3={(4,5^3),(5, 6^3)(6, $(-4)^3$)}={(4,125),(5,216),(6,-64)}

5. If f(x)=2x-1 and g(x)=x^2 then find (i) (3f-2g)(x) (ii) (fg)(x) (iii)$\frac{\sqrt{f}}{g}$(x) (iv) (f+g+2)(x)

Sol: Given f(x)=2x-1 and g(x)=x^2

(i) (3f-2g)(x)

(3f-2g)(x) =3f(x)-2g(x)=3(2x-1)-2(x^2)=6x-3-2x^2

(ii) (fg)(x)

(fg)(x) =f(x).g(x)=(2x-1)(x^2)=2x^3 -x^2

(iii)$\frac{\sqrt{f}}{g}$(x)

$\frac{\sqrt{f}}{g}(x) = \frac{\sqrt{f(x)}}{g(x)} = \frac{\sqrt{2x-1}}{x^2}$

(iv) (f+g+2)(x)

(f+g+2)(x)=f(x)+g(x)+2=2x-1+x^2+2=x^2+2x+1

6. If f(x) = $\frac{1-x^2}{1+x^2}$ then show that f(tan θ)=cos 2θ

Sol: Given f(x) = $\frac{1-x^2}{1+x^2}$

$f(\tan\theta)=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}=\cos 2\theta$

∴ f(tan θ)=cos 2θ

7. If f(x)=log $\left|\frac{1+x}{1-x}\right|$ then show that f($\frac{2x}{1+x^2}$) =2f(x)

Sol: Given f(x)=log $\left|\frac{1+x}{1-x}\right|$

$f(\frac{2x}{1+x^2})=\log\left|\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right|=\log\left|\frac{\frac{1+x^2+2x}{1+x^2}}{\frac{1+x^2-2x}{1+x^2}}\right|=\log\left|\frac{1+x^2+2x}{1+x^2-2x}\right|$

$=\log\left|\frac{(1+x)^2}{(1-x)^2}\right|=\log(\frac{1+x}{1-x})^2=2\log\left|\frac{1+x}{1-x}\right|=2f(x)$

∴ f($\frac{2x}{1+x^2}$) =2f(x)

8. If f(x) =4x-1; g(x)=x^2+2 then find

(i) (gof)(x) (ii) go(fof)(0) (iii) (gof)$\frac{a+1}{4}$ (iv) (fof)(x)

Sol: Given f(x) =4x-1; g(x)=x^2+2

(i) (gof)(x) =g[f(x)]=g(4x-1)

= $(4x-1)^2$+2=16x^2-8x+3

(ii) go(fof)(0) =

f[f(0)]=f(4(0)-1)=f(-1)=4(-1)-1=-4-1=-5

go(fof)(0) =g[(fof)(0)=g(-5) =$(-5)^2$+2 =25+2=27

(ii) (gof)$\frac{a+1}{4}$ =(gof)$\frac{a+1}{4}$=g[f($\frac{a+1}{4}$)]=g[4($\frac{a+1}{4}$)]g[a]= a^2+2

(iv) (fof)(x)=(fof)(x) =f[f(x)]=f(4x-1)

=4(4x-1)-1=16x-4-1=16x-5

9. f(x) =2, g(x) =x^2, h(x) =2x then find (fogoh)(x)

Sol: Given f(x) =2, g(x) =x^2, h(x) =2x

(fogoh)(x)=f[g{h(x)}]=f[g(2x)]=f[$(2x)^2$]=f[$4x^2$]=2

10. If f(x) = ax+b then find f^{-1}(x)

Sol: Given f(x) = ax+b ; Let f^{-1}(x)=t

Then x=f(t) ⇒x=at+b ⇒at=x-b

t=$\frac{x-b}{a}$ ∴ f^{-1}(x)= $\frac{x-b}{a}$

11.If f(x) =5^xthen find f^{-1}(x)

Sol: Given f(x) =5^x

Let f^{-1}(x)=t; Then x=f(t) ⇒x=5^t

⇒t=$\log_5 x$ ∴ f^{-1}(x)= $\log_5 x$

12. If f(x)=2x-3, g(x)=x^3+5 then find $(fog)^{-1}$(x).

Sol: Given f(x)=2x-3

(fog)(x)=f[g(x)]=f(x^3+5)=2(x^3+5)-3=2x^3+10-3=2x^3+7

$(fog)^{-1}$(x) =t

x=(fog)(t) = 2t^3+7

⇒x-7=2t^3

t^3=$\frac{x-7}{2}$

t=$(\frac{x-7}{2})^{\frac{1}{3}}$ ∴$(fog)^{-1}$(x) = $(\frac{x-7}{2})^{\frac{1}{3}}$

13. If f(x) = $\frac{x+1}{x-1}$ then find (fof)(x)

Sol: Given f(x) = $\frac{x+1}{x-1}$

(fof)(x) = f[f(x)]=f[$\frac{x+1}{x-1}$]= $\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$ =$\frac{x+1+x-1}{x+1-x+1}$ =$\frac{2x}{2}$=x

∴(fof)(x)=x

14. If f(x)=$\frac{1}{x}$; g(x)=$\sqrt{x}$ then find (gof)(x) and g$\sqrt{f}$ (x)

Sol: Given f(x)=$\frac{1}{x}$; g(x)=$\sqrt{x}$

(gof)(x) =g[f(x)]=g[$\frac{1}{x}$]=$\sqrt{\frac{1}{x}}$ = $\frac{1}{\sqrt{x}}$

g$\sqrt{f}$ (x) =g(x). $\sqrt{f}$ (x)= $\sqrt{x}$.($\frac{1}{\sqrt{x}}$)=1

15. If A={1,2,3,4} and f:A→R and f(x) = $\frac{x^2-x+1}{x+1}$ then find the range of f

Sol: Given A={1,2,3,4}; f(x) = $\frac{x^2-x+1}{x+1}$

f(1) =$\frac{1^2-1+1}{1+1}$=$\frac{1}{2}$ f(2) =$\frac{2^2-2+1}{2+1}$=1

f(3) =$\frac{3^2-3+1}{3+1}$=$\frac{7}{4}$ f(4) =$\frac{4^2-4+1}{4+1}$=$\frac{13}{5}$

∴ Range of f={f(1),f(2),f(3),f(4)}={ $\frac{1}{2}$,1, $\frac{7}{4}$, $\frac{13}{5}$}

Solved problems in the text book

1.If f(x) =4x-1 and g(x)=x^2+2, then find $(fog)^{-1}$(x).

Sol: Given f(x)=4x-1 ;g(x) = x^2+2

(fog)(x) =f(g(x))=f(x^2+2) =4(x^2+2)-1

=4x^2+8-1=4x^2+7

Put (fog)(x) =y ⇒4x^2+7=y

4x^2=y-7

x^2= $\frac{y-7}{4}$ ⇒x=$\sqrt{\frac{y-7}{4}}$

⇒$(fog)^{-1}$(y) = $\sqrt{\frac{y-7}{4}}$: ∴ $(fog)^{-1}$(x) = $\sqrt{\frac{y-7}{4}}$

2. If f(x) =$\frac{x+1}{x-1}$ and g(x) =x^2+2 then find(fog)(x).

Sol: Given f(x) =$\frac{x+1}{x-1}$ and g(x) =x^2+2

(fog)(x)=f(g(x))=f(x^2+2) = $\frac{x^2+2+1}{x^2+2-1}$ = $\frac{x^2+3}{x^2+1}$

@@

Exercise problems

1. If f(x) =e^x and g(x) =$\log_e x$, then show that gof=fog and f^{-1} and g^{-1}.

Sol: Given f(x) =e^x and g(x) =$\log_e x$

(fog)(x)=f(g(x))=f($\log_e x$)= $e^{\log_e x}$=x

(gof)(x)=g(f(x))=g(e^x)= $\log_e(e^x)$=x$\log_e e$=x(1)=x

∴(fog)=(gof)=x=1(x)

Hence f^{-1}(x)=g(x)= $\log_e x$

g^{-1}(x)=f(x)= e^x

2. If f(x)=2x-1 and g(x)=$\frac{x+1}{2}$, if x∈R, then find (gof)(x).

Sol: Given that f(x)=2x-1 and g(x)=$\frac{x+1}{2}$

∴(gof)(x) = g(f(x)) = g(2x-1) = $\frac{(2x-1)+1}{2}$ = $\frac{2x}{2}$ = x

3.If f:R→R;g:R→R are defined by f(x)=3x-1 and g(x)=x^2+1, then find

(i) (gof)(x) (ii) (gof)(2) (iii) (fof)(x^2+1)

Sol: Given f(x)=3x-1 and g(x)=x^2+1

(i) (gof)(x) =g(f(x))=g(3x-1)=$(3x-1)^2$+1

=9x^2-6x+1+1=9x^2-6x+2

(ii) (gof)(2) =g(f(2))=g(3(2)-1)=g(5)= 5^2+1=25+1=26

(iii) (fof)(x^2+1)=f(f(x^2+1)=f(3(x^2+1)-1=f(3x^2+3-1)

=f(3x^2+2)=[3(3x^2+2)-1]= 9x^2+6-1

=9x^2+5

4. If f:R→R;g:R→R are defined by f(x)=3x-2 and g(x)=x^2+1, then find

(i) (gof^{-1})((2) (ii)(gof)(x-1) (iii)(gof)(2a-3)

Sol: Given f(x)=3x-2 and g(x)=x^2+1

Let f(x)=y⇒y=3x-2⇒3x=y+2⇒x=$\frac{y+2}{3}$ ⇒ f^{-1}(y)= $\frac{y+2}{3}$

f^{-1}(x)= $\frac{x+2}{3}$

(i) (gof^{-1})(2)= g(f^{-1}(2))=g($\frac{2+2}{3}$)

= g($\frac{4}{3}$)= $(\frac{4}{3})^2$+1=$\frac{16+9}{9}$ =$\frac{25}{9}$

(ii) (gof)(x-1) =g(f (x-1))=g(3(x-1)-2)=g(3x-3-2)

=g(3x-5) = $(3x-5)^2$+1

= $9x^2$ -30x+25+1=$9x^2$ -30x+26

(iii) (gof)(2a-3) = g(f(2a-3))=g(3(2a-3)-2)=g(6a-9-2)

=g(6a-11)= $(6a-11)^2$+1

=$36a^2$ -132a+121+1=$36a^2$ -132a+122

5. If f:R→R;g:R→R are defined by f(x)=2x-3 and g(x)=x^3+5, then find

(i)(gof)(1) (ii) (gof^{-1})(2) (iii) (fog)(x)

Sol: Given f(x)=2x-3 and g(x)=x^3+5

(i)(gof)(1)= g(f91))=g(2(1)-3)=g(-1)

= $(-1)^3$+5 =-1+5=4

Let f(x)=y⇒y=2x-3⇒2x=y+3⇒x=$\frac{y+3}{2}$ ⇒ f^{-1}(y)= $\frac{y+3}{2}$

f^{-1}(x)= $\frac{x+3}{2}$

(ii) (gof^{-1})(2) = g(f^{-1}(2))=g($\frac{2+3}{2}$)

= g($\frac{5}{2}$)= $(\frac{5}{2})^3$+5=$\frac{125}{8}$+5 =$\frac{165}{8}$

(iii) (fog)(x) = f(g(x))=f(x^3+5)

=2(x^3+5)-3=$2x^3$+10-3=$2x^3$+7

6. If f=[(1,a)(2,c)(3,b)(4,d)] and g^{-1}={(1,c)(2,a)(3,d)(4,b)} then find (i) (g^{-1}o f^{-1}) (ii) $(gof)^{-1}$ (iii) $(fog)^{-1}$ (iv) (f^{-1}og^{-1})

Sol: Given that

f=[(1,a)(2,c)(3,b)(4,d)] ⇒ f^{-1} = [(a,1)(c,2)(b,3)(d,4)]

g^{-1}={(1,c)(2,a)(3,d)(4,b)} ⇒ g={(c,1)(a,2)(d,3)(b,4)}

(i) (g^{-1}o f^{-1})= g^{-1}(f^{-1}) = g^{-1}[(a,1)(c,2)(b,3)(d,4)]

= [(a,c)(b,d)(c,a)(d,b)]

gof=g(f)=(2,1)(1,2)(4,3)(1,4)

(ii) $(gof)^{-1}$={$(g(f)^{-1}$}= (1,2)(2,1)(3,4)(4,1)

fog=f(g)=(c,a)(d,b)(a,c)(b,d)

(iii) $(fog)^{-1}$={$(f(g)^{-1}$}=(a,c)(b,d)(c,a)(d,b)

(iv) (f^{-1}og^{-1})= f^{-1}(g^{-1})= (1,2)(2,1)(3,4)(4,1)

7. If f(x)=3^x then find f^{-1}(x).

Sol: Given f(x) =3^x

Let f^{-1}(x)=t; Then x=f(t) ⇒x=3^t

⇒t=$\log_3 x$ ∴ f^{-1}(x)= $\log_3 x$

8. If f(x)=3x + 5 then find f^{-1}(x).

Sol: f(x)=y ⇒3x+5=y⇒x=$\frac{y-5}{3}$ ⇒ f^{-1}(y)= $\frac{y-5}{3}$

∴ f^{-1}(x)= $\frac{x-5}{3}$

9.If A={0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$} and f:A→B f(x)=sin x then find B.

Sol: f(x)=sin x

F(0)=Sin(0)=0 f($\frac{\pi}{6}$)=sin $\frac{\pi}{6}$ =$\frac{1}{2}$

f($\frac{\pi}{4}$)=sin $\frac{\pi}{4}$ =$\frac{1}{\sqrt{2}}$ f($\frac{\pi}{3}$)=sin $\frac{\pi}{3}$=$\frac{\sqrt{3}}{2}$

f($\frac{\pi}{2}$)=sin $\frac{\pi}{2}$=1

∴B=f(A)={0, $\frac{1}{2}$, $\frac{1}{\sqrt{2}}$, $\frac{\sqrt{3}}{2}$,1}

10.if f:R-{0}→R is defined by f(x)=x+$\frac{1}{x}$ then prove that $(f(x))^2$ =f(x^2)+f(1)

Sol:Sol: RHS= f(x^2)+f(1)= x^2+$\frac{1}{x^2}$ +1^2+$\frac{1}{1^2}$=x^2+$\frac{1}{x^2}$ +2

=$(x+\frac{1}{x})^2$ =LHS

2. MATHEMATICAL INDUCTION

1. Prove that 1+2+3+.....+n=$\frac{n(n+1)}{2}$ using mathematical induction.

Sol: Let S(n)= 1+2+3+.....+n=$\frac{n(n+1)}{2}$

If n=1; LHS=1

RHS=$\frac{1(1+1)}{2}$=1

∴LHS=RHS, Hence S(1) is true.

Assume that S(k) is true for some k∈ N

i.e. 1+2+3+......+k=$\frac{k(k+1)}{2}$

on adding (k+1) to both sides of the above equation, we obtain

1+2+3+......+k+k+1=$\frac{k(k+1)}{2}$+k+1

=$\frac{k(k+1)+2(k+1)}{2}$ = $\frac{(k+1)(k+2)}{2}$ = $\frac{(k+1)(\overline{k+1}+1)}{2}$

∴S(k+1) is true, by the principle of mathematical induction the given statement is true

∴1+2+3+.....+n=$\frac{n(n+1)}{2}$ for all n∈ N

2. Prove that 1^2+2^2+3^2+.....+n^2=$\frac{n(n+1)(2n+1)}{6}$ using mathematical induction.

Sol: Let S(n) be the statement that

1^2+2^2+3^2+.....+n^2=$\frac{n(n+1)(2n+1)}{6}$

If n=1, then LHS=1^2=1

RHS=$\frac{1(1+1)(2(1)+1)}{6}$=1

∴LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

∴ 1^2+2^2+3^2+.....+k^2=$\frac{k(k+1)(2k+1)}{6}$

Adding both sides $(k+1)^2$, we get

1^2+2^2+3^2+.....+k^2 + $(k+1)^2$= $\frac{k(k+1)(2k+1)}{6}$+$(k+1)^2$

= $\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

= $\frac{(k+1)(2k^2+k+6k+6)}{6}$

= $\frac{(k+1)(2k^2+7k+6)}{6}$

= $\frac{(k+1)(k+2)(2k+3)}{6}$

= $\frac{(k+1)(\overline{k+1}+1)(2\overline{k+1}+1)}{6}$

∴S(k+1) is true, by the principle of mathematical induction the given statement is true

∴ 1^2+2^2+3^2+.....+n^2=$\frac{n(n+1)(2n+1)}{6}$ for all n∈ N

3. Prove that 1^3+2^3+3^3+.....+n^3=$\frac{n^2(n+1)^2}{4}$ using mathematical induction.

Sol: Let S(n) be the statement that

1^3+2^3+3^3+.....+n^3=$\frac{n^2(n+1)^2}{4}$

If n=1, then LHS=1^3=1

RHS=$\frac{1^2(1+1)^2}{4}$=1

∴LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

$\therefore 1^3+2^3+3^3+.....+k^3=\frac{k^2(k+1)^2}{4}$

Adding both sides $(k+1)^3$, we get

$1^3+2^3+3^3+.....+k^3+(k+1)^3=\frac{k^2(k+1)^2}{4}+(k+1)^3$

$=\frac{k^2(k+1)^2+4(k+1)^3}{4}$

$=\frac{(k+1)^2(k^2+4k+4)}{4}$

$=\frac{(k+1)^2(k+2)^2}{4}$

$=\frac{(k+1)^2(\overline{k+1}+1)^2}{4}$

∴S(k+1) is true, by the principle of mathematical induction the given statement is true

$\therefore 1^3+2^3+3^3+.....+n^3=\frac{n^2(n+1)^2}{4}$ for all n∈ N

4. Prove that 1+3+5+….+2n-1=n^2 using Mathematical induction.

Sol: Let S(n) be the statement that

$1+3+5+....+2n-1=n^2$

If n=1 then LHS=2(1)-1=1

RHS=1^2=1

∴LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

$\therefore 1+3+5+....+2k-1=k^2$

Adding both sides 2k+1, we get

$1+3+5+....+2k-1+2k+1=k^2+2k+1=(k+1)^2=(\overline{k+1})^2$

∴S(k+1) is true, by the principle of mathematical induction the given statement is true

$\therefore 1+3+5+\cdots.+2n-1=n^2$ for all n∈ N

5.Prove that a+(a+d)+(a+2d)+….+a+(n-1)d = $\frac{n}{2}$[2a+(n-1)d] using mathematical induction.

Sol: Let S(n) be the statement that

$a+(a+d)+(a+2d)+....+[a+(n-1)d]=\frac{n}{2}[2a+(n-1)d]$

If n=1, then LHS=a

RHS= $\frac{1}{2}[2a+(1-1)d]=a$

∴LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

$\therefore a+(a+d)+(a+2d)+....+[a+(k-1)d]=\frac{k}{2}[2a+(k-1)d]$

Adding both sides a+kd, we get

$a+(a+d)+(a+2d)+....+[a+(k-1)d]+[a+kd]$

$=\frac{k}{2}[2a+(k-1)d]+[a+kd]$

$=\frac{k[2a+(k-1)d]+2[a+kd]}{2}$

$=\frac{2ka+k(k-1)d]+2a+2kd}{2}$

$=\frac{k+1[2a+kd]}{2}$

∴S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore a+(a+d)+(a+2d)+\cdots.+a+(n-1)d=\frac{n}{2}[2a+(n-1)d]$ for all n∈ N

6. Prove that a +ar +ar^2+….+ar^{n-1}=$\frac{a(r^n-1)}{r-1}$ using mathematical induction.

Sol: Let S(n) be the statement that

$a+ar+ar^2+....+ar^{n-1}=\frac{a(r^n-1)}{r-1}$

n^{th} term of a +ar +ar^2+….is ar^{n-1}

If n=1, then LHS= ar^{1-1}=a

RHS=$\frac{a(r^1-1)}{r-1}=\frac{a(r-1)}{r-1}$=a

∴LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

$\therefore a+ar+ar^2+....+ar^{k-1}=\frac{a(r^k-1)}{r-1}$

Adding both sides ar^k, we get

$a+ar+ar^2+....+ar^{k-1}+ar^k=\frac{a(r^k-1)}{r-1}+ar^k$

$=\frac{a(r^k-1)+(r-1)ar^k}{r-1}$

$=\frac{ar^k-a+ar^{k+1}-ar^k}{r-1}$

$=\frac{a(r^{k+1}-1)}{r-1}$

∴S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore a+ar+ar^2+\cdots.+ar^{n-1}=\frac{a(r^n-1)}{r-1}$for all n∈ N

7. Prove that 3+3^2+3^3….3^n= $\frac{3}{2}$(3^n-1) using Mathematical induction.

Sol: Let S(n) be the statement that

$3+3^2+3^3....3^n=\frac{3}{2}(3^n-1)$

If n=1

LHS=3^1=3

RHS=$\frac{3}{2}(3^1-1)=\frac{3}{2}(3-1)=3$

∴LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

$\therefore 3+3^2+3^3....+3^k=\frac{3}{2}(3^k-1)$

Adding both sides 3^{k+1}, we get

$3+3^2+3^3....+3^k+3^{k+1}=\frac{3}{2}(3^k-1)+3^{k+1}$

$=\frac{3}{2}(3^k-1)+3.3^k$

$=\frac{3}{2}(3^k-1+2.3^k)$

$=\frac{3}{2}(3.3^k-1)=\frac{3}{2}(3^{k+1}-1)$

∴S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore 3+3^2+3^3....3^n=\frac{3}{2}(3^n-1)$ for all n∈ N

8. Prove that 1.2.3+2.3.4+3.4.5+…n terms = $\frac{n(n+1)(n+2)(n+3)}{4}$ using mathematical induction.

Sol: n^{th} term of 1.2.3+2.3.4+3.4.5 + is n(n+1)(n+2)

Let S(n) be the statement that

1.2.3+2.3.4+3.4.5 + ..+

n(n+1)(n+2)= $\frac{n(n+1)(n+2)(n+3)}{4}$

If n=1, then LHS=1.2.3=6

RHS=$\frac{1(1+1)(1+2)(1+3)}{4}$=6

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

$\therefore$ 1.2.3+2.3.4+3.4.5 + ..+

k(k+1)(k+2)= $\frac{k(k+1)(k+2)(k+3)}{4}$

Adding both sides $(k+1)(k+2)(k+3)$, we get

1.2.3+2.3.4+3.4.5 + ..+ k(k+1)(k+2)+ $(k+1)(k+2)(k+3)$

= $\frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)$

= $\frac{k(k+1)(k+2)(k+3)+4(k+1)(k+2)(k+3)}{4}$

= $\frac{(k+1)(k+2)(k+3)(k+4)}{4}=\frac{(k+1)(\overline{k+1}+1)(\overline{k+1}+2)(\overline{k+1}+3)}{4}$

$\therefore$S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore 1.2.3+2.3.4+3.4.5+\cdots n \text{ terms } = \frac{n(n+1)(n+2)(n+3)}{4}$ for all n∈ N

9. Prove that $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}$ +…. N terms=$\frac{n}{2n+1}$ using Mathematical induction.

Sol: $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}$ +…. N terms=$\frac{n}{2n+1}$ for all n∈ N

1,3,5,… is an A.P. its n^{th} term =1+(n-1)2=2n-1

$\therefore$ n^{th} term of $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}$ +…. Is $\frac{1}{(2n-1)(2n+1)}$

Let S(n) be the statement that

$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\dots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

If n=1, then LHS=$\frac{1}{1.3}=\frac{1}{3}$

RHS=$\frac{1}{2(1)+1}=\frac{1}{3}$

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true

$\therefore \frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\dots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

Adding both sides $\frac{1}{(2k+1)(2k+3)}$, we get

$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\dots+\frac{1}{(2n-1)(2n+1)}$

$+\frac{1}{(2k+1)(2k+3)}=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$

$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$

$=\frac{(k+1)(2k+1)}{(2k+1)(2k+3)}=\frac{k+1}{2k+3}=\frac{k+1}{2(k+1)+1}$

$\therefore$S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore \frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}$ +…. N terms=$\frac{n}{2n+1}$ for all n∈ N

10. Prove that $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}$ +…. N terms=$\frac{n}{3n+1}$ using Mathematical induction.

Sol: 1,4,7 are in A.P.

A=1,d=4-1=3

t_n=a+(n-1)d=1+(n-1)3=(3n-2)

4,7,10 are in A.P

A=4; d=(7-4)=3

t_n=a+(n-1)d=4+(n-1)3=(3n+1)

Given statement

$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots\frac{1}{3n-2}.\frac{1}{3n+1}=\frac{n}{3n+1}$

Let S(n)= $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$

Put n=1, LHS = $\frac{1}{1.4}=\frac{1}{4}$

RHS=$\frac{n}{3n+1}=\frac{1}{3(1)+1}=\frac{1}{4}$

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true for some k∈ N

$\therefore \frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1}$

Adding both sides $\frac{1}{(3k+1)(3k+4)}$, we get S(k+1)

$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots\frac{1}{(3k-2)(3k+1)}+$

$\frac{1}{(3k+1)(3k+4)}=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}$

$=\frac{1}{3k+1}\left[k+\frac{1}{(3k+4)}\right]$

$=\frac{1}{3k+1}\left[\frac{3k^2+4k+1}{(3k+4)}\right]$

$=\frac{1}{3k+1}\left[\frac{(k+1)(3k+1)}{(3k+4)}\right]$

$=\frac{(k+1)}{(3k+4)}=\frac{(k+1)}{(3k+3+1)}=\frac{(k+1)}{3(k+1)+1}$

$\therefore$S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore \frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}$ +…. N terms=$\frac{n}{3n+1}$ for all n∈ N

11. Prove that 2+7+12+….(5n-3) =$\frac{n(5n-1)}{2}$ using Mathematical induction.

Sol: Let S(n) be the statement that

2+7+12+….(5n-3) =$\frac{n(5n-1)}{2}$

If n=1, then LHS=(5n-3)=(5(1)-3)=(5-3)=2

RHS=$\frac{n(5n-1)}{2}=\frac{1(5-1)}{2}$=2

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true for some k∈ N

$\therefore$ 2+7+12+….(5k-3) =$\frac{k(5k-1)}{2}$

Adding both sides $5k+2$, we get S(k+1)

2+7+12+….(5k-3)+(5k+2) =$\frac{k(5k-1)}{2}$+(5k+2)

$=\frac{5k^2-k+10k+4}{2}=\frac{5k^2+9k+4}{2}=\frac{(k+1)(5k+4)}{2}=\frac{(k+1)(5(k+1)-1)}{2}$

$\therefore$S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore 2+7+12+\cdots.(5n-3) = \frac{n(5n-1)}{2}$ for all n∈ N

12. Prove that $4^3+8^3+12^3+\dots+n \text{ terms} =16n^2(n+1)^2$ using Mathematical induction.

Sol: It can be easily observed that

n^{th} term=$(4n)^3$

Let S(n) be the statement that
$4^3+8^3+12^3+.....+(4n)^3=16n^2(n+1)^2$
If n=1 then LHS=4^3=64
RHS=$16(1)^2(1+1)^2$=64
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true for some k∈ N
$4^3+8^3+12^3+.....+(4k)^3=16k^2(k+1)^2$
Adding both sides $[4(k+1)]^3$, we get
$4^3+8^3+12^3+.....+(4k)^3+[4(k+1)]^3$
$=16k^2(k+1)^2+[4(k+1)]^3$
$=16k^2(k+1)^2+64(k+1)^3$
$=16(k+1)^2[k^2+4(k+1)]$
$=16(k+1)^2[k^2+4k+4]$
$=16(k+1)^2(k+2)^2$
$=16(k+1)^2(k+1+1)^2$
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
∴ $4^3+8^3+12^3+.....+(4n)^3=16n^2(n+1)^2$ for all n∈ N

13. Prove that $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}$ using Mathematical induction.

Sol: Let the given statement
S(n)= $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}$
If n=1, then
LHS = $\frac{1}{1(1+1)}=\frac{1}{2}$
RHS = $\frac{1}{1+1}=\frac{1}{2}$
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true for some k∈ N
$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{k(k+1)}=\frac{k}{k+1}$
Adding both sides $\frac{1}{(k+1)(k+2)}$, we get
$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$
$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$
$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$
$=\frac{k(k+2)+1}{(k+1)(k+2)}$
$=\frac{k^2+2k+1}{(k+1)(k+2)}$
$=\frac{(k+1)^2}{(k+1)(k+2)}$
$=\frac{(k+1)}{(k+2)}=\frac{(k+1)}{(k+1+1)}$
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
∴ $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}$ for all n∈ N

14. Prove that $2+3.2+4.2^2....$n terms=$n2^n$ using Mathematical induction.

Sol: n^{th} term of $2+3.2+4.2^2+...$ is (n+1) 2^{n-1}
Let S(n) be the statement that
$2+3.2+4.2^2+...+(n+1)2^{n-1}=n2^n$
If n=1, then LHS=(1+1) 2^{1-1}=2
RHS= $(1)2^1$=2
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true for some k∈ N
$2+3.2+4.2^2+...+(k+1)2^{k-1}=k2^k$
Adding both sides (k+2) 2^k, we get
$+3.2+4.2^2+...+(k+1)2^{k-1}+(k+2)2^k=k2^k+(k+2)2^k$
$=(k+k+2)2^k$
$=(2k+2)2^k$
$=(k+1)2.2^k$
$=(k+1)2^{k+1}$
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
∴ $2+3.2+4.2^2....n\ terms=n2^n$ for all n∈ N

15. Prove that 2.3+3.4+4.5+…+ n terms $=\frac{n(n^2+6n+11)}{3}$ using Mathematical induction.

Sol: n^{th} term in the LHS of the given statement is (n+1)(n+2)
Let S(n) be the statement that
$2.3+3.4+4.5+...+(n+1)(n+2)=\frac{n(n^2+6n+11)}{3}$
If n=1 then LHS=(1+1)(1+2)=2.3=6
RHS =$\frac{1(1^2+6(1)+11)}{3}=\frac{1+6+11}{3}$=6
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true for some k∈ N
$2.3+3.4+4.5+...+(k+1)(k+2)=\frac{k(k^2+6k+11)}{3}$
Adding both sides (k+2)(k+3) we get
2.3+3.4+4.5+…+(k+1)(k+2)+ (k+2)(k+3) =
$\frac{k(k^2+6k+11)}{3}+(k+2)(k+3)$
$=\frac{(k^3+6k^2+11k+3k^2+15k+18)}{3}$
$=\frac{(k^3+9k^2+26k+18)}{3}$
$=\frac{(k+1)(k^2+8k+18)}{3}$
$=\frac{(k+1)[(k+1)^2+6(k+1)+11)}{3}$
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
∴ $2.3+3.4+4.5+\cdots+\ n\ terms=\frac{n(n^2+6n+11)}{3}$
for all n∈ N

$$

Solved problems in the text book

1.Show that 1.6+2.9+3.12+..+n(3n+3) =n(n+1)(n+2) for all n∈ N
Sol: Let S(n) be a statement that
1.6+2.9+3.12+..+n(3n+3) =n(n+1)(n+2)

If n=1 then LHS=1.6=6
RHS=1(1+1)(1+2)=1.2.3=6
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true for some k∈ N
∴1.6+2.9+3.12+..+k(3k+3) =k(k+1)(k+2)
Adding both sides (k+1)(3k+6), we get
1.6+2.9+3.12+..+k(3k+3)+ (k+1)(3k+6)
=k(k+1)(k+2)+(k+1)(3k+6)
= (k+1)(k+2)(k+3)
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
$\therefore 1.6+2.9+3.12+..+n(3n+3)=n(n+1)(n+2)$
for all n∈ N

2. Show that 1+(1+2)+(1+2+3) +..+upto n brackets = $\frac{n(n+1)(n+2)}{6}$ for all n∈ N.
Sol: n^{th} bracket is 1+2+3+…+n
Let S(n) be a statement that
1+(1+2)+(1+2+3) +..+(1+2+3+…+n) = $\frac{n(n+1)(n+2)}{6}$
If n=1 then LHS=1=1
RHS=$\frac{1(1+1)(1+2)}{6}$ = 1
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true for some k∈ N
∴1+(1+2)+(1+2+3) +..+(1+2+3+…+k) = $\frac{k(k+1)(k+2)}{6}$
Adding both sides (1+2+3+..+k+(k+1)
1+(1+2)+(1+2+3)+..+(1+2+3+.+k)+ (1+2+3+.+k+(k+1)
=$\frac{k(k+1)(k+2)}{6}$+ (1+2+3+..+k+(k+1)
= $\frac{k(k+1)(k+2)}{6}+\frac{(k+1)(k+2)}{6}$
= $\frac{(k+1)(k+2)}{2}(\frac{k}{3}+1)=\frac{(k+1)(k+2)}{2}(\frac{k+3}{3})$
= $\frac{(k+1)(k+2)(k+3)}{6}$
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
∴1+(1+2)+(1+2+3) +.+upto n brackets = $\frac{n(n+1)(n+2)}{6}$
for all n∈ N

3.Prove that by using Mathematical Induction:
1.3+2.4+3.5+…+n(n+2) = $\frac{n(n+1)(2n+7)}{6}$ for all n∈ N. **(mar20)**
Sol: Let S(n) be a statement that
1.3+2.4+3.5+…+n(n+2) = $\frac{n(n+1)(2n+7}{6}$
If n=1 then LHS=1.3=3
RHS=$\frac{1(1+1)(2.1+7)}{6}=\frac{18}{3}$=3
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true for some k∈ N
∴1.3+2.4+3.5+…+k(k+2) = $\frac{k(k+1)(2k+7)}{6}$
Adding both sides (k+1)(k+3), we get
1.3+2.4+3.5+…+k(k+2) +(k+1)(k+3)
= $\frac{k(k+1)(2k+7)}{6}$ +(k+1)(k+3)
= $\frac{k(k+1)(2k+7)+6(k+1)(k+3)}{6}$
= $\frac{(k+1)[k(2k+7)+6(k+3)]}{6}$
= $\frac{(k+1)[2k^2+7k+6k+18]}{6}$
= $\frac{(k+1)[2k^2+13k+18]}{6}$
= $\frac{(k+1)(k+2)(2k+9)}{6}$
= $\frac{(k+1)(k+1+1)(2(k+1)+7)}{6}$
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
$\therefore 1.3+2.4+3.5+\cdots+n(n+2)=\frac{n(n+1)(2n+7)}{6}$
for all n∈ N.

Exercise Questions

1. By principle of Mathematical Induction prove
1.2+2.3+3.4+..+n(n+1)=$\frac{n(n+1)(n+2)}{3}$ for all n∈ N.
Sol: Let S(n) be a statement that
1.2+2.3+3.4+..+n(n+1)=$\frac{n(n+1)(n+2)}{3}$
If n=1, then LHS=1.2=2
RHS=$\frac{1(1+1)(1+2)}{3}$=2
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true
∴ 1.2+2.3+3.4+ ..+ k(k+1) = $\frac{k(k+1)(k+2)}{3}$
Adding both sides $(k+1)(k+2)$, we get
1.2 +2.3+3.4 + ..+ k(k+1)+ $(k+1)(k+2)$
= $\frac{k(k+1)(k+2)}{3}+(k+1)(k+2)$
= $\frac{k(k+1)(k+2)+3(k+1)(k+2)}{3}$
= $\frac{(k+1)(k+2)(k+3)}{3}=\frac{(k+1)(\overline{k+1}+1)(\overline{k+1}+2)}{3}$
∴S(k+1) is true
by the principle of mathematical induction the given statement is true
$\therefore 1.2+2.3+3.4+..+n(n+1)=\frac{n(n+1)(n+2)}{3}$ for all n∈ N

2. By principle of Mathematical Induction prove
1.3+3.5+5.7+..+n(n+1)=$\frac{n(4n^2+6n-1)}{3}$ for all n∈ N.
Sol: Let S(n) be a statement that
1.3+3.5+5.7+..+n(n+1)=$\frac{n(4n^2+6n-1)}{3}$
If n=1, then LHS=1.3=3
RHS=$\frac{1(4(1)^2+6(1)-1)}{3}=\frac{4+6-1}{3}=\frac{9}{3}$ = 3
LHS=RHS, Hence S(1) is true.
Assume that S(k) is true
∴ 1.3+3.5+5.7+..+k(k+1)=$\frac{k(4k^2+6k-1)}{3}$
Adding both sides $(k+1)(k+3)$, we get
1.3+3.5+5.7+..+k(k+1)+(2k+1)(2k+3)
= $\frac{k(4k^2+6k-1)}{3}$ +(2k+1)(2k+3)
= $\frac{k(4k^2+6k-1)+3(2k+1)(2k+3)}{3}$

$= \frac{[4k^3+6k^2-k+12k^2+24k+9]}{3} = \frac{[4k^3+18k^2+23k+6]}{3}$

$= \frac{(k+1)[4k^2+14k+9]}{3} = \frac{(k+1)[4k^2+14k+9]}{3}$

$= \frac{(k+1)[4k^2+8k+4+6k+6-1]}{3}$

$= \frac{(k+1)[4(k^2+2k+1)+6(k+1)-1]}{3}$

$= \frac{(k+1)(4(k+1)^2+6(k+1)-1)}{3}$;

∴S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore 1.3 + 3.5 + 5.7 + .. + n(n+1) = \frac{n(4n^2+6n-1)}{3}$ for all n∈ N

3. By principle of Mathematical Induction prove $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} +.+ \frac{1}{(3n-1)(3n+2)} = \frac{n}{2(3n+2)}$ for all n∈ N

Sol: Let the given statement

S(n)= $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} +.+ \frac{1}{(3n-1)(3n+2)} = \frac{n}{2(3n+2)}$

If n=1, then LHS = $\frac{1}{(3(1)-1)(3(1)+2)} = \frac{1}{(2)(5)} = \frac{1}{10}$

RHS = $\frac{1}{2(3(1)+2)} = \frac{1}{10}$

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true for some k∈ N

$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} +.+ \frac{1}{(3k-1)(3k+2)} = \frac{k}{2(3k+2)}$

Adding both sides $\frac{1}{(3k+2)(3k+5)}$, we get

$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} +.+ \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3k+2)(3k+5)}$

$= \frac{k}{2(3k+2)} + \frac{1}{(3k+2)(3k+5)} = \frac{k(3k+5)+2}{2(3k+2)(3k+5)}$

$= \frac{3k^2+5k+2}{2(3k+2)(3k+5)} = \frac{(k+1)(3k+2)}{2(3k+2)(3k+5)} = \frac{(k+1)}{2(3k+5)}$

$= \frac{(k+1)}{2[3(k+1)+2]}$;

∴S(k+1) is true

by the principle of mathematical induction the given statement is true

$\therefore \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} +.+ \frac{1}{(3n-1)(3n+2)} = \frac{n}{2(3n+2)}$

for all n∈ N

4. By principle of Mathematical Induction prove $1^2+(1^2+2^2)+(1^2+2^2+3^2)+..$+upto n brackets $= \frac{n(n+1)^2(n+2)}{12}$ for all n∈ N

Sol: The n[th] term of the given series is `

$1^2+2^2+3^2+...+n^2 = \frac{n(n+1)(2n+1)}{6}$

Let S(n): $1^2+(1^2+2^2)+(1^2+2^2+3^2)+..+\frac{n(n+1)(2n+1)}{6}$

$= \frac{n(n+1)^2(n+2)}{12}$

If n=1, then ;LHS = 1^2=1

RHS = $\frac{1(1+1)^2(1+2)}{12} = \frac{1(2)^2(3)}{12} = \frac{12}{12} = 1$

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true for some k∈ N

S(k): $1^2+(1^2+2^2)+(1^2+2^2+3^2)+..+\frac{k(k+1)(2k+1)}{6} = \frac{k(k+1)^2(k+2)}{12}$

Adding both sides $\frac{(k+1)(k+2)(2k+3)}{6}$, we get

$1^2+(1^2+2^2)+(1^2+2^2+3^2)+..+\frac{k(k+1)(2k+1)}{6}+\frac{(k+1)(k+2)(2k+3)}{6}$

$= \frac{k(k+1)^2(k+2)}{12} + \frac{(k+1)(k+2)(2k+3)}{6}$

$= \frac{k(k+1)^2(k+2)+2(k+1)(k+2)(2k+3)}{12}$

$= \frac{(k+1)(k+2)[k(k+1)+2(2k+3)]}{12}$

$= \frac{(k+1)(k+2)[k^2+k+(4k+6)]}{12} = \frac{(k+1)(k+2)[k^2+5k+6]}{12}$

$= \frac{(k+1)(k+2)[(k+2)(k+3)]}{12} = \frac{(k+1)(k+2)^2(k+3)}{12}$

$= \frac{(k+1)(k+1+1)^2(k+1+2)}{12}$

S(k+1) is true

by the principle of mathematical induction the given statement is true

$1^2+(1^2+2^2)+(1^2+2^2+3^2)+..$+upto n brackets = $\frac{n(n+1)^2(n+2)}{12}$ for all n∈ N

5. By principle of Mathematical Induction prove $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} +..$+upto n brackets $= \frac{n(2n^2+9n+13)}{24}$ for all n∈ N

Sol: The n[th] term of the given series is `

$1^3+2^3+3^3+...+n^3 = \frac{n^2(n+1)^2}{4}$

Let S(n): $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} +..+ \frac{(n+1)^2}{4}$

$= \frac{n(2n^2+9n+13)}{24}$

If n=1, then

LHS = $\frac{1^3}{1}$=1

RHS = $\frac{1(2(1)^2+9(1)+13)}{24} = \frac{1(2+9+13)}{24} = \frac{24}{24} = 1$

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true for some k∈ N

S(k): $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} +.+ \frac{(k+1)^2}{4}$

$= \frac{k(2k^2+9k+13)}{24}$

Adding both sides $\frac{(k+2)^2}{4}$, we get

$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} +.+ \frac{(k+1)^2}{4} + \frac{(k+2)^2}{4}$

$= \frac{k(2k^2+9k+13)}{24} + \frac{(k+2)^2}{4}$

$= \frac{k(2k^2+9k+13)+6(k+2)^2}{24}$

$= \frac{(2k^3+9k^2+13k)+6(k^2+4k+4)}{24}$

$= \frac{(2k^3+15k^2+37k+24)}{24} = \frac{(k+1)(2k^3+13k+24)}{24}$

$= \frac{(k+1)(2k^3+4k+2+9k+9+13)}{24}$

$= \frac{(k+1)(2(k^3+2k+1)+9(k+1)+13)}{24}$

$= \frac{(k+1)(2(k+1)^2+9(k+1)+13)}{24}$

S(k+1) is true

by the principle of mathematical induction the given statement is true

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+..+$upto n brackets

$=\frac{n(2n^2+9n+13)}{24}$ for all n∈ N

6. By principle of Mathematical Induction prove

$\cos\theta\cos 2\theta\cos 4\theta....\cos 2^{n-1}\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$

for all n∈ N

Sol: Let the given statement

S(n)= $\cos\theta\cos 2\theta\cos 4\theta....\cos 2^{n-1}\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$

If n=1, then LHS = $\cos\theta=\cos\theta$

RHS = $\frac{\sin 2^1\theta}{2^1\sin\theta}=\frac{\sin 2\theta}{2\sin\theta}=\frac{2\sin\theta\cos\theta}{2\sin\theta}=\cos\theta$

LHS=RHS, Hence S(1) is true.

Assume that S(k) is true for some k∈ N

$\cos\theta\cos 2\theta\cos 4\theta....\cos 2^{k-1}\theta=\frac{\sin 2^k\theta}{2^k\sin\theta}$

multiply both sides $\cos 2^k\theta$, we get

$\cos\theta\cos 2\theta\cos 4\theta....\cos 2^{k-1}\theta\cos 2^k\theta$,

$=\frac{\sin 2^k\theta}{2^k\sin\theta}\cos 2^k\theta=\frac{\sin 2^k\theta\cos 2^k\theta}{2^k\sin\theta}=\frac{2\sin 2^k\theta\cos 2^k\theta}{2.2^k\sin\theta}$

$=\frac{\sin 2.2^k\theta}{2^{k+1}\sin\theta}=\frac{\sin 2^{k+1}\theta}{2^{k+1}\sin\theta}$

∴S(k+1) is true

by the principle of mathematical induction the given statement is true

∴ $\cos\theta\cos 2\theta\cos 4\theta....\cos 2^{n-1}\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$

for all n∈ N

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3. MATRICES

1. If A=$\begin{bmatrix}3&9&0\\1&8&-2\end{bmatrix}$ and B=$\begin{bmatrix}4&0&2\\7&1&4\end{bmatrix}$ then find A+B

Sol: A=$\begin{bmatrix}3&9&0\\1&8&-2\end{bmatrix}$ and B=$\begin{bmatrix}4&0&2\\7&1&4\end{bmatrix}$

A+B=$\begin{bmatrix}3&9&0\\1&8&-2\end{bmatrix}+\begin{bmatrix}4&0&2\\7&1&4\end{bmatrix}$

A+B=$\begin{bmatrix}3+4&9+0&0+2\\1+7&8+1&-2+4\end{bmatrix}=\begin{bmatrix}7&9&2\\8&9&2\end{bmatrix}$

2. If A=$\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}$ and B=$\begin{bmatrix}3&2&1\\1&2&3\end{bmatrix}$ then find 3B-2A

Sol: A=$\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}$ and B=$\begin{bmatrix}3&2&1\\1&2&3\end{bmatrix}$

3B-2A=$3\begin{bmatrix}3&2&1\\1&2&3\end{bmatrix}-2\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}$

3B-2A=$\begin{bmatrix}9&6&3\\3&6&9\end{bmatrix}-\begin{bmatrix}2&4&6\\6&4&2\end{bmatrix}$

3B-2A=$\begin{bmatrix}9-2&6-4&3-6\\3-6&6-4&9-2\end{bmatrix}=\begin{bmatrix}7&2&-3\\-3&2&7\end{bmatrix}$

3. If A=$\begin{bmatrix}0&1&2\\2&3&4\\4&5&6\end{bmatrix}$, B=$\begin{bmatrix}1&-2&0\\0&1&-1\\-1&0&3\end{bmatrix}$ then find A-B and 4B-3A.

Sol: A=$\begin{bmatrix}0&1&2\\2&3&4\\4&5&6\end{bmatrix}$, B=$\begin{bmatrix}1&-2&0\\0&1&-1\\-1&0&3\end{bmatrix}$

A-B=$\begin{bmatrix}0&1&2\\2&3&4\\4&5&6\end{bmatrix}-\begin{bmatrix}1&-2&0\\0&1&-1\\-1&0&3\end{bmatrix}$

A-B=$\begin{bmatrix}0-1&1-(-2)&2-0\\2-0&3-1&4-(-1)\\4-(-1)&5-0&6-3\end{bmatrix}=\begin{bmatrix}-1&3&2\\2&2&5\\5&5&3\end{bmatrix}$

4B-3A=$4\begin{bmatrix}1&-2&0\\0&1&-1\\-1&0&3\end{bmatrix}-3\begin{bmatrix}0&1&2\\2&3&4\\4&5&6\end{bmatrix}$

4B-3A=$\begin{bmatrix}4&-8&0\\0&4&-4\\-4&0&12\end{bmatrix}-\begin{bmatrix}0&3&6\\6&9&12\\12&15&18\end{bmatrix}$

$=\begin{bmatrix}4-0&-8-3&0-6\\0-6&4-9&-4-12\\-4-12&0-15&12-18\end{bmatrix}=\begin{bmatrix}4&-11&-6\\-6&-5&-16\\-16&-15&-6\end{bmatrix}$

4. If A=$\begin{bmatrix}2&3&1\\6&-1&5\end{bmatrix}$, B=$\begin{bmatrix}1&2&-1\\0&-1&3\end{bmatrix}$ and A+B-X=[0] Then find the matrix X.

Sol: A=$\begin{bmatrix}2&3&1\\6&-1&5\end{bmatrix}$, B=$\begin{bmatrix}1&2&-1\\0&-1&3\end{bmatrix}$, X=$\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\end{bmatrix}$

A+B-X=$\begin{bmatrix}2&3&1\\6&-1&5\end{bmatrix}+\begin{bmatrix}1&2&-1\\0&-1&3\end{bmatrix}-\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\end{bmatrix}=0$

A+B-X=$\begin{bmatrix}2+1-a_{11}&3+2-a_{12}&1-1-a_{13}\\6+0-a_{21}&-1-1-a_{22}&5+3-a_{23}\end{bmatrix}=0$

A+B-X=$\begin{bmatrix}3-a_{11}&5-a_{12}&0-a_{13}\\6-a_{21}&-2-a_{22}&8-a_{23}\end{bmatrix}=0$

⇒Matrix X=$\begin{bmatrix}3&5&0\\6&-2&8\end{bmatrix}$

5. Find the trace of the matrix $\begin{bmatrix}1&3&-5\\2&-1&5\\1&0&1\end{bmatrix}$

Sol: Trace of matrix=1+(-1)+1=1

The elements of the principle diagonal elements of A are 1,-1,1

6. If A=$\begin{bmatrix}2&3\\1&2\end{bmatrix}$ and B=$\begin{bmatrix}0&4\\-1&2\end{bmatrix}$ then find AB and BA

Sol: A=$\begin{bmatrix}2&3\\1&2\end{bmatrix}$; B=$\begin{bmatrix}0&4\\-1&2\end{bmatrix}$

AB=$\begin{bmatrix}2&3\\1&2\end{bmatrix}\begin{bmatrix}0&4\\-1&2\end{bmatrix}$

AB=$\begin{bmatrix}2x0+3(-1)&2x4+3x2\\1x0+2(-1)&1x4+2x2\end{bmatrix}=\begin{bmatrix}-3&14\\-21&8\end{bmatrix}$

BA=$\begin{bmatrix}0&4\\-1&2\end{bmatrix}\begin{bmatrix}2&3\\1&2\end{bmatrix}$

BA=$\begin{bmatrix}0x2+4x1&0x3+4x2\\-1x2+2x1&-1x3+2x2\end{bmatrix}=\begin{bmatrix}4&8\\0&1\end{bmatrix}$

7. If A=$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ then find A^2

Sol: A=$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

A^2=A.A=$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

=$\begin{bmatrix} 4x4+2(-1) & 4x2+2x1 \\ -1x4+1(-1) & -1x2+1x1 \end{bmatrix}$

=$\begin{bmatrix} 16-2 & 8+2 \\ -4-1 & -2+1 \end{bmatrix}$=$\begin{bmatrix} 14 & 10 \\ -5 & -1 \end{bmatrix}$

8. If A=$\begin{bmatrix} 2 & 4 \\ -1 & K \end{bmatrix}$ and A^2=[0] then find K.

Sol: A=$\begin{bmatrix} 2 & 4 \\ -1 & K \end{bmatrix}$

A^2=A.A=$\begin{bmatrix} 2 & 4 \\ -1 & K \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -1 & K \end{bmatrix}$=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

=$\begin{bmatrix} 2x2+4(-1) & 2x4+4(K) \\ -1x2+K(-1) & -1x4+k(K) \end{bmatrix}$=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

=$\begin{bmatrix} 4-4 & 8+4K \\ -2-K & -4+K^2 \end{bmatrix}$=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

⇒ 8+4K=0

4K= -8

K=$\frac{-8}{4}$=-2

9. If A=$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that A^2-4A-5I=[0]

Sol: A=$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

A^2=A.A=$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

=$\begin{bmatrix} 1x1+2x2+2x2 & 1x2+2x1+2x2 & 1x2+2x2+2x1 \\ 2x1+1x2+2x2 & 2x2+1x1+2x2 & 2x2+1x2+2x1 \\ 2x1+2x2+1x2 & 2x2+2x1+1x2 & 2x2+2x2+1x1 \end{bmatrix}$

=$\begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$=$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

A^2-4A-5I=$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}-4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}-5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A^2-4A-5I=$\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$-$\begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$-$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

A^2-4A-5I=$\begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$=$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

∴ A^2-4A-5I=[0]

10. If A=$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ then show that A^2= -I

Sol: A=$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

A^2=A.A=$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$=$\begin{bmatrix} i^2+0 & 0+0 \\ 0+0 & 0+i^2 \end{bmatrix}$=$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

= -$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$=-I

∴ A^2=-I

11. If A=$\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$ then find A+A^T

Sol: A=$\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$; A^T=$\begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$

A+A^T=$\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$+$\begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$=$\begin{bmatrix} 2+2 & -4-5 \\ -5-4 & 3+3 \end{bmatrix}$=$\begin{bmatrix} 4 & -9 \\ -9 & 6 \end{bmatrix}$

12. If A=$\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$ then find A.A^T

Sol: A=$\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$; A^T=$\begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$

A.A^T=$\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$=

$\begin{bmatrix} 2x2-4(-4) & 2(-5)-4(3) \\ -5x2+3(-4) & -5(-5)+3(3) \end{bmatrix}$

=$\begin{bmatrix} 4+16 & -10-12 \\ -10-12 & 25+9 \end{bmatrix}$ = $\begin{bmatrix} 20 & -22 \\ -22 & 34 \end{bmatrix}$

13. If A=$\begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$; B=$\begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ then find 2A+B^T

Sol: A=$\begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$; B=$\begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$; B^T=$\begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$

2A+B^T=2$\begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$+$\begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$=$\begin{bmatrix} -4 & 2 \\ 10 & 0 \\ -2 & 8 \end{bmatrix}$+$\begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$

2A+B^T=$\begin{bmatrix} -4-2 & 2+4 \\ 10+3 & 0+0 \\ -2+1 & 8+2 \end{bmatrix}$=$\begin{bmatrix} -6 & 6 \\ 13 & 0 \\ -1 & 10 \end{bmatrix}$

14. If A=$\begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$ then find A.A^T

Repeat Q.No.12

15. If A=$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, find the values of x.

Sol: A=$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix A=A^T

⇒ A^T=$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & x \\ 3 & 6 & 7 \end{bmatrix}$

A=A^T

$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$=$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & x \\ 3 & 6 & 7 \end{bmatrix}$

⇒x=6

16. A=$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then show that A.A^T=I

Sol: A=$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$; A^T=$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

A.A^T=$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

A.A^T=$\begin{bmatrix} \cos\alpha.\cos\alpha+\sin\alpha.\sin\alpha & \cos\alpha.-\sin\alpha+\sin\alpha.\cos\alpha \\ -\sin\alpha.\cos\alpha+\cos\alpha.\sin\alpha & -\sin\alpha.-\sin\alpha+\cos\alpha.\cos\alpha \end{bmatrix}$

A.A^T=$\begin{bmatrix} \cos^2\alpha+\sin^2\alpha & -\cos\alpha.\sin\alpha+\sin\alpha.\cos\alpha \\ -\sin\alpha.\cos\alpha+\cos\alpha.\sin\alpha & \sin^2\alpha+\cos^2\alpha \end{bmatrix}$

A.A^T=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$= I

∴ A.A^T= I

17. If A=$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$ then verify that $(A+B)^T$=A^T+B^T

Sol: Problem is not completed, B value not given.

18. If A=$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}$; B=$\begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{bmatrix}$ then find BA-4B^T

Sol: A=$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}$; B=$\begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{bmatrix}$; B^T=$\begin{bmatrix} 2 & 0 & 1 \\ -1 & -2 & 2 \\ 0 & 5 & 0 \end{bmatrix}$

BA=$\begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}$

BA=$\begin{bmatrix} 2x1-1x2+0x3 & 2x5-1x4+0x(-1) & 2x3-1x0+0x(-5) \\ 0x1-2x2+5x3 & 0x5-2x4+5x(-1) & 0x3-2x0+5x(-5) \\ 1x1+2x2+0x3 & 1x5+2x4+0x(-1) & 1x3+2x0+0x(-5) \end{bmatrix}$

BA=$\begin{bmatrix} 0 & 6 & 6 \\ 11 & -13 & -25 \\ 5 & 13 & 3 \end{bmatrix}$

BA-4B^T=$\begin{bmatrix} 0 & 6 & 6 \\ 11 & -13 & -25 \\ 5 & 13 & 3 \end{bmatrix}-4\begin{bmatrix} 2 & 0 & 1 \\ -1 & -2 & 2 \\ 0 & 5 & 0 \end{bmatrix}$

BA-4B^T=$\begin{bmatrix} 0-8 & 6-0 & 6-4 \\ 11+4 & -13+8 & -25-8 \\ 5-0 & 13-20 & 3-0 \end{bmatrix}$=$\begin{bmatrix} -8 & 6 & 2 \\ 15 & -5 & -33 \\ 5 & -7 & 3 \end{bmatrix}$

18. If A=$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}$; B=$\begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{bmatrix}$ then find 3A-4B^T

Sol: A=$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}$; B=$\begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{bmatrix}$; B^T=$\begin{bmatrix} 2 & 0 & 1 \\ -1 & -2 & 2 \\ 0 & 5 & 0 \end{bmatrix}$

3A-4B^T=3$\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}-4\begin{bmatrix} 2 & 0 & 1 \\ -1 & -2 & 2 \\ 0 & 5 & 0 \end{bmatrix}$

3A-4B^T=$\begin{bmatrix} 3-8 & 15-0 & 9-4 \\ 6+4 & 12+8 & 0-8 \\ 9-0 & -3-20 & -15-0 \end{bmatrix}$=$\begin{bmatrix} -5 & 15 & 5 \\ 10 & 20 & -8 \\ 9 & -23 & -15 \end{bmatrix}$

19. If A=$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$; B=$\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}$ then find BA^T

Sol: A=$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$; B=$\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}$: A^T=$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

BA^T=$\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

BA^T=$\begin{bmatrix} 0x2+4x3 & 0x1+4x2 \\ -1x2+2x3 & -1x1+2x2 \end{bmatrix}$=$\begin{bmatrix} 12 & 8 \\ 4 & 3 \end{bmatrix}$

20. If A=$\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}$ then find AA^T

Sol: A=$\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}$; A^T=$\begin{bmatrix} 0 & -1 \\ 4 & 2 \end{bmatrix}$

AA^T=$\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 4 & 2 \end{bmatrix}$

AA^T=$\begin{bmatrix} 0x0+4x4 & 0x(-1)+4x2 \\ -1x0+2x4 & -1x(-1)+2x2 \end{bmatrix}$=$\begin{bmatrix} 16 & 8 \\ 8 & 5 \end{bmatrix}$

21. Find the determinant A=$\begin{bmatrix} 2 & 1 \\ 41 & -5 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 2 & 1 \\ 41 & -5 \end{bmatrix}$

det A=2x(-5)-(1x1)= -10-1= -11

22. Find the determinant A=$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

Sol: A=$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

det A=ix(-i)-0x0=-i^2=1

23. Find the determinant of A=$\begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix}$

det A=1x1-(-1x-3)=1-3=-2

24. Find the determinant of A=$\begin{bmatrix} 2 & -1 & 4 \\ 0 & -2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 2 & -1 & 4 \\ 0 & -2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$

det A=[2(-2x3-1x5)]-(-1)[0x3-(-3x5)]+4[0x1-(-3x-2)]

det A=2[-6-5]+1[0+15]+4[0-6]=-22+15-24=-31

25. Find the determinant of A=$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

det A =[0(0-1)-1(0-1)+1(1-0)]=[0+1+1]=2

26. Find the determinant of A=$\begin{bmatrix} 2 & -1 & 4 \\ 4 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 2 & -1 & 4 \\ 4 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

det A=2[-3x1-2x1]-(-1)[4x1-1x1]+4[4x2-(1x-3)]

det A=2[-3-2]+[4-1]+4[8+3]=-10+3+44=37

27. Find the determinant of $\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & 4 \\ -3 & 7 & 6 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & 4 \\ -3 & 7 & 6 \end{bmatrix}$

det A=1[-1x6-7x4]-4[2x6-(-3x4)]+2[2x7-(-3x-1)]

det A=1[-6-28]-4[12+12]+2[14-3]=-34-96+22=-108

28. Find the determinant of $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 2 \\ 4 & 5 & 6 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 2 \\ 4 & 5 & 6 \end{bmatrix}$

det A=1[-1x6-5x2]-0[3x6-4x2]-2[3x5-4x-1]

det A=1[-6-10]-0-2[15+4]=-16-38=-54

29. Find the determinant of $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$=$\begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$

det A=1[9x25-16x16]-4[4x25-9x16]+9[4x16-9x9]

det A=1[225-256]-4[100-144]+9[64-81]

det A=-31+176-153=-8

30.Find the determinant of $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

Sol: A=$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

det A=a[c.b -a.a]-b[b.b-c.a]+c[b.a-c.c]

det A=a[bc -a^2]-b[b^2-ca]+c[ab-c^2]

det A=abc -a^3-b^3+abc +abc-c^3

det A=3abc -a^3- b^3 - c^3

31.Find the determinant of $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Sol: A=$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

det A=a[b.c -f.f]-h[h.c-g.f]+g[h.f-g.b]

det A=abc -af^2-ch^2+hgf +hgf-bg^2

det A=abc+2hgf -af^2-bg^2 - ch^2

32. Find the determinant of $\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}$ where 1,ω,ω^2 are cube roots of unity.

Sol:A=$\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}$

$C_1+C_2+C_3=\begin{bmatrix}1+\omega+\omega^2 & \omega & \omega^2\\ 1+\omega+\omega^2 & \omega^2 & 1\\ 1+\omega+\omega^2 & 1 & \omega\end{bmatrix}=\begin{bmatrix}0 & \omega & \omega^2\\ 0 & \omega^2 & 1\\ 0 & 1 & \omega\end{bmatrix}$

det A=0

33. Find x of $\begin{vmatrix}\mathbf{1} & \mathbf{0} & \mathbf{0}\\ \mathbf{2} & \mathbf{3} & \mathbf{4}\\ \mathbf{5} & \mathbf{-6} & \mathbf{x}\end{vmatrix}$**=45**

Sol: A=$\begin{vmatrix}1 & 0 & 0\\ 2 & 3 & 4\\ 5 & -6 & x\end{vmatrix}$=45

1[3x-(-6x4)]=45

3x+24=45

3x=45-24=21

$\therefore x=\frac{21}{3}=7$

34. Find $\begin{vmatrix}\mathbf{1} & \mathbf{-1} & \mathbf{2}\\ \mathbf{3} & \mathbf{0} & \mathbf{4}\\ \mathbf{-4} & \mathbf{-2} & \mathbf{5}\end{vmatrix}$

Sol: A=$\begin{vmatrix}1 & -1 & 2\\ 3 & 0 & 4\\ -4 & -2 & 5\end{vmatrix}$

det A=1[0x5-(-2x4)]-(-1)[3x5-(-4x4)]+2[3x-2-(-4x0)]

det A=1[0+8]+1[15+16]+2[-6+0]=8+31-12=27

35. Prove that $\begin{vmatrix}\mathbf{a-b} & \mathbf{b-c} & \mathbf{c-a}\\ \mathbf{b-c} & \mathbf{c-a} & \mathbf{a-b}\\ \mathbf{c-a} & \mathbf{a-b} & \mathbf{b-c}\end{vmatrix}$**=0**

Sol: $\begin{vmatrix}a-b & b-c & c-a\\ b-c & c-a & a-b\\ c-a & a-b & b-c\end{vmatrix}$

$R_1\to R_1+(R_2+R_3)=\begin{vmatrix}0 & 0 & 0\\ b-c & c-a & a-b\\ c-a & a-b & b-c\end{vmatrix}=0$

36. Prove that $\begin{vmatrix}\mathbf{b+c} & \mathbf{c+a} & \mathbf{a+b}\\ \mathbf{c+a} & \mathbf{a+b} & \mathbf{b+c}\\ \mathbf{a+b} & \mathbf{b+c} & \mathbf{c+a}\end{vmatrix}=\mathbf{2}\begin{vmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\\ \mathbf{b} & \mathbf{c} & \mathbf{a}\\ \mathbf{c} & \mathbf{a} & \mathbf{b}\end{vmatrix}$

Sol: LHS=$\begin{vmatrix}b+c & c+a & a+b\\ c+a & a+b & b+c\\ a+b & b+c & c+a\end{vmatrix}$

$R_1\to R_1+R_2+R_3=\begin{vmatrix}2(a+b+c) & 2(a+b+c) & 2(a+b+c)\\ c+a & a+b & b+c\\ a+b & b+c & c+a\end{vmatrix}$

$=2\begin{vmatrix}(a+b+c) & (a+b+c) & (a+b+c)\\ c+a & a+b & b+c\\ a+b & b+c & c+a\end{vmatrix}$

$R_2= R_2 - R_1;\ R_3= R_3 - R_1$

$=2\begin{vmatrix}(a+b+c) & (a+b+c) & (a+b+c)\\ -b & -c & -a\\ -c & -a & -b\end{vmatrix}$

$R_1\to R_1+(R_2+R_3)=2\begin{vmatrix}a & b & c\\ -b & -c & -a\\ -c & -a & -b\end{vmatrix}=2(-1)(-1)\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=2\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$=RHS

∴LHS=RHS

37. Prove that $\begin{vmatrix}\mathbf{b+c} & \mathbf{c+a} & \mathbf{a+b}\\ \mathbf{a+b} & \mathbf{b+c} & \mathbf{c+a}\\ \mathbf{a} & \mathbf{b} & \mathbf{c}\end{vmatrix}=\mathbf{a^3+b^3+c^3-3abc}$

Sol: LHS=$\begin{vmatrix}b+c & c+a & a+b\\ a+b & b+c & c+a\\ a & b & c\end{vmatrix}$

$R_1\to R_1+R_3=\begin{vmatrix}a+b+c & a+b+c & a+b+c\\ a+b & b+c & c+a\\ a & b & c\end{vmatrix}$

$R_2\to R_2-R_1=\begin{vmatrix}a+b+c & a+b+c & a+b+c\\ -c & -a & -b\\ a & b & c\end{vmatrix}=$

$(a+b+c)\begin{vmatrix}1 & 1 & 1\\ -c & -a & -b\\ a & b & c\end{vmatrix}$

$= (a+b+c)[1(-ac+b^2)-1(-c^2+ab)+1(-bc+a^2)]$

$= (a+b+c)[-ac+b^2+c^2-ab-bc+a^2]$

$= (a+b+c)[a^2+b^2+c^2-ab-bc-ca$

$=a^3+ab^2+ac^2-a^2b-abc-a^2c+a^2b+b^3+bc^2-ab^2-b^2c-abc+a^2c+b^2c+c^3-abc-b^2c-ac^2$

$]= a^3+b^3+c^3-3abc$

∴LHS=RHS

38. Prove that

$\begin{vmatrix}\mathbf{a-b-c} & \mathbf{2a} & \mathbf{2a}\\ \mathbf{2b} & \mathbf{b-c-a} & \mathbf{2b}\\ \mathbf{2c} & \mathbf{2c} & \mathbf{c-a-b}\end{vmatrix}=\mathbf{(a+b+c)^3}$

Sol: LHS=$\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$

$R_1\to R_1+R_2+R_3=\begin{vmatrix}a+b+c & a+b+c & a+b+c\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$

$=(a+b+c)\begin{vmatrix}1 & 1 & 1\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}$

$C_2\to C_2-C_1;\ C_3\to C_3-C_1$

$=\begin{vmatrix}1 & 0 & 0\\ 2b & -(b+c+a) & 0\\ 2c & 0 & -(a+b+c)\end{vmatrix}$

$=\begin{vmatrix}1 & 0 & 0\\ 2b & -(a+b+c) & 0\\ 2c & 0 & -(a+b+c)\end{vmatrix}$

$=(a+b+c)\{1[-(a+b+c)x[-(a+b+c)]\}$

$=(a+b+c)^3$

∴LHS=RHS

39. Prove that

$\begin{vmatrix}\mathbf{a+b+2c} & \mathbf{a} & \mathbf{b}\\ \mathbf{c} & \mathbf{b+c+2a} & \mathbf{b}\\ \mathbf{c} & \mathbf{a} & \mathbf{c+a+2b}\end{vmatrix}=\mathbf{2(a+b+c)^3}$

Sol: LHS=$\begin{vmatrix}a+b+2c & a & b\\ c & b+c+2a & b\\ c & a & c+a+2b\end{vmatrix}$

$C_1\to C_1+C_2+C_2=\begin{vmatrix}2a+2b+2c & a & b\\ 2a+2b+2c & b+c+2a & b\\ 2a+2b+2c & a & c+a+2b\end{vmatrix}$

$=2(a+b+c)\begin{vmatrix}1 & a & b\\ 1 & b+c+2a & b\\ 1 & a & c+a+2b\end{vmatrix}$

$R_2\to R_2-R_1;\ R_3\to R_3-R_1$

$=2(a+b+c)\begin{vmatrix}1 & a & b\\ 0 & a+b+c & 0\\ 0 & 0 & a+b+c\end{vmatrix}$

$=2(a+b+c)[1(a+b+c)(a+b+c)]=2(a+b+c)^3$=RHS

∴LHS=RHS

40. Prove that $\begin{vmatrix}\mathbf{1} & \mathbf{a} & \mathbf{a^2}\\ \mathbf{1} & \mathbf{b} & \mathbf{b^2}\\ \mathbf{1} & \mathbf{c} & \mathbf{c^2}\end{vmatrix}$**=(a-b)(b-c)(c-a)**

Sol: LHS=$\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$

$R_2\to R_2-R_1;\ R_3\to R_3-R_1$

$=\begin{vmatrix}1 & a & a^2\\ 0 & b-a & b^2-a^2\\ 0 & c-a & c^2-a^2\end{vmatrix}=\begin{vmatrix}1 & a & a^2\\ 0 & b-a & (b-a)(b+a)\\ 0 & c-a & (c-a)(c+a)\end{vmatrix}$

$=(b-a)(c-a)\begin{vmatrix}1 & a & a^2\\ 0 & 1 & (b+a)\\ 0 & 1 & (c+a)\end{vmatrix}$

=(b-a)(c-a){1[(c+a)-(b+a)]}=(b-a)(c-a)(c-b)

=(a-b)(b-c)(c-a)=RHS

∴LHS=RHS

41. Prove that $\begin{vmatrix}1 & a^2 & a^3\\ 1 & b^2 & b^3\\ 1 & c^2 & c^3\end{vmatrix}$=(a-b)(b-c)(c-a)(ab+bc+ca)

Sol: LHS=$\begin{vmatrix}1 & a^2 & a^3\\ 1 & b^2 & b^3\\ 1 & c^2 & c^3\end{vmatrix}$

$R_2\to R_2-R_1;\ R_3\to R_3-R_1$

$=\begin{vmatrix}1 & a^2 & a^3\\ 0 & b^2-a^2 & b^3-a^3\\ 0 & c^2-a^2 & c^3-a^3\end{vmatrix}=$

$\begin{vmatrix}1 & a^2 & a^3\\ 0 & (b-a)(b+a) & (b-a)(b^2+ab+a^2)\\ 0 & (c-a)(c+a) & (c-a)(c^2+ca+a^2)\end{vmatrix}$

$=(b-a)(c-a)\begin{vmatrix}1 & a^2 & a^3\\ 0 & (a+b) & (a^2+ab+b^2)\\ 0 & (c+a) & (c^2+ca+a^2)\end{vmatrix}$

$R_3\to R_3-R_2$

$=(b-a)(c-a)\begin{vmatrix}1 & a^2 & a^3\\ 0 & (a+b) & (a^2+ab+b^2)\\ 0 & (c-b) & (ac-ab+c^2-b^2)\end{vmatrix}$

$=(b-a)(c-a)\begin{vmatrix}1 & a^2 & a^3\\ 0 & (a+b) & (a^2+ab+b^2)\\ 0 & (c-b) & (c-b)(a+b+c)\end{vmatrix}$

$=(b-a)(c-a)(c-b)\begin{vmatrix}1 & a^2 & a^3\\ 0 & (a+b) & (a^2+ab+b^2)\\ 0 & 1 & (a+b+c)\end{vmatrix}$

=(b-a)(c-a)(c-b){1[(a+b)(a+b+c)-1(a^2+ab+b^2)]}

=(b-a)(c-a)(c-b)[a^2+ab+ac+ab+b^2+bc-a^2-ab-b^2]

=(a-b)(b-c)(c-a)(ab+bc+ca)=RHS

∴LHS=RHS

42. Show that $\begin{vmatrix}a & b & c\\ a^2 & b^2 & c^2\\ a^3 & b^3 & c^3\end{vmatrix}$ = abc(a-b)(b-c)(c-a)

Sol: LHS=$\begin{vmatrix}a & b & c\\ a^2 & b^2 & c^2\\ a^3 & b^3 & c^3\end{vmatrix}=abc\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}$

$C_2\to C_2-C_1;\ C_3\to C_3-C_1$

$=abc\begin{vmatrix}1 & 0 & 0\\ a & b-a & c-a\\ a^2 & b^2-a^2 & c^2-a^2\end{vmatrix}$

$=abc\begin{vmatrix}1 & 0 & 0\\ a & b-a & c-a\\ a^2 & (b-a)(b+a) & (c-a)(c+a)\end{vmatrix}$

$=(abc)(b-a)(c-a)\begin{vmatrix}1 & 0 & 0\\ a & 1 & 1\\ a^2 & b+a & c+a\end{vmatrix}$

=(abc)(b-a)(c-a)[1[(c+a)-(b+a)]]

=(abc)(b-a)(c-a)(c-b)

=(abc)(a-b)(b-c)(c-a)=RHS

∴LHS=RHS

43. Show that $\begin{vmatrix}1 & a & a^2-bc\\ 1 & b & b^2-ca\\ 1 & c & c^2-ab\end{vmatrix}$ = 0

Sol: LHS=$\begin{vmatrix}1 & a & a^2-bc\\ 1 & b & b^2-ca\\ 1 & c & c^2-ab\end{vmatrix}$

$R_2\to R_2-R_1;\ R_3\to R_3-R_1$

$=\begin{vmatrix}1 & a & a^2-bc\\ 1 & b-a & b^2-a^2+bc-ca\\ 1 & c-a & c^2-a^2-ab+bc\end{vmatrix}$

$=\begin{vmatrix}1 & a & a^2-bc\\ 1 & b-a & (b-a)(b+a)+c(b-a)\\ 1 & c-a & (c-a)(c+a)+b(c-a)\end{vmatrix}$

$=\begin{vmatrix}1 & a & a^2-bc\\ 1 & b-a & (b-a)(a+b+c)\\ 1 & c-a & (c-a)(a+b+c)\end{vmatrix}$

$=(b-a)(c-a)\begin{vmatrix}1 & a & a^2-bc\\ 1 & 1 & a+b+c\\ 1 & 1 & a+b+c\end{vmatrix}$

=(b-a)(c-a)(0)=0 =RHS R_2, R_3 same

∴LHS=RHS

44. Show that $\begin{vmatrix}ax & by & cz\\ x^2 & y^2 & z^2\\ 1 & 1 & 1\end{vmatrix}=\begin{vmatrix}a & b & c\\ x & y & z\\ yz & zx & xy\end{vmatrix}$ without expanding the matrix

Sol: LHS= $\begin{vmatrix}ax & by & cz\\ x^2 & y^2 & z^2\\ 1 & 1 & 1\end{vmatrix}=\begin{vmatrix}ax & by & cz\\ x^2 & y^2 & z^2\\ \frac{x}{x} & \frac{y}{y} & \frac{z}{z}\end{vmatrix}$

$=xyz\begin{vmatrix}a & b & c\\ x & y & z\\ \frac{1}{x} & \frac{1}{y} & \frac{1}{z}\end{vmatrix}=\begin{vmatrix}a & b & c\\ x & y & z\\ \frac{xyz}{x} & \frac{xyz}{y} & \frac{xyz}{z}\end{vmatrix}=\begin{vmatrix}a & b & c\\ x & y & z\\ yz & zx & xy\end{vmatrix}$=RHS

∴LHS=RHS

45. If A=$\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ then find A^{-1}

Sol: A= $\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

det A= $(\cos\alpha)(\cos\alpha)-(\sin\alpha)(-\sin\alpha)=\cos^2\alpha+\sin^2\alpha$

Adj A= $\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$A^{-1}=\frac{Adj\,A}{\det A}=\frac{1}{\cos^2\alpha+\sin^2\alpha}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

46. If A=$\begin{vmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{vmatrix}$ then find Adj (A)

Sol: Given A=$\begin{vmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{vmatrix}$

Det A=|A|=$1\begin{vmatrix}4 & 3\\ 3 & 4\end{vmatrix}-3\begin{vmatrix}1 & 3\\ 1 & 4\end{vmatrix}+3\begin{vmatrix}1 & 4\\ 1 & 3\end{vmatrix}$

=1(16-9)-3(4-3)+3(3-4)=7-3-3=1

Cofactors of elements of A are

$A_{11}=\begin{vmatrix}4 & 3\\ 3 & 4\end{vmatrix}$=16-9=7; $A_{12}=-\begin{vmatrix}1 & 3\\ 1 & 4\end{vmatrix}$=-(4-3) =-1;

$A_{13}=\begin{vmatrix}1 & 4\\ 1 & 3\end{vmatrix}$=3-4=-1 : $A_{21}=-\begin{vmatrix}3 & 3\\ 3 & 4\end{vmatrix}$= - (12-9)=-3

;$A_{22}=\begin{vmatrix}1 & 3\\ 1 & 4\end{vmatrix}$= (4-3) =1; $A_{23}=-\begin{vmatrix}1 & 3\\ 1 & 3\end{vmatrix}$=-(3-3)= 0

$A_{31}=\begin{vmatrix}3 & 3\\ 4 & 3\end{vmatrix}$= (9-12)=-3 ; $A_{32}=-\begin{vmatrix}1 & 3\\ 1 & 3\end{vmatrix}$=- (3-3) =0;

$A_{33}=\begin{vmatrix}1 & 3\\ 1 & 4\end{vmatrix}$= (4-3)= 1

∴If A= $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$

∴Adj A= $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

∴ $A^{-1}=\frac{\text{Adj A}}{\det A}=\frac{1}{\det A}$ Adj A $=\frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

47. If A= $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ then show that Adj A=3A^T

Sol: A= $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$; $A^T=\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$

Cofactors of elements of A are

$A_{11}=\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix}$=1-4=-3; $A_{12}=-\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix}$=-(2+4) =-6;

$A_{13}=\begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix}$=-4-2=-6 ; $A_{21}=-\begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix}$= - (-2-4)=6 ;

$A_{22}=\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix}$= (-1+4) =3; $A_{23}=-\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix}$=-(2+4)= -6

$A_{31}=\begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix}$= (4+2)=6 ; $A_{32}=-\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix}$=- (2+4) =-6;

$A_{33}=\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix}$= (-1+4)= 3

∴If A= $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}$

∴Adj A= $\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}=3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}=3A^T$

48. If A= $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then find $A^3=A^{-1}$

Sol: A= $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$A^2=\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$A^2=$

$$\begin{bmatrix} 3x3+(-3x2)+4x0 & 3x-3+(-3x-3)+4x-1 & 3x4+(-3x4)+4x1 \\ 2x3+(-3x2)+4x0 & 2x-3+(-3x-3)+4x-1 & 2x4+(-3x4)+4x1 \\ 0x3+(-1x2)+1x0 & 0x-3+(-1x-3)+1x-1 & 0x4+(-1x4)+1x1 \end{bmatrix}$$

$$=\begin{bmatrix} 9-6+0 & -9+9-4 & 12-12+4 \\ 6-6+0 & -6+9-4 & 8-12+4 \\ 0-2+0 & 0+3-1 & 0-4+1 \end{bmatrix}=\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$A^4=\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$A^4=$

$$\begin{bmatrix} 3x3+(-4x0)+(4x-2) & 3x-4+(-4x-1)+4x2 & 3x4+(-4x0)+4x-3 \\ 0x3+(-1x0)+(0x-2) & 0x-4+(-1x-1)+0x2 & 0x4+(-1x0)+0x-3 \\ -2x3+(2x0)+(-3x-2) & -2x-4+(2x-1)+(-3x2) & -2x4+(2x0)+(-3x-3) \end{bmatrix}$$

$$=\begin{bmatrix} 9+0-8 & -12+4+8 & 12+0-12 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ -6+0+6 & 8-2-6 & -8+0+9 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I$$

A^4=I

A. A^3=I⇒ $A^{-1}=A^3$

49. If A= $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ find $(A^T)^{-1}$

Sol: A= $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$; $A^T=\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

det A^T=1[-1-8]-0[-2-6]-2[-8+3]=-9+0+10=1

Cofactors of elements of A are

$A_{11}=\begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix}$=-1-8=-9; $A_{12}=-\begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix}$=-(-2-6) =8;

$A_{13}=\begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix}$=-8+3=-5

$A_{21}=-\begin{vmatrix} 0 & -2 \\ 4 & 1 \end{vmatrix}$= - (0+8)=-8 ; $A_{22}=\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix}$= (1+6) =7; $A_{23}=-\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix}$=-(4-0)= -4

$A_{31}=\begin{vmatrix} 0 & -2 \\ -1 & 2 \end{vmatrix}$= (0-2)=-2 ; $A_{32}=-\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix}$=- (2-4) =2;

$A_{33}=\begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix}$= (-1+0)= -1

∴If A= $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$

∴Adj A= $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

$(A^T)^{-1}=\frac{\text{Adj A}}{\det A}=1\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}=\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

50. Solve the following system of equation using matrix invariant.

x-y+3z=5; 4x+2y+z=0; -x+3y+z=5

Sol: x-y+3z=5

4x+2y+z=0

-x+3y+z=5

Let A= $\begin{bmatrix} 1 & -1 & 3 \\ 4 & 2 & -1 \\ -1 & 3 & 1 \end{bmatrix}$; X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; B= $\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$

Given equation can be written a AX=B

Matrix inversion method X=A^{-1}B is the solution.

det A=1(2+3)-(-1)[4-1]+3(12+2)=5+3+42=50

Cofactors of elements of A are

$A_{11}=\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$=2+3=5; $A_{12}=-\begin{vmatrix} 4 & -1 \\ -1 & 1 \end{vmatrix}$=-(4-1) =-3;

$A_{13}=\begin{vmatrix} 4 & 2 \\ -1 & 3 \end{vmatrix}$=12+2=14; $A_{21}=-\begin{vmatrix} -1 & 3 \\ 3 & 1 \end{vmatrix}$= - (-1-9)=10 ;

$A_{22}=\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}$= (1+3) =4; $A_{23}=-\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}$=-(1+3)= -4

$A_{31}=\begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix}$= (1-6)=-5 ; $A_{32}=-\begin{vmatrix} 1 & 3 \\ 4 & -1 \end{vmatrix}$=- (-1-12) =13;

$A_{33}=\begin{vmatrix} 1 & -1 \\ 4 & 2 \end{vmatrix}$= (2+4)= 6

∴If A= $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 5 & -3 & 14 \\ 10 & 4 & -4 \\ -5 & 13 & 6 \end{bmatrix}$

∴Adj A= $\begin{bmatrix} 5 & 10 & -5 \\ -3 & 4 & 13 \\ 14 & -4 & 6 \end{bmatrix}$

$A^{-1}=\frac{\text{Adj A}}{\det A}=\frac{1}{50}\begin{bmatrix} 5 & 10 & -5 \\ -3 & 4 & 13 \\ 14 & -4 & 6 \end{bmatrix}$

By matrix inversion method

$X=A^{-1}B=\frac{1}{50}\begin{bmatrix} 5 & 10 & -5 \\ -3 & 4 & 13 \\ 14 & -4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$

$=\frac{1}{50}\begin{bmatrix}25+0-25\\-15+0+65\\70+0+30\end{bmatrix}=\frac{1}{50}\begin{bmatrix}0\\50\\100\end{bmatrix}=\begin{bmatrix}0\\1\\2\end{bmatrix}$

∴x=0; y=1; z=2

51. Solve the following system of equation using matrix invariant.

2x-y+3z=8; -x+2y+z=4; 3x+y-4z=0

Sol: 2x-y+3z=8
-x+2y+z=4
3x+y-4z=0

Let A=$\begin{bmatrix}2&-1&3\\-1&2&1\\3&1&-4\end{bmatrix}$; X=$\begin{bmatrix}x\\y\\z\end{bmatrix}$; B=$\begin{bmatrix}8\\4\\0\end{bmatrix}$

Given equation can be written a AX=B

Matrix inversion method X=A^{-1}B is the solution.

det A=2(-8-1)-(-1)[4-3]+3(-1-6)=-18+1-21=-38

Cofactors of elements of A are

$A_{11}=\begin{vmatrix}2&1\\1&-4\end{vmatrix}$=-8-1=-9; $A_{12}=-\begin{vmatrix}-1&1\\3&-4\end{vmatrix}$=-(4-3) =-1;

$A_{13}=\begin{vmatrix}-1&2\\3&1\end{vmatrix}$=-1-6=-7

$A_{21}=-\begin{vmatrix}-1&3\\1&-4\end{vmatrix}$= - (4-3)=-1 ;$A_{22}=\begin{vmatrix}2&3\\3&-4\end{vmatrix}$= (-8-9) =-17;

$A_{23}=-\begin{vmatrix}2&-1\\3&1\end{vmatrix}$=-(2+3)= -5

$A_{31}=\begin{vmatrix}-1&3\\2&1\end{vmatrix}$= (-1-6)=-7 ;$A_{32}=-\begin{vmatrix}2&3\\-1&1\end{vmatrix}$=- (2+3) =-5;

$A_{33}=\begin{vmatrix}2&-1\\-1&2\end{vmatrix}$= (4-1)= 3

∴If A=$\begin{bmatrix}A_{11}&A_{12}&A_{13}\\A_{21}&A_{22}&A_{23}\\A_{31}&A_{32}&A_{33}\end{bmatrix}=\begin{bmatrix}-9&-1&-7\\-1&-17&-5\\-7&-5&3\end{bmatrix}$

∴Adj A=$\begin{bmatrix}-9&-1&-7\\-1&-17&-5\\-7&-5&3\end{bmatrix}$

$A^{-1}=\frac{\text{Adj A}}{\det A}=\frac{1}{-38}\begin{bmatrix}-9&-1&-7\\-1&-17&-5\\-7&-5&3\end{bmatrix}$

By matrix inversion method

X=A^{-1}B=$\frac{1}{-38}\begin{bmatrix}-9&-1&-7\\-1&-17&-5\\-7&-5&3\end{bmatrix}\begin{bmatrix}8\\4\\0\end{bmatrix}$

$=\frac{1}{-38}\begin{bmatrix}-72-4+0\\-8-68+0\\-56-20+0\end{bmatrix}=\frac{-1}{38}\begin{bmatrix}-76\\-76\\-76\end{bmatrix}=\begin{bmatrix}2\\2\\2\end{bmatrix}$

∴x=2; y=2; z=2

52. Solve the following system of equation using matrix invariant.

3x+4y+5z=18; 2x-y+8z=13; 5x-2y+7z=20

Sol: 3x+4y+5z=18
2x- y+ 8z=13
5x-2y+ 7z=20

Let A=$\begin{bmatrix}3&4&5\\2&-1&8\\5&-2&7\end{bmatrix}$; X=$\begin{bmatrix}x\\y\\z\end{bmatrix}$; B=$\begin{bmatrix}18\\13\\20\end{bmatrix}$

Given equation can be written a AX=B

Matrix inversion method X=A^{-1}B is the solution.

det A=3(-7+16)-4[14-40]+5(-4+5)=27+104+5=136

Cofactors of elements of A are

$A_{11}=\begin{vmatrix}-1&8\\-2&7\end{vmatrix}$=-7+16=9; $A_{12}=-\begin{vmatrix}2&8\\5&7\end{vmatrix}$=-(14-40) =26;

$A_{13}=\begin{vmatrix}2&-1\\5&-2\end{vmatrix}$=-4+5=1

$A_{21}=-\begin{vmatrix}4&5\\-2&7\end{vmatrix}$= - (28+10)=-38 ;$A_{22}=\begin{vmatrix}3&5\\5&7\end{vmatrix}$= (21-25) =-4;

$A_{23}=-\begin{vmatrix}3&4\\5&-2\end{vmatrix}$=-(-6-20)= 26

$A_{31}=\begin{vmatrix}4&5\\-1&8\end{vmatrix}$= (32+5)=37 ;$A_{32}=-\begin{vmatrix}3&2\\5&8\end{vmatrix}$=- (24-10) =-14;

$A_{33}=\begin{vmatrix}3&4\\2&-1\end{vmatrix}$= (-3-8)= -11

∴If A=$\begin{bmatrix}A_{11}&A_{12}&A_{13}\\A_{21}&A_{22}&A_{23}\\A_{31}&A_{32}&A_{33}\end{bmatrix}=\begin{bmatrix}9&26&1\\-38&-4&26\\37&-14&-11\end{bmatrix}$

∴Adj A=$\begin{bmatrix}9&-38&37\\26&-4&-14\\1&26&-11\end{bmatrix}$

$A^{-1}=\frac{\text{Adj A}}{\det A}=\frac{1}{136}\begin{bmatrix}9&-38&37\\26&-4&-14\\1&26&-11\end{bmatrix}$

By matrix inversion method

X=A^{-1}B=$\frac{1}{136}\begin{bmatrix}9&-38&37\\26&-4&-14\\1&26&-11\end{bmatrix}\begin{bmatrix}18\\13\\20\end{bmatrix}$

$=\frac{1}{136}\begin{bmatrix}9x18-38x13+37x20\\26x18-4x13-14x20\\1x18+26x13-11x20\end{bmatrix}$

$=\frac{1}{136}\begin{bmatrix}162-494+740\\468-52-280\\18+338-220\end{bmatrix}=\frac{1}{136}\begin{bmatrix}408\\136\\136\end{bmatrix}=\begin{bmatrix}3\\1\\1\end{bmatrix}$

∴x=3; y=1; z=1

53. Solve the following system of equation using Cramer's Rule.

x-y+3z=5; 4x+2y-z=0; -x+3y+z=5

Sol: x-y+3z=5
4x+2y+z=0
-x+3y+z=5

Let A=$\begin{bmatrix}1&-1&3\\4&2&-1\\-1&3&1\end{bmatrix}$; X=$\begin{bmatrix}x\\y\\z\end{bmatrix}$; B=$\begin{bmatrix}5\\0\\5\end{bmatrix}$

Now |A|=Δ=$\begin{vmatrix}1&-1&3\\4&2&-1\\-1&3&1\end{vmatrix}$

det A=1(2+3)-(-1)[4-1]+3(12+2)=5+3+42=50

$\Delta_1=\begin{vmatrix}5&-1&3\\0&2&-1\\5&3&1\end{vmatrix}$

= 5(2+3)-(-1)(0+5)+3(0-10)=25+5-30=0

$\Delta_2=\begin{vmatrix}1&5&3\\4&0&-1\\-1&5&1\end{vmatrix}$

=1(0+5)-5(4-1)+3(20+0)=5-15+60=50

$\Delta_3=\begin{vmatrix}1&-1&5\\4&2&0\\-1&3&5\end{vmatrix}$

=1(10-0)-(-1)(20-0)+5(12+2)=10+20+70=100

By Cramer's Rule

x=$\frac{\Delta_1}{\Delta}=\frac{0}{50}$=0: y=$\frac{\Delta_2}{\Delta}=\frac{50}{50}$=1; z=$\frac{\Delta_3}{\Delta}=\frac{100}{50}$=2

∴x=0; y=1; z=2

54. Solve the following system of equation using Cramer's Rule.

2x-y+3z=9; x+y+z=6; x-y+z=2

Sol: 2x-y+3z=9

x+y+z=6

x-y+z=2

Let A=$\begin{bmatrix}2 & -1 & 3\\1 & 1 & 1\\1 & -1 & 1\end{bmatrix}$; X=$\begin{bmatrix}x\\y\\z\end{bmatrix}$; B=$\begin{bmatrix}9\\6\\2\end{bmatrix}$

Now $|A|=\Delta=\begin{vmatrix}2 & -1 & 3\\1 & 1 & 1\\1 & -1 & 1\end{vmatrix}$

det A=2(1+1)-(-1)[1-1]+3(-1-1)=4+0-6=-2

$\Delta_1=\begin{vmatrix}9 & -1 & 3\\6 & 1 & 1\\2 & -1 & 1\end{vmatrix}$

= 9(1+1)-(-1)(6-2)+3(-6-2)=18+4-24=-2

$\Delta_2=\begin{vmatrix}2 & 9 & 3\\1 & 6 & 1\\1 & 2 & 1\end{vmatrix}$

=2(6-2)-9(1-1)+3(2-6)=8-0-12=-4

$\Delta_3=\begin{vmatrix}2 & -1 & 9\\1 & 1 & 6\\1 & -1 & 2\end{vmatrix}$

=2(2+6)-(-1)(2-6)+9(-1-1)=16-4-18=-6

By Cramer's Rule

$x=\frac{\Delta_1}{\Delta}=\frac{-2}{-2}=1$: $y=\frac{\Delta_2}{\Delta}=\frac{-4}{-2}=2$; $z=\frac{\Delta_3}{\Delta}=\frac{-6}{-2}=3$

∴x=1; y=2; z=3

55.Solve the following system of equation using Cramer's Rule.

2x-y+3z=8; -x+2y+z=4; 3x+y-4z=0

Sol: 2x-y+3z=8

-x+2y+z=4

3x+y-4z=0

Let A=$\begin{bmatrix}2 & -1 & 3\\-1 & 2 & 1\\3 & 1 & -4\end{bmatrix}$; X=$\begin{bmatrix}x\\y\\z\end{bmatrix}$; B=$\begin{bmatrix}8\\4\\0\end{bmatrix}$

Now $|A|=\Delta=\begin{vmatrix}2 & -1 & 3\\-1 & 2 & 1\\3 & 1 & -4\end{vmatrix}$

det A=2(-8-1)-(-1)[4-3]+3(-1-6)=-18+1-21=-38

$\Delta_1=\begin{vmatrix}8 & -1 & 3\\4 & 2 & 1\\0 & 1 & -4\end{vmatrix}$

= 8(-8-1)-(-1)(-16-0)+3(4-0)=-72-16+12=-76

$\Delta_2=\begin{vmatrix}2 & 8 & 3\\-1 & 4 & 1\\3 & 0 & -4\end{vmatrix}$

=2(-16-0)-8(4-3)+3(0-12)=-32-8-36=-76

$\Delta_3=\begin{vmatrix}2 & -1 & 8\\-1 & 2 & 4\\3 & 1 & 0\end{vmatrix}$

=2(0-4)-(-1)(0-12)+8(-1-6)=-8-12-56=-76

By Cramer's Rule

$x=\frac{\Delta_1}{\Delta}=\frac{-76}{-38}=2$: $y=\frac{\Delta_2}{\Delta}=\frac{-76}{-38}=2$; $z=\frac{\Delta_3}{\Delta}=\frac{-76}{-38}=2$

∴x=2; y=2; z=2

%%%

Exercise problems

1. If A=$\begin{bmatrix}2 & 3 & -1\\7 & 8 & 5\end{bmatrix}$ and B=$\begin{bmatrix}1 & 0 & 1\\2 & -4 & -1\end{bmatrix}$ then find A+B

Sol: A=$\begin{bmatrix}2 & 3 & -1\\7 & 8 & 5\end{bmatrix}$ and B=$\begin{bmatrix}1 & 0 & 1\\2 & -4 & -1\end{bmatrix}$

A+B=$\begin{bmatrix}2 & 3 & -1\\7 & 8 & 5\end{bmatrix}+\begin{bmatrix}1 & 0 & 1\\2 & -4 & -1\end{bmatrix}$

A+B=$\begin{bmatrix}2+1 & 3+0 & -1+1\\7+2 & 8-4 & 5-1\end{bmatrix}=\begin{bmatrix}3 & 3 & 0\\9 & 4 & 4\end{bmatrix}$

2. If A=$\begin{bmatrix}-1 & 3\\4 & 2\end{bmatrix}$, B=$\begin{bmatrix}2 & 1\\3 & -5\end{bmatrix}$ and A+B-X=[0] Then find the matrix X.

Sol: A=$\begin{bmatrix}-1 & 3\\4 & 2\end{bmatrix}$, B=$\begin{bmatrix}2 & 1\\3 & -5\end{bmatrix}$, X=$\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}$

A+B-X=$\begin{bmatrix}-1 & 3\\4 & 2\end{bmatrix}+\begin{bmatrix}2 & 1\\3 & -5\end{bmatrix}-\begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}$=0

A+B-X=$\begin{bmatrix}-1+2-a_{11} & 3+1-a_{12}\\4+3-a_{21} & 2-5-a_{22}\end{bmatrix}$=0

A+B-X=$\begin{bmatrix}1-a_{11} & 4-a_{12}\\7-a_{21} & -3-a_{22}\end{bmatrix}$=0

⇒Matrix X=$\begin{bmatrix}1 & 4\\7 & -3\end{bmatrix}$

3. If A=$\begin{bmatrix}3 & 2 & -1\\2 & -2 & 0\\1 & 3 & 1\end{bmatrix}$, B=$\begin{bmatrix}-3 & -1 & 0\\2 & 1 & 3\\4 & -1 & 2\end{bmatrix}$ and X=A+B Then find the matrix X.

Sol: A=$\begin{bmatrix}3 & 2 & -1\\2 & -2 & 0\\1 & 3 & 1\end{bmatrix}$, B=$\begin{bmatrix}-3 & -1 & 0\\2 & 1 & 3\\4 & -1 & 2\end{bmatrix}$,

X=$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix}$

A+B-X=$\begin{bmatrix}3 & 2 & -1\\2 & -2 & 0\\1 & 3 & 1\end{bmatrix}+\begin{bmatrix}-3 & -1 & 0\\2 & 1 & 3\\4 & -1 & 2\end{bmatrix}-\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix}$=0

A+B-X=$\begin{bmatrix}3-3-a_{11} & 2-1-a_{12} & -1+0-a_{13}\\2+2-a_{21} & -2+1-a_{22} & 0+3-a_{23}\\1+4-a_{31} & 3-1-a_{32} & 1+2-a_{33}\end{bmatrix}$=0

A+B-X=$\begin{bmatrix}0-a_{11} & 1-a_{12} & -1-a_{13}\\4-a_{21} & -1-a_{22} & 3-a_{23}\\5-a_{31} & 2-a_{32} & 3-a_{33}\end{bmatrix}$=0

⇒Matrix X=$\begin{bmatrix}0 & 1 & -1\\4 & -1 & 3\\5 & 2 & 3\end{bmatrix}$

4. If $\begin{bmatrix}x-1 & 2 & y-5\\z & 0 & 0\\1 & -1 & 1+w\end{bmatrix}=\begin{bmatrix}1-x & 2 & -y\\2 & 0 & 2\\1 & -1 & 1\end{bmatrix}$ then find the values of x,y,z,w.

Sol: $\begin{bmatrix}x-1 & 2 & y-5\\z & 0 & 0\\1 & -1 & 1+w\end{bmatrix}=\begin{bmatrix}1-x & 2 & -y\\2 & 0 & 2\\1 & -1 & 1\end{bmatrix}$

Equating the corresponding elements

x-1=1-x ⇒2x=2⇒x=1

y-5=-y ⇒2y=5⇒y=$\frac{5}{2}$

z=2⇒z=2

1+w=1⇒w=0

∴x=1; y=$\frac{5}{2}$;z=2,w=0

5. If $\begin{bmatrix}x-1 & 2 & 5-y\\0 & z-1 & 7\\1 & 0 & w-5\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\0 & 4 & 7\\1 & 0 & 0\end{bmatrix}$ then find the values of x,y,z,w.

Sol: $\begin{bmatrix}x-1 & 2 & 5-y\\0 & z-1 & 7\\1 & 0 & w-5\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\0 & 4 & 7\\1 & 0 & 0\end{bmatrix}$

Equating the corresponding elements

x-1=1⇒x=2

5-y=3⇒y=2

z-1=4⇒z=5

w-5=0⇒w=5

∴x=2;y=2;z=5;w=5

6. Find the trace of the matrix $\begin{bmatrix} 1 & 2 & -\frac{1}{3} \\ 0 & -1 & 2 \\ -\frac{1}{2} & 2 & 1 \end{bmatrix}$

Sol: Given matrix $\begin{bmatrix} 1 & 2 & -\frac{1}{3} \\ 0 & -1 & 2 \\ -\frac{1}{2} & 2 & 1 \end{bmatrix}$

The trace of a square matrix is the sum of elements in the principal diagonal.

∴The trace of the given matrix is 1+(-1)+1=1

7.If A=$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$, B=$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$verify that $(A+B)^T = A^T + B^T$

Sol: Given A=$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$, B=$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

$A^T=\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$; $B^T=\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ -1 & 3 \end{bmatrix}$

A+B=$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}+\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}=\begin{bmatrix} 1+1 & 4+2 & 7-1 \\ 2+0 & 5-1 & 8+3 \end{bmatrix}$

$=\begin{bmatrix} 2 & 6 & 6 \\ 2 & 4 & 11 \end{bmatrix}$

LHS=$(A+B)^T=\begin{bmatrix} 2 & 2 \\ 6 & 4 \\ 6 & 11 \end{bmatrix}$

RHS=$A^T+B^T=\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}+\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ -1 & 3 \end{bmatrix}=\begin{bmatrix} 1+1 & 2+0 \\ 4+2 & 5-1 \\ 7-1 & 8+3 \end{bmatrix}=\begin{bmatrix} 2 & 2 \\ 6 & 4 \\ 6 & 11 \end{bmatrix}$

∴ $(A+B)^T = A^T + B^T$

8.If A=$\begin{bmatrix} -2 & 5 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$;B=$\begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$then find $2A+B^T$

Sol: Given A=$\begin{bmatrix} -2 & 5 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$;B=$\begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$

2A=$2\begin{bmatrix} -2 & 5 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}=\begin{bmatrix} -4 & 10 \\ 10 & 0 \\ -2 & 8 \end{bmatrix}$

$B^T=\begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$

∴$2A+B^T=\begin{bmatrix} -4 & 10 \\ 10 & 0 \\ -2 & 8 \end{bmatrix}+\begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} -4-2 & 10+4 \\ 10+3 & 0+0 \\ -2+1 & 8+2 \end{bmatrix}$

$=\begin{bmatrix} -6 & 14 \\ 13 & 0 \\ -1 & 10 \end{bmatrix}$

9. Find the determinant $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \\ -4 & -2 & 5 \end{bmatrix}$

Sol: A=$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \\ -4 & -2 & 5 \end{bmatrix}$

det A=1[(0x5-(-2)x4]-(-1)[(3x5)-(-4x4)+2[(3x-2)-(-4x0)]

=1[0+8]+[15+16]+2[-6-0]=8+31-12=27

10. Show that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$=(a-b)(b-c)(c-a)

Sol: LHS=$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$R_2 \to R_2-R_1$; $R_3 \to R_3-R_1$

$=\begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-bc \end{vmatrix}=\begin{vmatrix} 1 & a & bc \\ 0 & -(a-b) & c(a-b) \\ 0 & -(a-c) & b(a-c) \end{vmatrix}$

=(a-b)(a-c) $\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix}$

= (a-b)(c-a)[1[-b+c]=(a-b)(b-c)(c-a)=RHS

∴LHS=RHS

11. Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$=(a-b)(b-c)(c-a) **(mar19)**

Sol: LHS=$\begin{vmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{a^2} & \mathbf{b^2} & \mathbf{c^2} \end{vmatrix}$

$C_2 \to C_2-C_1$; $C_3 \to C_3-C_1$

$=\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ \mathbf{a^2} & b^2-a^2 & c^2-a^2 \end{vmatrix}$

$=\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ \mathbf{a^2} & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix}$

=(b-a)(c-a) $\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ \mathbf{a^2} & (b+a) & (c+a) \end{vmatrix}$

=(b-a)(c-a){1[(c+a)-(b+a)]}=(b-a)(c-a)(c-b)

=(a-b)(b-c)(c-a)=RHS

∴LHS=RHS

12. Find the inverse of $\begin{bmatrix} 2 & 1 \\ 1 & -5 \end{bmatrix}$

Sol: Given A=$\begin{bmatrix} 2 & 1 \\ 1 & -5 \end{bmatrix}$; det A=2(-5)-1(1)=-10-1=-11

Adj A=$\begin{bmatrix} -5 & -1 \\ -1 & 2 \end{bmatrix}$

∴ $A^{-1}=\frac{adj\,A}{\det A}=\frac{1}{-11}\begin{bmatrix} -5 & -1 \\ -1 & 2 \end{bmatrix}$

13. Find the inverse of $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

Sol: Given A=$\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

det A=2(6)-4(-3)= 12+12=24

Adj A=$\begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}$

∴ $A^{-1}=\frac{adj\,A}{\det A}=\frac{1}{24}\begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}=\begin{bmatrix} \frac{1}{4} & \frac{1}{8} \\ \frac{-1}{6} & \frac{1}{12} \end{bmatrix}$

14. Find the inverse of$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

det A=1[(2x2)-(1x3)]-2[(3x2)-(1x3)]+1[(3x1)-1x2)]

= 1[4-3]-2[6-3]+1[3-2]=1-6+1=-4

Cofactors of elements of A are

$A_{11}=\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$= 4-3=1; $A_{12}=-\begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix}$=-(6-3) =-3;

$A_{13}=\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$= 3-2=1; $A_{21}=-\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$= - (4-1)=-3

$A_{22}=\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$= (2-1) =1; $A_{23}=-\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$=-(1-2)= 1

$A_{31}=\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}$= (6-2)=4 ;$A_{32}=-\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix}$=- (3-3) = 0;

$A_{33}=\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$= (2-6)= -4

If A=$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}=\begin{bmatrix} 1 & -3 & 1 \\ -3 & 1 & 1 \\ 4 & 0 & -4 \end{bmatrix}$

∴Adj A=$\begin{bmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$

$A^{-1}=\frac{\text{Adj A}}{\det A}=\frac{1}{-4}\begin{bmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$

15. Find the inverse of $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

det A=1[(1x1)-(2x0)]-0[(2x1)-(3x0)]+2[(2x2)-3x1)]

= 1[1-0]-0[2-0]+2[4-3]=1-0+2= 3

Cofactors of elements of A are

$A_{11}=\begin{vmatrix}1 & 0\\2 & 1\end{vmatrix}$= 1-0=1; $A_{12}=-\begin{vmatrix}2 & 0\\3 & 1\end{vmatrix}$=-(2-0) =-2;

$A_{13}=\begin{vmatrix}2 & 1\\3 & 2\end{vmatrix}$= 4-3=1; $A_{21}=-\begin{vmatrix}0 & 2\\2 & 1\end{vmatrix}$= - (0-4)= 4

$A_{22}=\begin{vmatrix}1 & 2\\3 & 1\end{vmatrix}$= (1-6) =-5; $A_{23}=-\begin{vmatrix}1 & 0\\3 & 2\end{vmatrix}$=-(2-0)= -2

$A_{31}=\begin{vmatrix}0 & 2\\1 & 0\end{vmatrix}$= (0-2)=-2 ;$A_{32}=-\begin{vmatrix}1 & 2\\2 & 0\end{vmatrix}$=- (0-4) = 4;

$A_{33}=\begin{vmatrix}1 & 0\\2 & 1\end{vmatrix}$= (1-0)= 1

If $A=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\A_{21} & A_{22} & A_{23}\\A_{31} & A_{32} & A_{33}\end{bmatrix}=\begin{bmatrix}1 & -2 & 1\\4 & -5 & -2\\-2 & 4 & 1\end{bmatrix}$

∴Adj $A=\begin{bmatrix}1 & 4 & -2\\-2 & -5 & 4\\1 & -2 & 1\end{bmatrix}$

$A^{-1}=\frac{\text{Adj }A}{\det A}=\frac{1}{3}\begin{bmatrix}1 & 4 & -2\\-2 & -5 & 4\\1 & -2 & 1\end{bmatrix}$

16. Solve the following system of equation using Cramer's Rule.

3x+4y+5z=18; 2x-y+8z=13; 5x-2y+7z=20

Sol: 3x+4y+5z=18

2x - y+8z=13

5x -2y+7z=20

Let $A=\begin{bmatrix}3 & 4 & 5\\2 & -1 & 8\\5 & -2 & 7\end{bmatrix}$; $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$; $B=\begin{bmatrix}18\\13\\20\end{bmatrix}$

Now $|A|=\Delta=\begin{vmatrix}3 & 4 & 5\\2 & -1 & 8\\5 & -2 & 7\end{vmatrix}$

det A=3(-7+16)- 4[14-40]+5(-4+5)= 27+104+5= 136

$\Delta_1=\begin{vmatrix}18 & 4 & 5\\13 & -1 & 8\\20 & -2 & 7\end{vmatrix}$

= 18(-7+16)-4(91-160)+5(-26+20)=162+276-30 = 408

$\Delta_2=\begin{vmatrix}3 & 18 & 5\\2 & 13 & 8\\5 & 20 & 7\end{vmatrix}$

=3(91-160)-18(14-40)+5(40-65)=-207+468-125 = 136

$\Delta_3=\begin{vmatrix}3 & 4 & 18\\2 & -1 & 13\\5 & -2 & 20\end{vmatrix}$

=3(-20+26)-4(40-65)+18(-4+5)=18+100+18= 136

By Cramer's Rule

$x=\frac{\Delta_1}{\Delta}=\frac{408}{136}$=3: $y=\frac{\Delta_2}{\Delta}=\frac{136}{136}$=1; $z=\frac{\Delta_3}{\Delta}=\frac{136}{136}$=1; ∴x=3; y=1; z=1

17. Solve the system following of equations using crammer's rule

x + y + z = 3, 2x + 2y − z = 3, x + y − z = 1.

Sol: x + y + z = 3,

2x + 2y − z = 3,

x + y − z = 1

Let $A=\begin{bmatrix}1 & 1 & 1\\2 & 2 & -1\\1 & 1 & -1\end{bmatrix}$; $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$; $B=\begin{bmatrix}3\\3\\1\end{bmatrix}$

Now $|A|=\Delta=\begin{vmatrix}1 & 1 & 1\\2 & 2 & -1\\1 & 1 & -1\end{vmatrix}$

det A=1(-2+1) -1[-2+1]+1(2-2)= -1+1+5= 136

$\Delta_1=\begin{vmatrix}3 & 1 & 1\\3 & 2 & -1\\1 & 1 & -1\end{vmatrix}$ = 3(-2+1)-1(-3+1)+(3-2)=-3+2+1

$\Delta_2=\begin{vmatrix}1 & 3 & 1\\2 & 3 & -1\\1 & 1 & -1\end{vmatrix}$ =1(-3+1)-3(-2+1)+1(2-3)= -2+3-1

$\Delta_3=\begin{vmatrix}1 & 1 & 3\\2 & 2 & 3\\1 & 1 & 1\end{vmatrix}$ =1(2-3)-1(2-3)+3(2-2)=-1+1+0=

By Cramer's Rule

$x=\frac{\Delta_1}{\Delta}$=—=3: $y=\frac{\Delta_2}{\Delta}$=—=1; $z=\frac{\Delta_3}{\Delta}$=—=1∴x=3; y=1; z=1

18.Solve 2x+4y-z=0; x+2y+2z=5; 3x+6y-7z=2 by matrix Inversion method.

Sol: 2x+4y - z=0;

x+2y+2z=5;

3x+6y-7z=2

Let $A=\begin{bmatrix}2 & 4 & -1\\1 & 2 & 2\\3 & 6 & -7\end{bmatrix}$; $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$; $B=\begin{bmatrix}0\\5\\2\end{bmatrix}$

Given equation can be written a AX=B

Matrix inversion method $X=A^{-1}B$ is the solution.

det A=2(-14-12)-4[-7-6]-1(6-6)=-52+52-0= 0

det A=0 the problem has no solution.

1.Solve 2x-y+3z=9; x+y+z=6; x-y+z=2 by matrix Inversion method.

Sol: 2x-y+3z=9

x+y+z=6

x-y+z=2

Let $A=\begin{bmatrix}2 & -1 & 3\\1 & 1 & 1\\1 & -1 & 1\end{bmatrix}$; $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$; $B=\begin{bmatrix}9\\6\\2\end{bmatrix}$

Given equation can be written a AX=B

Matrix inversion method $X=A^{-1}B$ is the solution.

det A=2(1+1)-(-1)[1-1]+3(-1-1)=4+0-6= -2

Cofactors of elements of A are

$A_{11}=\begin{vmatrix}1 & 1\\-1 & 1\end{vmatrix}$=1+1=2; $A_{12}=-\begin{vmatrix}1 & 1\\1 & 1\end{vmatrix}$=-(1-1) =0;

$A_{13}=\begin{vmatrix}1 & 1\\1 & -1\end{vmatrix}$=-1-1=-2; $A_{21}=-\begin{vmatrix}-1 & 3\\-1 & 1\end{vmatrix}$= - (-1+3)=-2 ;

$A_{22}=\begin{vmatrix}2 & 3\\1 & 1\end{vmatrix}$= (2-3) =-1; $A_{23}=-\begin{vmatrix}2 & -1\\1 & -1\end{vmatrix}$=-(-2+1)= 1

$A_{31}=\begin{vmatrix}-1 & 3\\1 & 1\end{vmatrix}$= (-1-3)=-4 ;$A_{32}=-\begin{vmatrix}2 & 3\\1 & 1\end{vmatrix}$=- (2-3) =1;

$A_{33}=\begin{vmatrix}2 & -1\\1 & 1\end{vmatrix}$= (2+1)= 3

∴If $A=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\A_{21} & A_{22} & A_{23}\\A_{31} & A_{32} & A_{33}\end{bmatrix}=\begin{bmatrix}2 & 0 & -2\\-2 & -1 & 1\\-4 & 1 & 3\end{bmatrix}$

∴Adj $A=\begin{bmatrix}2 & -2 & -4\\0 & -1 & 1\\-2 & 1 & 3\end{bmatrix}$

$A^{-1}=\frac{\text{Adj }A}{\det A}=\frac{1}{-2}\begin{bmatrix}2 & -2 & -4\\0 & -1 & 1\\-2 & 1 & 3\end{bmatrix}$

By matrix inversion method

$X=A^{-1}B=\frac{1}{-2}\begin{bmatrix}2 & -2 & -4\\0 & -1 & 1\\-2 & 1 & 3\end{bmatrix}\begin{bmatrix}9\\6\\2\end{bmatrix}$

$=\frac{1}{-2}\begin{bmatrix}18-12-8\\0-6+2\\-18+6+6\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}-2\\-4\\-6\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

∴x=1; y=2; z=3

4&5 VECTOR ALGEBRA

1. Let $\bar{a}=\bar{i}+2\bar{j}+3\bar{k}$ and $\bar{b}=3\bar{i}+\bar{j}$ find the unit vector in the direction of $\bar{a}+\bar{b}$

Sol: $\bar{a}=\bar{i}+2\bar{j}+3\bar{k}$ and $\bar{b}=3\bar{i}+\bar{j}$

$\bar{a}+\bar{b}=\bar{i}+2\bar{j}+3\bar{k}+3\bar{i}+\bar{j}=4\bar{i}+3\bar{j}+3\bar{k}$

$|\bar{a}+\bar{b}|=\sqrt{(4)^2+(3)^2+(3)^2}=\sqrt{16+9+9}=\sqrt{34}$

∴ Unit vector in the direction of $\bar{a}+\bar{b}$ is

$\frac{\bar{a}+\bar{b}}{|\bar{a}+\bar{b}|}=\frac{4\bar{i}+3\bar{j}+3\bar{k}}{\sqrt{34}}$

2. If the vectors $-3\bar{i}+4\bar{j}+\lambda\bar{k}$ and $\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear then find λ and μ.

Sol: $-3\bar{i}+4\bar{j}+\lambda\bar{k}$, $\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear

$\Rightarrow \mu\bar{i}+8\bar{j}+6\bar{k}=m(-3\bar{i}+4\bar{j}+\lambda\bar{k})$

$\mu=-3m$; $8=4m \Rightarrow m=\frac{8}{4}=2$; $6=\lambda m$

∴ $\mu=-3m=-3(2)=-6$; $6=\lambda m \Rightarrow \lambda=\frac{6}{m}=\frac{6}{2}=3$

∴ $\lambda=3$; $\mu=-6$

3. If the points whose position vectors are $3\bar{i}-2\bar{j}-\bar{k}$, $2\bar{i}+3\bar{j}-4\bar{k}$, $-\bar{i}+\bar{j}+2\bar{k}$ and $4\bar{i}+5\bar{j}+\lambda\bar{k}$ are coplanar then show that $\lambda=\frac{-146}{47}$.

Sol: If A,B,C,D are the given points respectively, then

$\overrightarrow{OA}=3\bar{i}-2\bar{j}-\bar{k}$

$\overrightarrow{OB}=2\bar{i}+3\bar{j}-4\bar{k}$

$\overrightarrow{OC}=-\bar{i}+\bar{j}+2\bar{k}$

$\overrightarrow{OD}=4\bar{i}+5\bar{j}+\lambda\bar{k}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(2\bar{i}+3\bar{j}-4\bar{k})-(3\bar{i}-2\bar{j}-\bar{k})=-\bar{i}+5\bar{j}-3\bar{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(-\bar{i}+\bar{j}+2\bar{k})-(3\bar{i}-2\bar{j}-\bar{k})=-4\bar{i}+3\bar{j}+3\bar{k}$

$\overrightarrow{AD}=\overrightarrow{OC}-\overrightarrow{OA}=(4\bar{i}+5\bar{j}+\lambda\bar{k})-(3\bar{i}-2\bar{j}-\bar{k})=\bar{i}+7\bar{j}+(\lambda+1)\bar{k}$

Given points are coplanar $[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}]=0$

$\Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & (\lambda+1) \end{vmatrix}=0$

$=-1[3((\lambda+1)-21]-5[-4(\lambda+1)-3]-3[-28-3]=0$

$=-3\lambda-3+21+20\lambda+20+15+84+9=0$

=17V+146=0

$\Rightarrow \lambda=\frac{-146}{17}$

4. If OA=$\bar{i}+\bar{j}+\bar{k}$, AB=$3\bar{i}-2\bar{j}+\bar{k}$, BC=$\bar{i}+2\bar{j}-2\bar{k}$ and CD=$2\bar{i}+\bar{j}+3\bar{k}$. Then find the vector OD

Sol: OA=$\bar{i}+\bar{j}+\bar{k}$

AB=$3\bar{i}-2\bar{j}+\bar{k}$

BC=$\bar{i}+2\bar{j}-2\bar{k}$

CD=$2\bar{i}+\bar{j}+3\bar{k}$

OD=OA+AB+BC+CD=$(\bar{i}+\bar{j}+\bar{k})+(3\bar{i}-2\bar{j}+\bar{k})+(\bar{i}+2\bar{j}-2\bar{k})+(2\bar{i}+\bar{j}+3\bar{k})$

∴OD=$7\bar{i}+2\bar{j}+3\bar{k}$

5. Let a=$2\bar{i}+4\bar{j}-5\bar{k}$, b=$\bar{i}+\bar{j}+\bar{k}$ and c=$\bar{j}+2\bar{k}$. Find the unit vector in the opposite direction of a+b+c.

Sol: Let a,b,c be the given vectors respectively.

∴a+b+c=$(2\bar{i}+4\bar{j}-5\bar{k})+(\bar{i}+\bar{j}+\bar{k})+(\bar{j}+2\bar{k})=3\bar{i}+6\bar{j}-2\bar{k}$

$|a+b+c|=\sqrt{(3)^2+(6)^2+(-2)^2}=\sqrt{9+36+4}=\sqrt{49}=7$

Required unit vector=$\frac{-(a+b+c)}{|a+b+c|}=\frac{-(3\bar{i}+6\bar{j}-2\bar{k})}{7}$

6. OABC is a parallelogram, if $\overline{OA}=\bar{a}$ and $\overline{OC}=\bar{c}$. Find the vector equation of the side BC.

Sol: OABC is a parallelogram ⇒ $\overline{CB}=\overline{OA}$=a

Equation of $\overline{BC}$ is r=C+ta, t∈R

7. Find the vector equation of the plane passing through the points $\bar{i}-2\bar{j}+5\bar{k}$,$-5\bar{j}-\bar{k}and-3\bar{i}+5\bar{j}$.

Sol: The vector equation of the plane is

$\bar{r}=(1-S-t)\bar{a}+S\bar{b}+t\bar{c}$

S,t are scalars

A($\bar{a}$)= $\bar{i}-2\bar{j}+5\bar{k}$

B($\bar{b}$)= $-5\bar{j}-\bar{k}$

C($\bar{c}$)= $-3\bar{i}+5\bar{j}$

$\bar{r}=(1-S-t)(\bar{i}-2\bar{j}+5\bar{k})+S(-5\bar{j}-\bar{k})+t(-3\bar{i}+5\bar{j})$

8. If $\bar{a}=6\bar{i}+2\bar{j}+3\bar{k}$ and $\bar{b}=2\bar{i}-9\bar{j}+6\bar{k}$, then find the angle between the vectors$\bar{a}$and $\bar{b}$.

Sol: Let $\bar{a}=6\bar{i}+2\bar{j}+3\bar{k}$

$\bar{b}=2\bar{i}-9\bar{j}+6\bar{k}$

a.b =$(6\bar{i}+2\bar{j}+3\bar{k}).(2\bar{i}-9\bar{j}+6\bar{k})$=12-18+18=12

$|a|=\sqrt{(6)^2+(2)^2+(3)^2}=\sqrt{36+4+9}=\sqrt{49}=7$

$|b|=\sqrt{(2)^2+(-9)^2+(6)^2}=\sqrt{4+81+36}=\sqrt{121}=11$

$\cos(a.b)=\frac{a.b}{|a||b|}=\frac{12}{7x11}=\frac{12}{77}$

$\Rightarrow a.b=\cos^{-1}\frac{12}{77}$

Angle between the vectors $\bar{a}$ and $\bar{b}$ is $\cos^{-1}\frac{12}{77}$

9. If|a|=11, |b|=23 and |a − b|=30. Then find the angle between the vectors $\bar{a}$ and $\bar{b}$also find |a + b|.

Sol: Given |a|=11, |b|=23 and |a − b|=30

Let θbe the angle between $\bar{a}$ and $\bar{b}$

∴ $|\bar{a}-\bar{b}|^2=\bar{a}^2-2\bar{a}\bar{b}+\bar{b}^2$

$30^2=11^2-2x11x23x\cos\theta+23^2$

$900=121-506\cos\theta+529$

$\Rightarrow \cos\theta=\frac{250}{506}=\frac{125}{253}$

∴ $\theta=\cos^{-1}\frac{125}{253}$

$|\bar{a}+\bar{b}|^2=\bar{a}^2+2\bar{a}\bar{b}+\bar{b}^2=11^2+2x11x23(\frac{-125}{253})+23^2$

$|\bar{a}+\bar{b}|^2$=121-250+529=400

$|\bar{a}+\bar{b}|=\sqrt{400}=20$

10. If the vectors $\lambda\bar{i}-3\bar{j}+5\bar{k}$ and $2\lambda\bar{i}-\lambda\bar{j}-\bar{k}$ are perpendicular to each other, find λ.

Sol: Given vectors $\lambda\bar{i}-3\bar{j}+5\bar{k}$ and $2\lambda\bar{i}-\lambda\bar{j}-\bar{k}$

Given vectors are perpendicular

$(\lambda\bar{i}-3\bar{j}+5\bar{k}).(2\lambda\bar{i}-\lambda\bar{j}-\bar{k})=2\lambda^2+3\lambda-5=0$

$(2\lambda+5)(\lambda-1)=0$

∴ $\lambda=1$ or $\frac{5}{2}$

11.If $\bar{a}=2\bar{i}-\bar{j}+3\bar{k}$ and $\bar{b}=\bar{i}-3\bar{j}-5\bar{k}$. Find the vector$\bar{c}$, such that $\bar{a}$, $\bar{b}$ and $\bar{c}$ form the sides of a triangle.

Sol: Given $\bar{a}=2\bar{i}-\bar{j}+3\bar{k}$

$\bar{b}=\bar{i}-3\bar{j}-5\bar{k}$

$\bar{c}$ =?

a,b and c are the sides of a triangle

a+b+c=0

$(2\bar{\imath}-\bar{\jmath}+3\bar{k})+(\bar{\imath}-3\bar{\jmath}-5\bar{k})+c=0$

$(3\bar{\imath}-4\bar{\jmath}-4\bar{k})+c=0$

$\Rightarrow c=-3\bar{\imath}+4\bar{\jmath}+4\bar{k}$

∴Vector $C=-3\bar{\imath}+4\bar{\jmath}+4\bar{k}$

12. If |a|=2, |b|=3 and |c|=4 and each of $\bar{a}$, $\bar{b}$ and $\bar{c}$ are perpendicular to the sum of the other two vectors then find the magnitude of a+b+c.

Sol: |a|=2, |b|=3 and |c|=4

a is perpendicular to b+c

⇒a.(b+c)=0

⇒a.b+a.c=0 ...(1)

b is perpendicular to c+a

⇒b.(c+a)=0

⇒b.c+b.a=0 ...(2)

c is perpendicular to a+b

⇒c.(a+b)=0

⇒c.a+c.b=0 ...(3)

Eq(1)+eq(2)+eq(3)

=(a.b+a.c)+(b.c+b.a)+(c.a+c.b)=0

=2a.b+2b.c+2a.c=0

$|a+b+c|^2=(a+b+c)^2$

$=a^2+b^2+c^2+2a.b+2b.c+2a.c$

$=|a|^2+|b|^2+|c|^2+0$

$=2^2+3^2+4^2=4+9+16=29$

∴ $|a+b+c|=\sqrt{29}$

13. Find the area of the parallelogram for which the vector . $\bar{a}=2\bar{\imath}-3\bar{\jmath}$ and $\bar{b}=3\bar{\imath}-\bar{k}$ are adjacent sides.

Sol: $\bar{a}=2\bar{\imath}-3\bar{\jmath}$ and $\bar{b}=3\bar{\imath}-\bar{k}$

Area of the parallelogram

$\bar{a}X\bar{b}=\begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 3 & 0 & -1 \end{vmatrix}$=i(3-0)-j(-2-0)+k(0+9)=3i+2j+9k

Area of the parallelogram is

$|\bar{a}X\bar{b}|=\sqrt{(3)^2+(2)^2+(9)^2}=\sqrt{9+4+81}=\sqrt{94}$ units.

14. If $4\bar{\imath}+\frac{2P}{3}\bar{\jmath}+P\bar{k}$ is parallel to the vector $\bar{\imath}+2\bar{\jmath}+3\bar{k}$. Find the value of P.

Sol: Vectors $4\bar{\imath}+\frac{2P}{3}\bar{\jmath}+P\bar{k}$; $\bar{\imath}+2\bar{\jmath}+3\bar{k}$

Vectors are parallel

$4\bar{\imath}+\frac{2P}{3}\bar{\jmath}+P\bar{k}=m(\bar{\imath}+2\bar{\jmath}+3\bar{k})$

⇒4=m ; $\frac{2P}{3}$ = 2m ; P=3m

⇒m=4 ; $\frac{2P}{3}$ = 2x4 ;P=3x4=12

⇒P=12

15. If |a|=13, |b|=5 and a.b=60. Then find $|\bar{a}X\bar{b}|$

Sol: Given |a|=13, |b|=5 and a.b=60

$|\bar{a}x\bar{b}|^2=|a|^2.|b|^2-(a.b)^2$

$=13^2.5^2-60^2$=169X25-3600=4225-3600=625

∴ $|\bar{a}x\bar{b}|=\sqrt{625}$=25

16.If $a=7\bar{\imath}-2\bar{\jmath}+3\bar{k}$, $b=2\bar{\imath}+8\bar{k}$ and $c=\bar{\imath}+\bar{\jmath}+\bar{k}$, then compute $\bar{a}x\bar{b}$, $\bar{a}x\bar{c}$ and $\bar{a}x(\bar{b}+\bar{c})$. Verify whether the cross product is distributive over vector addition.

Sol: Given $a=7\bar{\imath}-2\bar{\jmath}+3\bar{k}$,

$b=2\bar{\imath}+8\bar{k}$

$c=\bar{\imath}+\bar{\jmath}+\bar{k}$

$\bar{b}+\bar{c}=(2\bar{\imath}+8\bar{k})+(\bar{\imath}+\bar{\jmath}+\bar{k})=3\bar{\imath}+\bar{\jmath}+9\bar{k}$

$\bar{a}x(\bar{b}+\bar{c})=\begin{vmatrix} i & j & k \\ 7 & -2 & 3 \\ 3 & 1 & 9 \end{vmatrix}$=i(-18-3)-j(63-9)+k(7+6)=-21i-54j+13k

$\bar{a}x\bar{b}=\begin{vmatrix} i & j & k \\ 7 & -2 & 3 \\ 2 & 0 & 8 \end{vmatrix}$=i(-16-0)-j(56-6)+k(0+4)= -16i-50j+4k

$\bar{a}x\bar{c}=\begin{vmatrix} i & j & k \\ 7 & -2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$=i(-2-3)-j(7-3)+k(7+2)= -5i-4j+9k

$(\bar{a}x\bar{b})+(\bar{a}x\bar{c})$ =(-16i-50j+4k)+(-5i-4j+9k)= -21i-54j+13k

∴ $\bar{a}x(\bar{b}+\bar{c})=(\bar{a}x\bar{b})+(\bar{a}x\bar{c})$

17. If $a=3\bar{\imath}-\bar{\jmath}+2\bar{k}$, $b=-\bar{\imath}+3\bar{\jmath}+2\bar{k}$, $c=4\bar{\imath}+5\bar{\jmath}-2\bar{k}$ and $d=\bar{\imath}+3\bar{\jmath}+5\bar{k}$. Then compute the following.

(i) $(\bar{a}x\bar{b})X(\bar{c}x\bar{d})$ (ii) $(\bar{a}x\bar{b}).\bar{c}-(\bar{a}x\bar{d}).\bar{b}$

Sol: Given $a=3\bar{\imath}-\bar{\jmath}+2\bar{k}$, $b=-\bar{\imath}+3\bar{\jmath}+2\bar{k}$, $c=4\bar{\imath}+5\bar{\jmath}-2\bar{k}$ and $d=\bar{\imath}+3\bar{\jmath}+5\bar{k}$

(i) $\bar{a}x\bar{b}=\begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ -1 & 3 & 2 \end{vmatrix}$=i(-2-6)-j(6+2)+k(9-1)= -8i-8j+8k

$\bar{c}x\bar{d}=\begin{vmatrix} i & j & k \\ 4 & 5 & -2 \\ 1 & 3 & 5 \end{vmatrix}$=i(25+6)-j(20+2)+k(12-5)

= 31i-22j+7k

(i) $(\bar{a}x\bar{b})X(\bar{c}x\bar{d})=\begin{vmatrix} i & j & k \\ -8 & -8 & 8 \\ 31 & -22 & 7 \end{vmatrix}$

=i(-56+176)-j(-56-248)+k(176+248)

= 120i-304j+424k

$\bar{a}x\bar{d}=\begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 1 & 3 & 5 \end{vmatrix}$=i(-5-6)-j(15-2)+k(9+1)= -11i-13j+10k

(ii) $(\bar{a}x\bar{b}).\bar{c}-(\bar{a}x\bar{d}).\bar{b}$

= (-8i-8j+8k).($4\bar{\imath}+5\bar{\jmath}-2\bar{k}$)-(-11i-13j+10k).($-\bar{\imath}+3\bar{\jmath}+2\bar{k}$)

= (-32-40-16) – (11-39+20) = -88+8 = -80

18. If the vectors $a=2\bar{\imath}-\bar{\jmath}+\bar{k}$, $b=\bar{\imath}+2\bar{\jmath}-3\bar{k}$ and $c=3\bar{\imath}+P\bar{\jmath}+5\bar{k}$ are coplanar, then find P.

Sol: $a=2\bar{\imath}-\bar{\jmath}+\bar{k}$,

$b=\bar{\imath}+2\bar{\jmath}-3\bar{k}$

$c=3\bar{\imath}+P\bar{\jmath}+5\bar{k}$

$\bar{a},\bar{b},\bar{c}$ are coplanar $|\bar{a}.\bar{b}.\bar{c}|=0$

$|\bar{a}.\bar{b}.\bar{c}|=\begin{vmatrix}2 & -1 & 1\\1 & 2 & -3\\3 & P & 5\end{vmatrix}=0$

= 2(10+3P) +1(5+9)+1(P-6)=0

= 20+6P+14+P-6=0

=7P+28=0⇒P=$\frac{-28}{7}$=-4

19. Find the equation of the plane passing through the points A(2,3,-1), B(4,5,2) and C(3,6,5)

Sol: Given points A(2,3,-1), B(4,5,2) and C(3,6,5)

Let P(x,y,z) be a point on the plane passing through A,B,C

For all positions of P on the plane, the three vectors AP, AB, AC are coplanar

AB=$\bar{B}$ - $\bar{A}$ and AC=$\bar{C}$ - $\bar{A}$

⇒ $|\overline{AP}.\overline{AB}.\overline{AC}|$=0

$\Rightarrow \begin{vmatrix}x-2 & y-3 & z+1\\2 & 2 & 3\\1 & 3 & 6\end{vmatrix}=0$

=(x-2)[12-9)-(y-3)[12-3]+(z+1)[6-2]=0

=3x-6-9y+27+4z+4=0

=3x-9y+4z+25=0

Equation of the plane passing through the points is 3x-9y+4z+25=0

20. Find the shortest distance between the skew lines r=(6ī+2j̄+2k̄)+t(ī-2j̄+2k̄) and r=(-4ī-k̄)+s(3ī-2j̄-2k̄)

Sol: Given skew lines

$\bar{r}$ = (6ī+2j̄+2k̄)+t(ī-2j̄+2k̄)

$\bar{r}$ =(-4ī-k̄)+s(3ī-2j̄-2k̄)

$\bar{r}=\bar{a}+t\bar{b}$; $\bar{r}=\bar{c}+s\bar{d}$

Shortest distance = $\frac{|(\bar{a}-\bar{c}).\bar{b}X\bar{d}|}{|\bar{b}X\bar{d}|}$

∴a=6ī+2j̄+2k̄ b=ī-2j̄+2k̄

c=-4ī-k̄ d=3ī-2j̄-2k̄

$\bar{a}-\bar{c}$ =(6ī+2j̄+2k̄)-(-4ī-k̄)= 10ī+2j̄+3k̄

$\bar{b}X\bar{d}=\begin{vmatrix}i & j & k\\1 & -2 & 2\\3 & -2 & -2\end{vmatrix}$=i(4+4)-j(-2-6)+k(-2+6)= 8i+8j+4k

∴ $(\bar{a}-\bar{c}).\bar{b}X\bar{d}$ =(10ī+2j̄+3k̄).(8i+8j+4k)

=10X8+2X8+3X4=80+16+12=108

$|\bar{b}X\bar{d}|=\sqrt{(8)^2+(8)^2+(4)^2}=\sqrt{64+64+16}=\sqrt{144}$ =12

∴Shortest distance=$\frac{|(\bar{a}-\bar{c}).\bar{b}X\bar{d}|}{|\bar{b}X\bar{d}|}=\frac{108}{12}$ = 9

21. Simplify the following

(i) (ī-2j̄+3k̄)X(2ī+j̄-k̄). (j̄ + k̄)

(ii)(2ī-3j̄+k̄). $(\bar{\imath}-\bar{\jmath}+2\bar{k})X(2\bar{\imath}+\bar{\jmath}+\bar{k})$

(i)Sol: (ī-2j̄+3k̄)X(2ī+j̄-k̄). (j̄ + k̄)

$=\begin{vmatrix}1 & -2 & 3\\2 & 1 & -1\\0 & 1 & 1\end{vmatrix}$=1(1+1)- (-2)[2-0]+3(2-0)=2+4+6=12

(ii) (2ī-3j̄+k̄). $(\bar{\imath}-\bar{\jmath}+2\bar{k})X(2\bar{\imath}+\bar{\jmath}+\bar{k})$

$==\begin{vmatrix}2 & -3 & 1\\1 & -1 & 2\\2 & 1 & 1\end{vmatrix}$=2(-1-2)- (-3)[1-4]+1(1+2)=-6-9+3=-12

22. Find λ in order that the four points A(3,2,1), B(4,λ,5), C(4,2,-2) and D(6,5,-1) be coplanar.

Sol: A(3,2,1), B(4,λ,5), C(4,2,-2) and D(6,5,-1)

A,B,C,D be coplanar

$\overline{AB}=\overline{OB}-\overline{OA}$ =(4-3)i+(λ-2)j+(5-1)k= i+(λ-2)j+4k

$\overline{AC}=\overline{OC}-\overline{OA}$ =(4-3)i+(2-2)j+(-2-1)k= i -3k

$\overline{AD}=\overline{OD}-\overline{OA}$ =(6-3)i+(5-2)j+(-1-1)k= 3i3j-2k

A,B,C,D are coplanar $|\overline{AB}\ \overline{AC}\ \overline{AD}|$=0

$\Rightarrow \begin{vmatrix}1 & \lambda-2 & 4\\1 & 0 & -3\\3 & 3 & -2\end{vmatrix}=0$

=1(0+9) - (λ-2)[-2+9]+4(3+0)=0

=9-7λ+14+12=0

=7λ=35 ⇒ λ=$\frac{35}{7}$=5

23. Find the volume of the tetrahedron having the edges ī+j̄+k̄, ī-j̄ and ī+2j̄+k̄

Sol: $[\bar{\imath}+\bar{\jmath}+\bar{k}\quad \bar{\imath}-\bar{\jmath}\quad \bar{\imath}+2\bar{\jmath}+\bar{k}]=\begin{vmatrix}1 & 1 & 1\\1 & -1 & 0\\1 & 2 & 1\end{vmatrix}$

= 1(-1+0)- 1(1-0)+1(2+1)=-1-1+3=1

∴Volume of the tetrahedron = $\frac{1}{6}$(1)= $\frac{1}{6}$ cubic units

24.Compute $[\bar{\imath}-\bar{\jmath}\quad \bar{\imath}-\bar{k}\quad \bar{k}-\bar{\imath}]$

Sol: $\begin{vmatrix}1 & -1 & 0\\1 & 0 & -1\\-1 & 0 & 1\end{vmatrix}$=1(0-0)- 1(1-1]+0(0+0)= 0+0+0=0

25. If ā=(1,-2,1); b̄=(2,1,1) and c̄=(1,2,-1) then find $|\bar{a}\ x(\bar{b}\ x\ \bar{c})|$and $|(\bar{a}\ x\ \bar{b})x\bar{c}|$

Sol: ā=(1,-2,1); b̄=(2,1,1) and c̄=(1,2,-1)

ā=ī-2j̄+k̄,

b̄=2ī+j̄+k̄

c̄=ī+2j̄-k̄

$\bar{a}\ x\ \bar{b}=\begin{vmatrix}i & j & k\\1 & -2 & 1\\2 & 1 & 1\end{vmatrix}$=i(-2-1)-j(1-2)+k(1+4)= -3i+j+5k

$\bar{b}\ x\ \bar{c}=\begin{vmatrix}i & j & k\\2 & 1 & 1\\1 & 2 & -1\end{vmatrix}$=i(-1-2)-j(-2-1)+k(4-1)= -3i+3j+3k

$\bar{a}\ x(\bar{b}\ x\ \bar{c})=\begin{vmatrix}i & j & k\\1 & -2 & 1\\-3 & 3 & 3\end{vmatrix}$=i(-6-3)-j(3+3)+k(3-6)

= -9i-6j-3k

$(\bar{a}\ x\ \bar{b})x\bar{c}=\begin{vmatrix}i & j & k\\-3 & 1 & 5\\1 & 2 & -1\end{vmatrix}$ = i(-1-10)-j(3-5)+k(-6-1)

= -11i+2j-7k

$|\bar{a}\ x(\bar{b}\ x\ \bar{c})|=|-9i-6j-3k|$

$=\sqrt{(-9)^2+(-6)^2+(-3)^2}=\sqrt{81+36+9}=\sqrt{126}$

$|(\bar{a}\ x\ \bar{b})x\bar{c}|=|-11i+2j-7k|$

$=\sqrt{(-11)^2+(2)^2+(-7)^2}=\sqrt{121+4+49}=\sqrt{174}$

26. If ā=2ī+2j̄-3k̄, b̄=3ī-j̄+2k̄, then find the angle between (2ā+b̄) and (ā+2b̄).

Sol: ā=2ī+2j̄-3k̄,

b̄=3ī-j̄+2k̄

∴2ā+b̄=2(2ī+2j̄-3k̄)+(3ī-j̄+2k̄)= 7ī+3j̄-4k̄

$|2\bar{a}+\bar{b}|=|7\bar{\imath}+3\bar{\jmath}-4\bar{k}|$

$=\sqrt{(-7)^2+(3)^2+(-4)^2}=\sqrt{49+9+16}=\sqrt{74}$

ā+2b̄=(2ī+2j̄-3k̄)+2(3ī-j̄+2k̄) = 8ī+k̄

$|\bar{a}+2\bar{b}|=|8\bar{\imath}+\bar{k}|$

$=\sqrt{(8)^2+(1)^2}=\sqrt{64+1}=\sqrt{65}$

Angle between $(2\bar{a}+\bar{b})$ and $(\bar{a}+2\bar{b})$

$\Rightarrow \cos\theta=\frac{(2\bar{a}+\bar{b}).(\bar{a}+2\bar{b})}{|2\bar{a}+\bar{b}||\bar{a}+2\bar{b}|}=\frac{(7\bar{\imath}+3\bar{\jmath}-4\bar{k}).(8\bar{\imath}+\bar{k})}{\sqrt{74}\sqrt{65}}=\frac{56+0+(-4)}{\sqrt{74}\sqrt{65}}$

$=\frac{52}{\sqrt{74}\sqrt{65}} \Rightarrow \theta=\cos^{-1}\frac{52}{\sqrt{74}\sqrt{65}}$

27. Simplify the following

(a) $(\bar{\imath}-2\bar{\jmath}+3\bar{k})X(2\bar{\imath}+\bar{\jmath}-\bar{k})x(\bar{\jmath}+\bar{k})$

(b) $(2\bar{\imath}-3\bar{\jmath}+\bar{k}).(\bar{\imath}-\bar{\jmath}+2\bar{k})x(2\bar{\imath}+\bar{\jmath}+\bar{k})$

Repeated Q.No.21

28.If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors, then find the value of $\frac{(\bar{a}+2\bar{b}-\bar{c})[(\bar{a}-\bar{b})x(\bar{a}-\bar{b}-\bar{c})]}{[\bar{a}\ \ \bar{b}\ \ \bar{c}]}$

Sol: $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors $\Rightarrow[\bar{a}\ \bar{b}\ \bar{c}]\neq0$

We have $(\bar{a}+2\bar{b}-\bar{c}).[(\bar{a}-\bar{b})x(\bar{a}-\bar{b}-\bar{c})]$

$=[\bar{a}+2\bar{b}-\bar{c}\ \ \ \bar{a}-\bar{b}\ \ \ \bar{a}-\bar{b}-\bar{c}]$

$=\begin{vmatrix}1 & 2 & -1\\ 1 & -1 & 0\\ 1 & -1 & -1\end{vmatrix}[\bar{a}\ \bar{b}\ \bar{c}]=1(1+0)-2(-1-0)-1(-1+1)[\bar{a}\ \bar{b}\ \bar{c}]$

$=(1+2-0)[\bar{a}\ \bar{b}\ \bar{c}]=3[\bar{a}\ \bar{b}\ \bar{c}]$

Now $\frac{(\bar{a}+2\bar{b}-\bar{c})[(\bar{a}-\bar{b})x(\bar{a}-\bar{b}-\bar{c})]}{[\bar{a}\ \ \bar{b}\ \ \bar{c}]}=\frac{3[\bar{a}\ \bar{b}\ \bar{c}]}{[\bar{a}\ \ \bar{b}\ \ \bar{c}]}=3$

Solved problems in the Text book

1.Find the point of intersection of the line $\bar{r}=2\bar{a}+\bar{b}+t(\bar{b}-\bar{c})$ and the plane $\bar{r}=\bar{a}+x(\bar{b}+\bar{c})+y(\bar{a}+2\bar{b}-\bar{c})$ where $\bar{a}$, $\bar{b}$ and $\bar{c}$ are non coplanar vectors.

Sol: At the point of intersection of the line and plane, we have

$2\bar{a}+\bar{b}+t(\bar{b}-\bar{c})=\bar{a}+x(\bar{b}+\bar{c})+y(\bar{a}+2\bar{b}-\bar{c})$

$2\bar{a}+(1+t)\bar{b}-t\bar{c}=(1+y)\bar{a}+(x+2y)\bar{b}+(x-y)\bar{c}$

On comparing the corresponding coefficients

$\Rightarrow2=1+y;\ 1+t=x+2y;\ -t=x-y$

$\Rightarrow1=y;\ t=x+2y-1;\ -t=x-y$

$\Rightarrow y=1,\ x=0;\ t=1$

∴The point of intersection is $2\bar{a}+2\bar{b}-\bar{c}$

2.If $\bar{a}=2\bar{\imath}+3\bar{\jmath}+4\bar{k}$; $\bar{b}=\bar{\imath}+\bar{\jmath}-\bar{k}$ and $\bar{c}=\bar{\imath}-\bar{\jmath}+\bar{k}$, then find $\bar{a}\ x(\bar{b}\ x\ \bar{c})$ (mar19)

Sol: Given $\bar{a}=2\bar{\imath}+3\bar{\jmath}+4\bar{k}$; $\bar{b}=\bar{\imath}+\bar{\jmath}-\bar{k}$ and $\bar{c}=\bar{\imath}-\bar{\jmath}+\bar{k}$

$\bar{b}\ x\ \bar{c}=\begin{vmatrix}i & j & k\\ 1 & 1 & -1\\ 1 & -1 & 1\end{vmatrix}=i(1-1)-j(1+1)+k(-1-1)=-2j-2k$

$\bar{a}\ x(\bar{b}\ x\ \bar{c})=\begin{vmatrix}i & j & k\\ 2 & 3 & 4\\ 0 & -2 & -2\end{vmatrix}=i(-6+8)-j(-4-0)+k(-4-0)$

$=2i+4j-4k$

∴ $\bar{a}\ x(\bar{b}\ x\ \bar{c})=2i+4j-4k$

Exercise problems

1. Find the unit vector in the direction of $\bar{a}=2\bar{\imath}+3\bar{\jmath}+\bar{k}$

Sol: Given $\bar{a}=2\bar{\imath}+3\bar{\jmath}+\bar{k}$

unit vector in the direction of $\bar{a}=\frac{\bar{a}}{|\bar{a}|}=\frac{2\bar{\imath}+3\bar{\jmath}+\bar{k}}{\sqrt{(2)^2+(3)^2+(1)^2}}$

$=\frac{2\bar{\imath}+3\bar{\jmath}+\bar{k}}{\sqrt{4+9+1}}=\frac{1}{\sqrt{14}}[2\bar{\imath}+3\bar{\jmath}+\bar{k}]$

2. Find a vector in the direction of vector $\bar{a}=\bar{\imath}-2\bar{\jmath}$ that has magnitude 7 units.

Sol: $\bar{a}=\bar{\imath}-2\bar{\jmath} \Rightarrow |\bar{a}|=\sqrt{(1)^2+(-2)^2}=\sqrt{1+4}=\sqrt{5}$

unit vector in the direction of $\bar{a}=\frac{\bar{a}}{|\bar{a}|}=\frac{\bar{\imath}-2\bar{\jmath}}{\sqrt{5}}$

Therefore the vector having magnitude 7 units in the direction of $\bar{a}=7\bar{a}$

$\therefore7\bar{a}=7\{\frac{\bar{\imath}-2\bar{\jmath}}{\sqrt{5}}\}=\frac{7}{\sqrt{5}}\bar{\imath}-\frac{14}{\sqrt{5}}\bar{\jmath}$

3. Find the unit vector in the direction of the sum of the vectors $\bar{a}=2\bar{\imath}+2\bar{\jmath}-5\bar{k}$ and $\bar{b}=2\bar{\imath}+\bar{\jmath}+3\bar{k}$.

Sol: Given $\bar{a}=2\bar{\imath}+2\bar{\jmath}-5\bar{k}$ and $\bar{b}=2\bar{\imath}+\bar{\jmath}+3\bar{k}$

$\bar{a}+\bar{b}=(2\bar{\imath}+2\bar{\jmath}-5\bar{k})+(2\bar{\imath}+\bar{\jmath}+3\bar{k})=4\bar{\imath}+3\bar{\jmath}-2\bar{k}$

$\Rightarrow|\bar{a}+\bar{b}|=\sqrt{(4)^2+(3)^2+(-2)^2}=\sqrt{16+9+4}=\sqrt{29}$

∴ unit vector in the direction of $\bar{a}+\bar{b}=\frac{\bar{a}+\bar{b}}{|\bar{a}+\bar{b}|}$

$=\frac{4\bar{\imath}+3\bar{\jmath}-2\bar{k}}{\sqrt{29}}=\frac{1}{\sqrt{29}}(4\bar{\imath}+3\bar{\jmath}-2\bar{k})$

4.Find the direction ratio and direction cosines of the vector $\bar{\imath}+\bar{\jmath}-2\bar{k}$.

Sol: Let α,β,γ are the angles made by the vector $\overline{OP}=\bar{r}=\bar{\imath}+\bar{\jmath}-2\bar{k}$

$|\bar{r}|=\sqrt{(1)^2+(1)^2+(-2)^2}=\sqrt{1+1+4}=\sqrt{6}$

Direction cosines of the vector $\bar{r}$ are

$l=\cos\alpha=\frac{x}{|\bar{r}|}=\frac{1}{\sqrt{6}}$; $m=\cos\beta=\frac{y}{|\bar{r}|}=\frac{1}{\sqrt{6}}$;

$n=\cos\gamma=\frac{z}{|\bar{r}|}=-\frac{2}{\sqrt{6}}$

∴direction cosines of the vector is $(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},-\frac{2}{\sqrt{6}})$

5. Consider two points P and Q with position vectors $\overline{OP}=3\bar{a}-2\bar{b}$ and $\overline{OQ}=\bar{a}+\bar{b}$. Find the position vector of a point R which divides the line joining P and Q in the ratio 2:1,

(i) internally and (ii) externally

(i) Sol: The position vector of the point R dividing the join of P and Q internally in the ratio 2:1 is

$\overline{OR}=\frac{2(\overline{OQ})+1(\overline{OP})}{2+1}=\frac{2(\bar{a}+\bar{b})+1(3\bar{a}-2\bar{b})}{3}=\frac{5\bar{a}}{3}$

(ii) Sol: The position vector of the point R dividing the join of P and Q externally in the ratio 2:1 is

$\overline{OR}=\frac{2(\overline{OQ})-1(\overline{OP})}{2-1}=\frac{2(\bar{a}+\bar{b})-1(3\bar{a}-2\bar{b})}{1}=4\bar{b}-\bar{a}$

6. Show that the points $A(2\bar{\imath}-\bar{\jmath}+\bar{k})$, $B(\bar{\imath}-3\bar{\jmath}-5\bar{k})$, $C(3\bar{\imath}-4\bar{\jmath}-4\bar{k})$ are the right angled triangle.

Sol: $\overline{AB}=\overline{OB}-\overline{OA}=(\bar{\imath}-3\bar{\jmath}-5\bar{k})-(2\bar{\imath}-\bar{\jmath}+\bar{k})$

$=(1-2)\bar{\imath}+(-3+1)\bar{\jmath}+(-5-1)\bar{k}=-\bar{\imath}-2\bar{\jmath}-6\bar{k}$

$\Rightarrow|\overline{AB}|=\sqrt{(-1)^2+(-2)^2+(-6)^2}=\sqrt{1+4+36}=\sqrt{41}$

$\overline{BC}=\overline{OC}-\overline{OB}=(3\bar{\imath}-4\bar{\jmath}-4\bar{k})-(\bar{\imath}-3\bar{\jmath}-5\bar{k})$

$=(3-1)\bar{\imath}+(-4+3)\bar{\jmath}+(-4+5)\bar{k}=2\bar{\imath}-\bar{\jmath}+\bar{k}$

$\Rightarrow|\overline{BC}|=\sqrt{(2)^2+(-1)^2+(1)^2}=\sqrt{4+1+1}=\sqrt{6}$

$\overline{CA}=\overline{OA}-\overline{OC}=(2\bar{\imath}-\bar{\jmath}+\bar{k})-(3\bar{\imath}-4\bar{\jmath}-4\bar{k})$

$=(2-3)\bar{\imath}+(-1+4)\bar{\jmath}+(1+4)\bar{k}=-\bar{\imath}+3\bar{\jmath}+5\bar{k}$

$\Rightarrow|\overline{CA}|=\sqrt{(-1)^2+(3)^2+(5)^2}=\sqrt{1+9+25}=\sqrt{35}$

Here, $|\overline{AB}|^2=41$; $|\overline{BC}|^2+|\overline{CA}|^2=6+35=41$

$\Rightarrow|\overline{AB}|^2=|\overline{BC}|^2+|\overline{CA}|^2$

7. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar vectors, prove that $-\bar{a}+4\bar{b}-3\bar{c}$, $3\bar{a}+2\bar{b}-5\bar{c}$, $-3\bar{a}+8\bar{b}-5\bar{c}$, $-3\bar{a}+2\bar{b}+\bar{c}$ are coplanar. **(mar19)**

Sol: Let O be the origin of reference so that

$\overline{OP}= -\bar{a}+4\bar{b}-3\bar{c}$, $\overline{OQ}=3\bar{a}+2\bar{b}-5\bar{c}$, $\overline{OR}= -3\bar{a}+8\bar{b}-5\bar{c}$,

$\overline{OS} = -3\bar{a}+2\bar{b}+\bar{c}$

$\overline{PQ} = \overline{OQ} - \overline{OP}=(3\bar{a}+2\bar{b}-5\bar{c}) -(-\bar{a}+4\bar{b}-3\bar{c})= 4\bar{a} -2\bar{b} -2\bar{c}$

$\overline{PR} = \overline{OR} - \overline{OP}=(-3\bar{a}+8\bar{b}-5\bar{c}) -(-\bar{a}+4\bar{b}-3\bar{c})= -2\bar{a}+4\bar{b}-2\bar{c}$

$\overline{PS} = \overline{OS} - \overline{OP}=(-3\bar{a}+2\bar{b}+\bar{c}) -(-\bar{a}+4\bar{b}-3\bar{c})= -2\bar{a}-2\bar{b}+4\bar{c}$

Now, $[\overline{PQ} \quad \overline{PR} \quad \overline{PS}] = \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix} [\bar{a} \quad \bar{b} \quad \bar{c}]$

$=[4(16-4)-(-2)[-8-4)-2(4+8)] [\bar{a} \quad \bar{b} \quad \bar{c}]=[48-24-24] [\bar{a} \quad \bar{b} \quad \bar{c}]=0$

Hence, the three vectors $\overline{PQ}, \overline{PR}, \overline{PS}$ are coplanar

∴The four points, P,Q,R,S are coplanar.

8. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.

Sol: The vector equation of the line through the point A($\bar{a}$) and parallel to the vector $\bar{b}$ is

$\bar{r}=\bar{a}+t\bar{b}$, t∈R

Here, the point A($\bar{a}$)= $2\bar{i} + 3\bar{j} + \bar{k}$ and the vector $\bar{b}=4\bar{i} - 2\bar{j} + 3\bar{k}$

∴Vector equation of the line is

$\bar{r}=(2\bar{i} + 3\bar{j} + \bar{k})+t(4\bar{i} - 2\bar{j} + 3\bar{k})$, t∈R

$=(2 + 4t)\bar{i} + (3 - 2t)\bar{j} + (1 + 3t)\bar{k}$, t∈R

9.Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$.

Sol: The vector equation of the line through the point A($\bar{a}$), B($\bar{b}$) is

$\bar{r}=(1-t)\bar{a}+t\bar{b}$, t∈R

Here, the point A($\bar{a}$)= $2\bar{i} + \bar{j} + 3\bar{k}$,

B($\bar{b}$)= $-4\bar{i} + 3\bar{j} - \bar{k}$

∴Vector equation of the line is

$\bar{r}=(1-t)(2\bar{i} + \bar{j} + 3\bar{k})+t(-4\bar{i} + 3\bar{j} - \bar{k})$, t∈R

$=(2 - 6t)\bar{i} + (1 + 2t)\bar{j} + (3 - 4t)\bar{k}$, t∈R

10.Find the vector equation of the plane passing through the points $\bar{i} - 2\bar{j} + 5\bar{k}$, $-5\bar{j} - \bar{k}$, $3\bar{i} + 5\bar{j}$.

Sol: The vector equation of the plane passing through the points A($\bar{a}$), B($\bar{b}$), C($\bar{c}$) is

$\bar{r}=(1-s-t)\bar{a}+s\bar{b}+t\bar{c}$, t∈R

Here, A($\bar{a}$)= $\bar{i} - 2\bar{j} + 5\bar{k}$

B($\bar{b}$)= $-5\bar{j} - \bar{k}$

C($\bar{c}$)= $3\bar{i} + 5\bar{j}$

∴Vector equation of the plane is

$\bar{r}=(1-s-t)(\bar{i} - 2\bar{j} + 5\bar{k})+s(-5\bar{j} - \bar{k}) +t(3\bar{i} + 5\bar{j})$, t∈R

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1.Find the cosine angle between the vectors $2\bar{i} - \bar{j} + \bar{k}$ and $3\bar{i} + 4\bar{j} - \bar{k}$

Sol: Sol: Let $\bar{a}=2\bar{i} - \bar{j} + \bar{k}$

$\bar{b}= 3\bar{i} + 4\bar{j} - \bar{k}$

$\bar{a}.\bar{b} =(2\bar{i} - \bar{j} + \bar{k}).(3\bar{i} + 4\bar{j} - \bar{k})=6-4-1=1$

$|\bar{a}|=\sqrt{(2)^2 + (-1)^2 + (1)^2} =\sqrt{4 + 1 + 1} =\sqrt{6}$

$|\bar{b}|=\sqrt{(3)^2 + (4)^2 + (-1)^2} =\sqrt{9 + 16 + 1} =\sqrt{26}$

$\cos(\bar{a}.\bar{b})= \frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|} = \frac{1}{\sqrt{6}\sqrt{26}} = \frac{1}{\sqrt{153}}$

$\Rightarrow \bar{a}.\bar{b}=\cos^{-1}\frac{1}{\sqrt{153}}$

Angle between the vectors $\bar{a}$ and $\bar{b}$ is $\cos^{-1}(\frac{1}{\sqrt{153}})$

2. If the vectors $2\bar{i} +\lambda\bar{j} - \bar{k}$ and $4\bar{i} -2\bar{j} +2\bar{k}$ are perpendicular to each other, find λ.

Sol: Let $\bar{a}=2\bar{i} +\lambda\bar{j} - \bar{k}$

$\bar{b}= 4\bar{i} -2\bar{j} +2\bar{k}$

If $\bar{a}, \bar{b}$ are perpendicular then $\bar{a}.\bar{b} = 0$

$\bar{a}.\bar{b}=(2\bar{i} +\lambda\bar{j} - \bar{k}).(4\bar{i} -2\bar{j} +2\bar{k})=0$

$\Rightarrow 8-2\lambda-2=0$

$\Rightarrow 2\lambda =6; \Rightarrow \lambda = 3$

3. If $\bar{a}+\bar{b}+\bar{c}=0$, $|\bar{a}|=3$, $|\bar{b}|=5$, $|\bar{c}|=7$,then find the cosine angle between vectors $\bar{a}$ and $\bar{b}$.

Sol: $\bar{a}+\bar{b}+\bar{c}=0 \Rightarrow \bar{a}+\bar{b}= -\bar{c} \Rightarrow |\bar{a} + \bar{b}|=|-\bar{c}|$

$\Rightarrow |\bar{a} + \bar{b}|^2=|-\bar{c}|^2$

$\Rightarrow \bar{a}^2+\bar{b}^2+2\bar{a}.\bar{b}=\bar{c}^2$

$\Rightarrow \bar{a}^2+\bar{b}^2+2|\bar{a}|.|\bar{b}|\cos\theta=\bar{c}^2$

$\Rightarrow 3^2+5^2+2.3.5\cos\theta=7^2$

$\Rightarrow 9+25+30\cos\theta=49$

$\Rightarrow \cos\theta = \frac{15}{30} = \frac{1}{2} \Rightarrow \theta=60°$

∴ Cosine angle between vectors $\bar{a}$ and $\bar{b}$ is $\theta=60°$

4.If $|\bar{a}|=2$, $|\bar{b}|=3$ and$(\bar{a}, \bar{b})=30°$, then find $|\bar{a}x\bar{b}|^2$

Sol: Given $|\bar{a}|=2$, $|\bar{b}|=3$ and$(\bar{a}, \bar{b})=30°$

$|\bar{a}x\bar{b}|^2=|\bar{a}|^2|\bar{b}|^2\sin^2\theta =2^2.3^2\sin^2 30°=4.9.(\frac{1}{2})^2=9$

5. Find the unit vector perpendicular to both $\bar{i} + \bar{j} +\bar{k}$ and $2\bar{i} + \bar{j} +3\bar{k}$

Sol: Given $\bar{a}=\bar{i} + \bar{j} +\bar{k}$: $\bar{b}=2\bar{i} + \bar{j} +3\bar{k}$

$\bar{a}x\bar{b} =\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \bar{i}(3-1) - \bar{j}(3-2)+ \bar{k}(1-2)= 2\bar{i} - \bar{j} - \bar{k}$

$|\bar{a}x\bar{b}|=\sqrt{(2)^2 + (-1)^2 + (-1)^2} =\sqrt{4 + 1 + 1} =\sqrt{6}$

∴ unit vector perpendicular to $\bar{a}$ and $\bar{b}$ is $=\pm\frac{\bar{a}x\bar{b}}{|\bar{a}x\bar{b}|}$

$=\pm\frac{2\bar{i} - \bar{j} - \bar{k}}{\sqrt{6}} =\pm\frac{1}{\sqrt{6}}(2\bar{i} - \bar{j} - \bar{k})$

6. If θ is the angle between $\bar{i} + \bar{j}$ and $\bar{j} +\bar{k}$ then find $\sin\theta$.

Sol: Given $\bar{a}=\bar{i} + \bar{j}$: $\bar{b}=\bar{j} +\bar{k}$

$|\bar{a}|=\sqrt{(1)^2 + (1)^2} =\sqrt{1 + 1} =\sqrt{2}$

$|\bar{b}|=\sqrt{(1)^2 + (1)^2} =\sqrt{1 + 1} =\sqrt{2}$

$\bar{a}x\bar{b} =\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \bar{i}(1-0) - \bar{j}(1-0)+ \bar{k}(1-0)= \bar{i} - \bar{j} + \bar{k}$

$|\bar{a}x\bar{b}|=\sqrt{(1)^2 + (-1)^2 + (1)^2} =\sqrt{1 + 1 + 1} =\sqrt{3}$

∴ $\sin\theta = \frac{|\bar{a}x\bar{b}|}{|\bar{a}||\bar{b}|} = \frac{\sqrt{3}}{\sqrt{2}\sqrt{2}} =\frac{\sqrt{3}}{2}$

7. Find the area of the parallelogram whose diagonals are $3\bar{i} + \bar{j} -2\bar{k}$ and $\bar{i} -3\bar{j} +4\bar{k}$.

Sol: Let $\overline{d_1} =3\bar{i} + \bar{j} -2\bar{k}$ and $\overline{d_2} = \bar{i} -3\bar{j} +4\bar{k}$

$\overline{d_1} \, x \overline{d_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = \bar{i}(4\text{-}6) - \bar{j}(12\text{+}2) + \bar{k}(\text{-}9\text{-}1)$

$= -2\bar{i} - 14\bar{j} - 10\bar{k}$

∴Area of parallelogram,

$\frac{1}{2}|\overline{d_1} \, x\overline{d_2}| = \frac{1}{2}\sqrt{(-2)^2 + (-14)^2 + (-10)^2}$

$= \frac{1}{2}\sqrt{4 + 196 + 100} = \frac{1}{2}\sqrt{300} = \frac{10}{2}\sqrt{3} = 5\sqrt{3}$ sq.units

8.Find the area of the triangle having $3\bar{i} + 4\bar{j}$ and $-5\bar{i} + 7\bar{j}$ as two of its edges.

Sol: Let $\bar{a} = 3\bar{i} + 4\bar{j}$ $\quad \bar{b} = -5\bar{i} + 7\bar{j}$

$\bar{a}x\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix} = \bar{i}(0\text{-}0) - \bar{j}(0\text{-}0) + \bar{k}(21\text{+}20) = 41\bar{k}$

∴Area of the given triangle $= \frac{1}{2}|\bar{a} \, x\bar{b}| = \frac{1}{2}|41\,\bar{k}|$

$= \frac{41}{2}|\bar{k}| = \frac{41}{2}(1) = 20.5$ sq.units

9.Find the equation of the plane passing through the points A(1,2,3),B(2,3,1) and C(3,1,2).

Sol: Given points A(1,2,3),B(2,3,1) and C(3,1,2).

Let P(x,y,z) be a point on the plane passing through A,B,C

For all positions of P on the plane, the three vectors AP, AB, AC are coplanar

$AB = \bar{B} - \bar{A}$ and $AC = \bar{C} - \bar{A}$

$\Rightarrow |\overline{AP}.\overline{AB}.\overline{AC}| = 0$

$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{vmatrix} = 0$

=(x-1)[-1-2]-(y-2)[-1+4]+(z-3)[-1-2]=0

=-3x+3-3y+6-3z+9=0

=3x+3y+3z-18=0

Equation of the plane passing through the points is x+y+z-6=0

10.If $\bar{a}+\bar{b}+\bar{c}=0$ then prove that $\bar{a}x\bar{b} = \bar{b}x\bar{c} = \bar{c}x\bar{a}$

(may19)

Sol: Given that $\bar{a}+\bar{b} = -\bar{c}$ ...(1)

Now, cross multiplying eq(1) with $\bar{a}$, we have

$\bar{a}x(\bar{a}+\bar{b}) = \bar{a}x(-\bar{c}) \Rightarrow \bar{a}x\bar{a} + \bar{a}x\bar{b} = -(\bar{a}x\bar{c})$

$\Rightarrow \bar{0} + (\bar{a}x\bar{b}) = (\bar{c}x\bar{a}) \Rightarrow \bar{a}x\bar{b} = \bar{c}x\bar{a}$ ...(2)

Again cross multiplying eq(1) with $\bar{b}$, we have

$\bar{b}x(\bar{a}+\bar{b}) = \bar{b}x(-\bar{c}) \Rightarrow \bar{b}x\bar{a} + \bar{b}x\bar{b} = -(\bar{b}x\bar{c})$

$\Rightarrow \bar{b}x\bar{a} + \bar{0} = (\bar{c}x\bar{b}) \Rightarrow \bar{b}x\bar{a} = \bar{c}x\bar{b} \Rightarrow \bar{a}x\bar{b} = \bar{b}x\bar{c}$...(3)

From eq(2) & eq(3) we have, $\bar{a}x\bar{b} = \bar{b}x\bar{c} = \bar{c}x\bar{a}$

11. If $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{b} = -\bar{i} + 2\bar{j} - 4\bar{k}$, $\bar{c} = \bar{i} + \bar{j} + \bar{k}$ then find $(\bar{a}x\bar{b}).(\bar{b}x\bar{c})$

Sol: Given that $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{b} = -\bar{i} + 2\bar{j} - 4\bar{k}$, $\bar{c} = \bar{i} + \bar{j} + \bar{k}$

$\bar{a}x\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & -1 \\ -1 & 2 & -4 \end{vmatrix} = \bar{i}(-4+2) - \bar{j}(-8-1) + \bar{k}(4+1) = -2\bar{i} + 9\bar{j} + 5\bar{k}$

$\bar{b}x\bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 2 & -4 \\ 1 & 1 & 1 \end{vmatrix} = \bar{i}(2+4) - \bar{j}(-1+4) + \bar{k}(-1-2) = 6\bar{i} - 3\bar{j} - 3\bar{k}$

$(\bar{a}x\bar{b}).(\bar{b}x\bar{c}) = (-2\bar{i} + 9\bar{j} + 5\bar{k}).(6\bar{i} - 3\bar{j} - 3\bar{k})$

= (-2)(6)+(9)(-3)+(5)(-3)= -12-27-15= -54

∴$(\bar{a}x\bar{b}).(\bar{b}x\bar{c}) = -54$

12.Prove that the vectors $2\bar{i} - \bar{j} + \bar{k}$, $\bar{i} - 3\bar{j} - 5\bar{k}$, $3\bar{i} - 4\bar{j} - 4\bar{k}$ are coplanar.

Sol: We know that, if the vectors $\bar{a},\bar{b},\bar{c}$ are coplanar Then $[\bar{a} \;\; \bar{b} \;\; \bar{c}] = 0$:

Given $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$; $\bar{b} = \bar{i} - 3\bar{j} - 5\bar{k}$; $\bar{c} = 3\bar{i} - 4\bar{j} - 4\bar{k}$

$[\bar{a} \;\; \bar{b} \;\; \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -3 & -5 \\ 3 & -4 & -4 \end{vmatrix} = 2(12\text{-}20) - 1(\text{-}1)(\text{-}4\text{+}15) + 1(\text{-}4\text{+}9)$

=-16+11+5=0

∴The vectors $\bar{a},\bar{b},\bar{c}$ are coplanar

13. If $\bar{a},\bar{b}$ and $\bar{c}$ are unit coplanar vectors, then find $[2\bar{a} - \bar{b} \quad 2\bar{b} - \bar{c} \quad 2\bar{c} - \bar{a}]$

Sol: $[2\bar{a} - \bar{b} \quad 2\bar{b} - \bar{c} \quad 2\bar{c} - \bar{a}]$

$= (2\bar{a} - \bar{b}).\{(2\bar{b} - \bar{c})x(2\bar{c} - \bar{a})$

$= (2\bar{a} - \bar{b})\{4(\bar{b}x\bar{c}) - 2(\bar{b}x\bar{a}) + (\bar{c}x\bar{a})\}$

$= 8\{\bar{a}.(\bar{b}x\bar{c}) - \bar{b}.(\bar{c}x\bar{a})$

$= 8[\bar{a} \;\; \bar{b} \;\; \bar{c}] - [\bar{a} \;\; \bar{b} \;\; \bar{c}] = 7[\bar{a} \;\; \bar{b} \;\; \bar{c}] = 0$

$\therefore [2\bar{a} - \bar{b} \quad 2\bar{b} - \bar{c} \quad 2\bar{c} - \bar{a}] = 0$

14.Find the value of t, if the vectors $\bar{a} = 2\bar{i} - 3\bar{j} + \bar{k}$; $\bar{b} = \bar{i} + 2\bar{j} - 3\bar{k}$; $\bar{c} = \bar{j} - t\bar{k}$ are coplanar.

Sol: $\bar{a},\bar{b}$ and $\bar{c}$ are coplanar vectors, then

$[\bar{a} \;\; \bar{b} \;\; \bar{c}] = 0$

$[\bar{a} \;\; \bar{b} \;\; \bar{c}] = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -3 \\ 0 & 1 & -t \end{vmatrix} = 2(\text{-}2t\text{+}3) + 3(\text{-}t\text{-}0) + 1(1\text{-}0)$

= -4t+6-3t+1=0

= -7t+7=0 ⇒t=1

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6. TRIGONOMETRIC RATIOS AND FUNCTIONS

1. Find the value of

$\cos 225°$- $\sin 225°$+$\tan 495°$- $\cot 495°$

Sol: $\cos 225°$- $\sin 225°$+$\tan 495°$- $\cot 495°$

=$\cos(180+45)°$- $\sin(180+45)°$+$\tan(360+135)°$- $\cot(360+135)°$

= -$\cos 45°$+$\sin 45°$+$\tan 135°$- $\cot 135°$

= $-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$-1+1=0

2. Find the value of

$\sin^2\frac{\pi}{10}+\sin^2\frac{4\pi}{10}+\sin^2\frac{6\pi}{10}+\sin^2\frac{9\pi}{10}$

Sol: $\sin^2\frac{\pi}{10}+\sin^2\frac{4\pi}{10}+\sin^2\frac{6\pi}{10}+\sin^2\frac{9\pi}{10}$

$=\sin^2\frac{\pi}{10}+\sin^2(\frac{5\pi-\pi}{10})+\sin^2(\frac{5\pi+\pi}{10})+\sin^2(\frac{10\pi-\pi}{10})$

$=\sin^2\frac{\pi}{10}+\sin^2(\frac{\pi}{2}-\frac{\pi}{10})+\sin^2(\frac{\pi}{2}+\frac{\pi}{10})+\sin^2(\pi-\frac{\pi}{10})$

$=\sin^2\frac{\pi}{10}+\cos^2\frac{\pi}{10}+\cos^2\frac{\pi}{10}+\sin^2\frac{\pi}{10}$

$=2(\sin^2\frac{\pi}{10}+\cos^2\frac{\pi}{10})=2$

3. Find the value of

$\cos^2 45°+\cos^2 135°+\cos^2 225°+\cos^2 315°$

Sol: $\cos^2 45°+\cos^2 135°+\cos^2 225°+\cos^2 315°$

$=\cos^2 45°+\cos^2(180-45)°+\cos^2(180°+45°)+\cos^2(360-45)°$

$=\cos^2 45°+\cos^2 45°+\cos^2 45°+\cos^2 45°$

$=4\cos^2 45°=4(\frac{1}{\sqrt{2}})^2=4(\frac{1}{2})=2$

4. Find the value of $\sin^2\frac{2\pi}{3}+\cos^2\frac{5\pi}{6}-\tan^2\frac{3\pi}{4}$

Sol: $\sin^2\frac{2\pi}{3}+\cos^2\frac{5\pi}{6}-\tan^2\frac{3\pi}{4}$

$=\sin^2 120°+\cos^2 150°-\tan^2 135°$

$=\sin^2(90+30)°+\cos^2(90+60)°-\tan^2(90+45)°$

$=\cos^2 30°+\sin^2 60°-\cot^2 45°=(\frac{\sqrt{3}}{2})^2+(\frac{\sqrt{3}}{2})^2-1$

$=\frac{3}{4}+\frac{3}{4}-1=\frac{1}{2}$

5. If $\tan 20°$=P, then prove that $\frac{\tan 610°+\tan 700°}{\tan 560°-\tan 470°}=-\frac{1-P^2}{1+P^2}$

Sol: $\frac{\tan 610°+\tan 700°}{\tan 560°-\tan 470°}$

$\Rightarrow \tan 610°=\tan(\frac{7\pi}{2}-20)=\cot 20°=\frac{1}{\tan 20°}$

$\tan 700°=\tan(4\pi-20)=-\tan 20°$

$\tan 560°=\tan(3\pi+20)=-\tan 20°$

$\tan 470°=\tan(\frac{5\pi}{2}+20)=\cot 20°=\frac{1}{\tan 20°}$

$\therefore \frac{\tan 610°+\tan 700°}{\tan 560°-\tan 470°}=\frac{\frac{1}{\tan 20°}-\tan 20°}{-\tan 20°-\frac{1}{\tan 20°}}=\frac{\frac{1}{P}-P}{-P-\frac{1}{P}}=\frac{\frac{1-P^2}{P}}{\frac{P^2+1}{P}}$

$=-\frac{1-P^2}{1+P^2}$

6. Show that $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}=1$

Sol: LHS $=\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$

$=(\cot\frac{\pi}{20}\cot\frac{9\pi}{20})(\cot\frac{3\pi}{20}\cot\frac{7\pi}{20})\cot\frac{5\pi}{20}$

$=\cot\frac{\pi}{2}\cot\frac{\pi}{2}\cot\frac{\pi}{4}$=(1)(1)(1)=1=RHS

$\because$A+B = $\frac{\pi}{2}$

$\cot A\cot B=1$

$\frac{\pi}{20}+\frac{9\pi}{20}=\frac{3\pi}{20}+\frac{7\pi}{20}=\frac{\pi}{2}$

7. If $\tan 20°=\lambda$, then prove that $\frac{\tan 160°-\tan 110°}{1+\tan 160°\tan 110°}=\frac{1-\lambda^2}{2\lambda}$

Sol: $\tan 20°=\lambda$

$\tan 160°=\tan(180°-20)=-\tan 20°=-\lambda$

$\tan 110°=\tan(90°+20)=-\cot 20°=-\frac{1}{\tan 20°}=\frac{-1}{\lambda}$

$\therefore$LHS$=\frac{\tan 160°-\tan 110°}{1+\tan 160°\tan 110°}=\frac{-\lambda-(\frac{-1}{\lambda})}{1+(-\lambda)(\frac{-1}{\lambda})}=\frac{\frac{1}{\lambda}-\lambda}{1+1}=\frac{1-\lambda^2}{2\lambda}$ = RHS

$\therefore$LHS = RHS

8. Prove that

$(\sin\theta+\cos\theta)^2+(\cos\theta+\sec\theta)^2-(\tan^2\theta+\cot^2\theta)=7$

Sol: LHS

$=(\sin\theta+\mathrm{cosec}\,\theta)^2+(\cos\theta+\sec\theta)^2-(\tan^2\theta+\cot^2\theta)$

$=(\sin^2\theta+\mathrm{cosec}^2\theta+2\sin\theta.\mathrm{cosec}\,\theta)+$

$(\cos^2\theta+\sec^2\theta+2\cos\theta.\sec\theta)-(\tan^2\theta+\cot^2\theta)$

$=(\sin^2\theta+\cos^2\theta)+(\mathrm{cosec}^2\theta-\cot^2\theta)+$

$(\sec^2\theta-\tan^2\theta)+2\sin\theta.\frac{1}{\sin\theta}+2\cos\theta.\frac{1}{\cos\theta}$

=1+1+1+2+2=7=RHS

$\therefore$LHS = RHS

9. Prove that $\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta+\cos\theta)^2}=\frac{1-\cos\theta}{1+\cos\theta}$

Sol: LHS$=\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta+\cos\theta)^2}$

$=\frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\cos\theta-2\sin\theta\cos\theta}{1+\sin^2\theta+\cos^2\theta+2\sin\theta+2\cos\theta+2\sin\theta\cos\theta}$

$=\frac{1+1+2\sin\theta-2\cos\theta-2\sin\theta\cos\theta}{1+1+2\sin\theta+2\cos\theta+2\sin\theta\cos\theta}$

$=\frac{2(1+\sin\theta-\cos\theta-\sin\theta\cos\theta)}{2(1+\sin\theta+\cos\theta+\sin\theta\cos\theta)}$

$=\frac{(1+\sin\theta)(1-\cos\theta)}{(1+\sin\theta)(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}$=RHS

$\therefore$LHS = RHS

10. If $\frac{2\sin\theta}{1+\cos\theta+\sin\theta}$ = x then prove that $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$ = x

Sol: $x=\frac{2\sin\theta}{1+\cos\theta+\sin\theta}=\frac{2\sin\theta}{1+\sin\theta+\cos\theta}$

$=\frac{2\sin\theta}{1+\sin\theta+\cos\theta}.\frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}$

$=\frac{2\sin\theta(1+\sin\theta-\cos\theta)}{(1+\sin\theta)^2-\cos^2\theta}=\frac{2\sin\theta(1+\sin\theta-\cos\theta)}{1+\sin^2\theta+2\sin\theta-\cos^2\theta}$

$=\frac{2\sin\theta(1+\sin\theta-\cos\theta)}{2\sin^2\theta+2\sin\theta}$

$=\frac{2\sin\theta(1+\sin\theta-\cos\theta)}{2\sin\theta(1+\sin\theta)}=\frac{1+\sin\theta-\cos\theta}{1+\sin\theta}=\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}=x$

=RHS

$\therefore$LHS = RHS

11.Show that $\cos^4\alpha+2\cos^2\alpha(1-\frac{1}{\sec^2\alpha})=1-\sin^4\alpha$

Sol: LHS = $\cos^4\alpha+2\cos^2\alpha(1-\frac{1}{\sec^2\alpha})$

$=\cos^4\alpha+2\cos^2\alpha(1-\cos^2\alpha)$

$=\cos^4\alpha+2\cos^2\alpha\sin^2\alpha$

$=\cos^2\alpha[\cos^2\alpha+2\sin^2\alpha]$

$=(1-\sin^2\alpha)[\cos^2\alpha+\sin^2\alpha+\sin^2\alpha]$

$=(1-\sin^2\alpha)(1+\sin^2\alpha)=1-\sin^4\alpha$=RHS

∴LHS = RHS

12. Prove that

$$2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$$

Sol: LHS$=2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1$

$=2[(\sin^2\theta)^3+(\cos^2\theta)^3]-3[(\sin^2\theta)^2+(\cos^2\theta)^2]+1$

$=2[(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)]-3[(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta]+1$

$=2[1-3\sin^2\theta\cos^2\theta]-3[1-2\sin^2\theta\cos^2\theta]+1$

$=2-6\sin^2\theta\cos^2\theta-3+6\sin^2\theta\cos^2\theta]+1=0$=RHS

∴LHS = RHS

13. Prove that

$$(\tan\theta+\cot\theta)^2=\sec^2\theta+\text{cosec}^2\theta=\sec^2\theta\,.\,\text{cosec}^2\theta$$

Sol: LHS $=(\tan\theta+\cot\theta)^2$

$=\tan^2\theta+\cot^2\theta+2\tan\theta.\cot\theta$

$=\sec^2\theta-1+\text{cosec}^2\theta-1+2=\sec^2\theta+\text{cosec}^2\theta$=RHS

∴LHS = RHS

14. If $\tan^2\theta=1-e^2$ then show that

$$\sec\theta+\tan^3\theta\,\text{cosec}\,\theta=(2-e^2)^{\frac{3}{2}}$$

Sol: LHS = $\sec\theta+\tan^3\theta\,\text{cosec}\,\theta$

$=\frac{1}{\cos\theta}+\tan^2\theta.\tan\theta\frac{1}{\sin\theta}$

$=\frac{1}{\cos\theta}+\tan^2\theta.\frac{\sin\theta}{\cos\theta}\frac{1}{\sin\theta}$

$=\frac{1}{\cos\theta}[1+\tan^2\theta]=\sec\theta[1+\tan^2\theta]$

$=\sqrt{1+\tan^2\theta}\,(1+\tan^2\theta)$

$=(1+\tan^2\theta)^{\frac{3}{2}}=(1+1-e^2)^{\frac{3}{2}}$

$=(2-e^2)^{\frac{3}{2}}$=RHS

∴LHS = RHS

15. Prove that

$$3(\sin x-\cos x)^4+6(\sin x+\cos x)^2+4(\sin^6x+\cos^6x)=13$$

Sol: Consider

$(\sin\theta-\cos\theta)^2=\sin^2\theta+\cos^2\theta-2\sin\theta.\cos\theta$

$=1-2\sin\theta.\cos\theta$

$(\sin\theta-\cos\theta)^4=[(\sin\theta-\cos\theta)^2]^2$

$=[1-2\sin\theta.\cos\theta]^2$

$=1+4\sin^2\theta.\cos^2\theta-4\sin\theta.\cos\theta$

$(\sin\theta+\cos\theta)^2=\sin^2\theta+\cos^2\theta+2\sin\theta.\cos\theta$

$=1+2\sin\theta.\cos\theta$

$\sin^6\theta+\cos^6\theta=(\sin^2\theta)^3+(\cos^2\theta)^3$

$=[(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta.\cos^2\theta(\sin^2\theta+\cos^2\theta)$

$=1-3\sin^2\theta.\cos^2\theta$

$\therefore 3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4(\sin^6\theta+\cos^6\theta)$

$=3[1+4\sin^2\theta.\cos^2\theta-4\sin\theta.\cos\theta]+6[1+2\sin\theta.\cos\theta]+4[1-3\sin^2\theta.\cos^2\theta]$

$=3+12\sin^2\theta.\cos^2\theta-12\sin\theta.\cos\theta+6+12\sin\theta.\cos\theta+4-12\sin^2\theta.\cos^2\theta$

=3+6+4=13 = RHS

∴LHS = RHS

16. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$

Sol: LHS$=\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$

$=\frac{\tan\theta+\sec\theta-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$

$=\frac{(\tan\theta+\sec\theta)[1-(\sec\theta-\tan\theta)]}{\tan\theta-\sec\theta+1}$

$=\frac{(\tan\theta+\sec\theta)[1-\sec\theta+\tan\theta]}{\tan\theta-\sec\theta+1}=\tan\theta+\sec\theta$

$=\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}=\frac{\sin\theta+1}{\cos\theta}=\frac{1+\sin\theta}{\cos\theta}$=RHS

∴LHS = RHS

17. If 3sin θ+4cos θ=5 then find the value of 4sin θ-3cos θ.

Sol: Let $4\sin\theta-3\cos\theta=x$ ....(1)

$3\sin\theta+4\cos\theta=5$ ....(2)

Square eq(1) and eq(2) and add

$x^2+5^2=(4\sin\theta-3\cos\theta)^2+(3\sin\theta+4\cos\theta)^2$

$=16\sin^2\theta-24\sin\theta.\cos\theta+9\cos^2\theta+9\sin^2\theta-24\sin\theta.\cos\theta+16\cos^2\theta$

$=25\sin^2\theta+25\cos^2\theta=25(\sin^2\theta+\cos^2\theta)=25$

$x^2+25=25$

$x^2=0$; $x=0$

$\therefore 4\sin\theta-3\cos\theta=0$

18. If 3sin A+5cos A =5 then show that 5sin A-3cos A =±3

Sol: Let $5\sin A-3\cos A=x$ ....(1)

$3\sin A+5\cos A=5$ ....(2)

Square eq(1) and eq(2) and add

$x^2+5^2=(5\sin A-3\cos A)^2+(3\sin A+5\cos A)^2$

$=25\sin^2A-30\sin A.\cos A+9\cos^2A+9\sin^2A-30\sin A.\cos A+25\cos^2A$

$=34\sin^2A+34\cos^2A=34(\sin^2\theta+\cos^2\theta)=24$

$x^2+25=34$

$x^2=9$; x==±3

$\therefore 5\sin A-3\cos A=\pm3$

19. If acosθ -b sinθ= C then show that

$$a\sin\theta+b\cos\theta=\pm\sqrt{a^2+b^2-c^2}$$

Sol: $a\sin\theta+b\cos\theta=x$ ....(1)

$a\cos\theta-b\sin\theta=C$ ....(2)

Square eq(1) and eq(2) and add

$x^2+c^2=(a\sin\theta+b\cos\theta)^2+(a\cos\theta-b\sin\theta)^2$

$=a^2\sin^2\theta+2ab\sin\theta.\cos\theta+b^2\cos^2\theta+a^2\cos^2\theta-ab\sin\theta.\cos\theta+b^2\sin^2\theta$

$=(a^2+b^2)\sin^2\theta+(a^2+b^2)\cos^2\theta=(a^2+b^2)(\sin^2\theta+\cos^2\theta)=(a^2+b^2)$

$x^2+c^2=(a^2+b^2)$

$x^2=a^2+b^2-c^2$; $x=\pm\sqrt{a^2+b^2-c^2}$

$\therefore a\sin\theta+b\cos\theta=\pm\sqrt{a^2+b^2-c^2}$

20. If cos θ+sin θ =$\sqrt{2}$ cos θ, then prove that cos θ-sin θ=$\sqrt{2}$ sin θ.

Sol: $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$

$\sqrt{2}\cos\theta-\cos\theta=\sin\theta$

$(\sqrt{2}-1)\cos\theta=\sin\theta$

$\cos\theta=\frac{\sin\theta}{\sqrt{2}-1}=$

$\frac{\sin\theta}{\sqrt{2}-1}\frac{\sqrt{2}+1}{\sqrt{2}+1}=\frac{\sin\theta(\sqrt{2}+1)}{\sqrt{2}-1\sqrt{2}+1}=\frac{\sin\theta(\sqrt{2}+1)}{2-1}=\sin\theta(\sqrt{2}+1)$

$= \sqrt{2}\sin\theta + \sin\theta$

$\therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

21. If $x = a\cos^3\theta$; $y = b\sin^3\theta$ then eliminate θ.

Sol: $x = a\cos^3\theta$; $y = b\sin^3\theta$

$\cos\theta = (\frac{x}{a})^{\frac{1}{3}}$ $\sin\theta = (\frac{y}{b})^{\frac{1}{3}}$

$\therefore \cos^2\theta + \sin^2\theta = 1$

$(\frac{x}{a})^{\frac{2}{3}} + (\frac{y}{b})^{\frac{2}{3}} = 1$

$\frac{x^{\frac{2}{3}}}{a^{\frac{2}{3}}} + \frac{y^{\frac{2}{3}}}{b^{\frac{2}{3}}} = 1$

22.Prove that

$\sin 780° \sin 480° + \cos 240° \cos 300° = \frac{1}{2}$

Sol: LHS=$\sin 780° \sin 480° + \cos 240° \cos 300°$

$= \sin(2\times 360° + 60°)\sin(450° + 30°) + \cos(180° + 60°)\cos(360° - 60°)$

$= \sin 60°(-\cos 30°) + (-\cos 60°)\cos 60°$

$= \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} - \frac{1}{2}\frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

$\therefore \sin 780° \sin 480° + \cos 240° \cos 300° = \frac{1}{2}$

23. Find the value of

$\sin 330° \cos 120° + \cos 210° \sin 300°$

Sol: $\sin 330° \cos 120° + \cos 210° \sin 300°$

$= \sin(360° - 30°)\cos(180° - 60°) + \cos(180° + 30°)\sin(360° - 60°)$

$= -\sin 30°(-\cos 60°) + (-\cos 30°)(-\sin 60°)$

$= (-\frac{1}{2})\left(-\frac{1}{2}\right) + (-\frac{\sqrt{3}}{2})(-\frac{\sqrt{3}}{2}) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

Exercise problems

1(i). Find the value of $\sin\frac{5\pi}{3}$.

Sol: $\sin\frac{5\pi}{3} = \sin(2\pi - \frac{\pi}{3}) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

1(ii). Find the value of $\sec\frac{13\pi}{3}$

Sol: $\sec\frac{13\pi}{3} = \sec(4\pi + \frac{\pi}{3}) = \sec\frac{\pi}{3} = 2$

1(iii). Find the value of $\cos(-\frac{7\pi}{2})$

Sol: $\cos(-\frac{7\pi}{2}) = \cos(4\pi - \frac{\pi}{2}) = \cos\frac{\pi}{2} = 0$

1(iv). Find the value of $\tan 855°$

Sol: $\tan 855° = \tan((900° - 45°) = -\tan 45° = -1$

1(v). Find the value of $\sec 2100°$

Sol: $\sec 2100° = \sec((2160° - 60°) = \sec 60° = 2$

1(vi). Find the value of $\cot(-315°)$

Sol: $\cot(-315°) = \cot(360° - 45°) = \cot 45° = 1$

2. Prove the following

$$\frac{\cos(\pi-\theta)\cot(\frac{\pi}{2}+\theta)\cos(-\theta)}{\tan(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)\sin(2\pi-\theta)} = \cos\theta$$

Sol: LHS = $\frac{\cos(\pi-\theta)\cot(\frac{\pi}{2}+\theta)\cos(-\theta)}{\tan(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)\sin(2\pi-\theta)}$

$= \frac{-\cos\theta\,(-\tan\theta)\,(\cos\theta)}{\tan\theta(-\cot\theta)\,(-\sin\theta)} = \cos\theta = \text{RHS}$

$\therefore$LHS = RHS

3. Prove the following

$$\frac{\sin(3\pi-\theta)\cos(\theta-\frac{\pi}{2})\tan(\frac{3\pi}{2}-\theta)}{\sec(3\pi+\theta)\,\mathrm{cosec}(\frac{13\pi}{2}+\theta)\cot(\theta-\frac{\pi}{2})} = \cos^4\theta$$

Sol: LHS=$\frac{\sin(3\pi-\theta)\cos(\theta-\frac{\pi}{2})\tan(\frac{3\pi}{2}-\theta)}{\sec(3\pi+\theta)\,\mathrm{cosec}(\frac{13\pi}{2}+\theta)\cot(\theta-\frac{\pi}{2})}$

$= \frac{\sin\theta\sin\theta\cot\theta}{-\sec\theta\sec\theta\,(-\tan\theta)} =$

$= \frac{\sin\theta\sin\theta\,\frac{\cos\theta}{\sin\theta}}{-\frac{1}{\cos\theta}\frac{1}{\cos\theta}\,(-\frac{\sin\theta}{\cos\theta})} = \cos^4\theta = \text{RHS}$

$\therefore$LHS = RHS

4. Prove the following

$\cot\frac{\pi}{16}\cot\frac{2\pi}{16}\cot\frac{3\pi}{16}\ldots.\cot\frac{7\pi}{16} = 1$

Sol: LHS=$\cot\frac{\pi}{16}\cot\frac{2\pi}{16}\cot\frac{3\pi}{16}\cot\frac{4\pi}{16}\cot\frac{5\pi}{16}\cot\frac{6\pi}{16}\cot\frac{7\pi}{16}$

$= (\cot\frac{\pi}{16}\cot\frac{7\pi}{16})(\cot\frac{2\pi}{16}\cot\frac{6\pi}{16})(\cot\frac{3\pi}{16}\cot\frac{5\pi}{16})\cot\frac{4\pi}{16}$

$= (\cot\frac{\pi}{16}\cot(\frac{8\pi-\pi}{16}))(\cot\frac{2\pi}{16}\cot(\frac{8\pi-2\pi}{16}))(\cot\frac{3\pi}{16}\cot(\frac{8\pi-3\pi}{16}))\cot\frac{\pi}{4}$

$= (\cot\frac{\pi}{16}\cot(\frac{\pi}{2} - \frac{\pi}{16}))(\cot\frac{2\pi}{16}\cot(\frac{\pi}{2} - \frac{2\pi}{16}))(\cot\frac{3\pi}{16}\cot(\frac{\pi}{2} - \frac{3\pi}{16}))1$

$= (\cot\frac{\pi}{16}\tan\frac{\pi}{16})(\cot\frac{2\pi}{16}\tan\frac{2\pi}{16})(\cot\frac{3\pi}{16}\tan\frac{3\pi}{16}) = 1.1.1 = \text{RHS}$

$\therefore$LHS = RHS

5(i) Eliminate θ from the following $x = a\cos^4\theta$; $y = b\sin^4\theta$

Sol: $x = a\cos^4\theta$; $y = b\sin^4\theta$

$\cos\theta = (\frac{x}{a})^{\frac{1}{4}}$ $\sin\theta = (\frac{y}{b})^{\frac{1}{4}}$

$\therefore \cos^2\theta + \sin^2\theta = 1$

$(\frac{x}{a})^{\frac{2}{4}} + (\frac{y}{b})^{\frac{2}{4}} = 1$

$(\frac{x}{a})^{\frac{1}{2}} + (\frac{y}{b})^{\frac{1}{2}} = 1$

$\frac{\sqrt{x}}{\sqrt{a}} + \frac{\sqrt{y}}{\sqrt{b}} = 1$

5(ii) Eliminate θ from the following

$x = a(\sec\theta + \tan\theta)$; $y = b(\sec\theta - \tan\theta)$

Sol: Given $x = a(\sec\theta + \tan\theta)$; $y = b(\sec\theta - \tan\theta)$

$xy = a(\sec\theta + \tan\theta) \times b(\sec\theta - \tan\theta)$

$= ab(\sec^2\theta - \tan^2\theta) = ab.1 = ab$

5(iii) Eliminate θ from the following

$x = (\cot\theta + \tan\theta)$; $y = (\sec\theta - \cos\theta)$

Sol: Given $x = (\cot\theta + \tan\theta)$; $y = (\sec\theta - \cos\theta)$

$x = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$; $y = \frac{1}{\cos\theta} - \cos\theta$

$x = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$ $y = \frac{1 - \cos^2\theta}{\cos\theta}$

6. If $\sin\alpha = -\frac{1}{3}$ and α does not lie in the third quadrant, then find the values of $\cot\alpha$ and $\cos\alpha$.

Sol: We know that $\sin\alpha$ is negative in third and fourth quadrant. Since α is not in third quadrant (Q_3). We have $\alpha \in Q_4$

$\because \sin\alpha = -\frac{1}{3}$

$\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - (-\frac{1}{3})^2} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\sqrt{\frac{8}{9}}}{-\frac{1}{3}} = -\sqrt{8} = -2\sqrt{2}$

7. If $\sin\theta = \frac{4}{5}$ and θ does not lie in the first quadrant, then find the values of $\cos\theta$.

Sol: We know that $\sin\theta$ is positive in first and second quadrant. Since θ is not in first quadrant (Q_1). We have $\theta \in Q_2$

$\because \sin\theta = \frac{4}{5}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(\frac{4}{5})^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\cos\theta$ in second quadrant is negative ,so $\cos\theta = \frac{3}{5}$

COMPOUND ANGLES

1.Find the value of $\sin 75°$, $\cos 75°$, $\tan 75°$

Sol: $\sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

$\cos 75° = \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\tan 75° = \tan(45° + 30°)$

$= \frac{\tan 45° + \tan 30°}{1-\tan 45° \tan 30°} = \frac{1+\frac{1}{\sqrt{3}}}{1-1.\frac{1}{\sqrt{2}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

2. Prove that $\cos 100° \cos 40° + \sin 100° \sin 40° = \frac{1}{2}$

Sol: LHS $= \cos 100° \cos 40° + \sin 100° \sin 40°$

$= \cos(100° - 40°) = \cos 60° = \frac{1}{2}$

$\because \cos A \cos B + \sin A \sin B = \cos(A-B)$

3. Prove that $\tan 75° + \cot 75° = 4$

Sol: LHS $= \tan 75° + \cot 75°$

$= \tan(45°+75°) + \cot(45°+30°)$

$= \frac{\tan 45° + \tan 30°}{1-\tan 45° \tan 30°} + \frac{\cot 45° \cot 30° - 1}{\cot 45° + \cot 30°}$

$= \frac{1+\frac{1}{\sqrt{3}}}{1-1.\frac{1}{\sqrt{2}}} + \frac{1.\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{(\sqrt{3}+1)^2(\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+1+2\sqrt{3}+3+1-2\sqrt{3}}{3-1} = \frac{8}{2} = 4 =$ RHS

$\therefore$ LHS = RHS

4. Prove that $\cos 100° \cos 40° + \sin 100° \sin 40° = \frac{1}{2}$

Repeated Q.No.2

5. Show that $\cos 42° + \cos 78° + \cos 162° = 0$

Sol: LHS $= \cos 42° + \cos 78° + \cos 162°$

$= \cos(60°-18°) + \cos(60°+18°) + \cos(180°-18°)$

$= \cos 60°.\cos 18° + \sin 60°.\sin 18°$

$+ \cos 60°.\cos 18° - \sin 60°.\sin 18° - \cos 18°$

$= 2\cos 60°.\cos 18° - \cos 18°$

$= 2.\frac{1}{2}\cos 18° - \cos 18° = \cos 18° - \cos 18° = 0 =$ RHS

$\therefore$ LHS = RHS

6. If $\sin(\theta+\alpha) = \cos(\theta+\alpha)$ then find $\tan\theta$ in term of $\tan\alpha$

Sol: Given $\sin(\theta+\alpha) = \cos(\theta+\alpha)$

$\sin\theta.\cos\alpha + \cos\theta.\sin\alpha = \cos\theta.\cos\alpha - \sin\theta.\sin\alpha$

$\sin\theta(\cos\alpha + \sin\alpha) = \cos\theta(\cos\alpha - \sin\alpha)$

$\frac{\sin\theta}{\cos\theta} = \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} = \tan\theta$

$\therefore \tan\theta = \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} = \frac{\frac{\cos\alpha}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}}{\frac{\cos\alpha}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}} = \frac{1-\tan\alpha}{1+\tan\alpha}$

7. Find the value of $\sin^2 82\frac{1}{2}° - \sin^2 22\frac{1}{2}°$

Sol: $\sin^2 82\frac{1}{2}° - \sin^2 22\frac{1}{2}°$

$\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

$= \sin(82\frac{1}{2}° + 22\frac{1}{2}°)\sin(82\frac{1}{2}° - 22\frac{1}{2}°)$

$= \sin 105° \sin 60°$

$= \sin(60° + 45°)\sin 60° = \sin(60° + 45°)\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}[\sin 60°.\cos 45° + \cos 60°.\sin 45°]$

$= \frac{\sqrt{3}}{2}[\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}}] = \frac{\sqrt{3}}{2}[\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}] = \frac{\sqrt{3}(\sqrt{3}+1)}{4\sqrt{2}}$

8. Find the value of $\cos^2 112\frac{1}{2}° - \sin^2 52\frac{1}{2}°$

Sol: $\cos^2 112\frac{1}{2}° - \sin^2 52\frac{1}{2}°$ $\because \cos^2 A - \sin^2 B$

$= \cos(A+B)\cos(A-B)$

$= \cos(112\frac{1}{2}° + 52\frac{1}{2}°)\cos(112\frac{1}{2}° - 52\frac{1}{2}°)$

$= \cos 165° \cos 60° = \cos(90° + 75°)\cos 60°$

$= -\sin 75° \cos 60° = -\sin(45° + 30°)\frac{1}{2}$

$= -\frac{1}{2}[\sin 45°.\cos 30° + \cos 45°.\sin 30°]$

$= -\frac{1}{2}[\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}] = -\frac{1}{2}[\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}] = \frac{-(\sqrt{3}+1)}{4\sqrt{2}}$

9. Find the value of

$\tan 20° + \tan 40° + \sqrt{3}\tan 20° \tan 40°$

Sol: $\tan 40° = \tan(60° - 20°)$

$= \frac{\tan 60° - \tan 20°}{1+\tan 60° \tan 20°} = \frac{\sqrt{3} - \tan 20°}{1+\sqrt{3}\tan 20°}$

$\therefore \tan 20° + \tan 40° + \sqrt{3}\tan 20° \tan 40°$

$= \tan 20° + \tan 40°(1+\sqrt{3}\tan 20°)$

$= \tan 20° + \frac{\sqrt{3} - \tan 20°}{1+\sqrt{3}\tan 20°}(1+\sqrt{3}\tan 20°)$

$= \tan 20° + \sqrt{3} - \tan 20° = \sqrt{3}$

10. Find the value of $\tan 56° - \tan 11° - \tan 56° \tan 11°$

Sol: $\tan 56° = \tan(45° + 11°)$

$= \frac{\tan 45° + \tan 11°}{1-\tan 45° \tan 11°} = \frac{1+\tan 11°}{1-\tan 11°}$

$\therefore \tan 56° - \tan 11° - \tan 56° \tan 11° =$

$\tan 56°(1-\tan 11°) - \tan 11°$

$= \frac{1+\tan 11°}{1-\tan 11°}(1-\tan 11°) - \tan 11°$

$= 1 + \tan 11° - \tan 11° = 1$

11. If $\sin\alpha = \frac{1}{\sqrt{10}}$; $\sin\beta = \frac{1}{\sqrt{5}}$ and α and β are acute, then show that $\alpha + \beta = \frac{\pi}{4}$.

Sol: Given α is acute and $\sin\alpha = \frac{1}{\sqrt{10}} \Rightarrow \tan\alpha = \frac{1}{3}$

B is acute and $\sin\beta = \frac{1}{\sqrt{5}} \Rightarrow \tan\beta = \frac{1}{2}$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\frac{1}{2}}=\frac{2+3}{6-1}=\frac{5}{5}=1$

$\therefore\ \alpha+\beta=\tan^{-1}1=\frac{\pi}{4}$

12. If $\sin(A+B)=\frac{24}{25}$, $\tan A=\frac{3}{4}$A, A+B are acute then find the value of $\cos B$.

Sol: $\sin(A+B)=\frac{24}{25}$; $\Rightarrow \cos(A+B)=\frac{7}{25}$

$\tan A=\frac{3}{4}$;$\Rightarrow \sin A=\frac{3}{5}$; $\cos A=\frac{4}{5}$

$\cos B=\cos(A+B-A)$

$=\cos(A+B).\cos A+\sin(A+B).\sin A$

$=\frac{7}{25}\text{x}\frac{4}{5}+\frac{24}{25}\text{x}\frac{3}{5}=\frac{28+72}{125}=\frac{100}{125}=\frac{4}{5}$

$\therefore \cos B=\frac{4}{5}$

13. If A+B = $45°$, then prove that (1+tan A)(1+tan B)=2

Sol: A+B = $45°$,

$\tan(A+B)=\tan 45°$

$\tan(A+B)=1$

$=\frac{\tan A+\tan B}{1-\tan A\tan B}=1$

$=\tan A+\tan B=1-\tan A\tan B$

$=\tan A+\tan B+\tan A\tan B=1$

$=1+\tan A+\tan B+\tan A\tan B=1+1$

$=(1+\tan A)(1+\tan B)=2$

$\therefore(1+\tan A)(1+\tan B)=2$

14. If A+B = $225°$, then prove that $\frac{\cot A+\cot B}{(1+\cot A)(1+\cot B)}=2$

Sol: A+B = $225°=(180°+45°)$

$\cot(A+B)=\cot(180°+45°)=\cot 45°=1$

$=\frac{\cot A\cot B-1}{\cot A+\cot B}=1$

$=\cot A\cot B-1=\cot A+\cot B$

$=\cot A\cot B=1+\cot A+\cot B$

$=2\cot A\cot B=1+\cot A+\cot B+\cot A\cot B$

$=(1+\cot A)(1+\cot B)=2\cot A\cot B$

15.If A-B=$\frac{3\pi}{4}$, then show that $(1-\tan t A)(1+\tan B)=2$

Sol: A-B=$\frac{3\pi}{4}=135°$

$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$\tan 135°=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$-1=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$-1-\tan A\tan B=\tan A-\tan B$

$-\tan A+\tan B-\tan A\tan B=1$

$1-\tan A+\tan B-\tan A\tan B=1+1=2$

$(1-\tan t A)(1+\tan B)=2$

$\therefore(1-\tan t A)(1+\tan B)=2$

16. If A+B+C = $\frac{\pi}{2}$, then prove that $\cot A+\cot B+\cot C=\cot A\cot B\cot C$

Sol: A+B+C = $\frac{\pi}{2}$

A+B = $\frac{\pi}{2}-C$

$\cot(A+B)=\cot(\frac{\pi}{2}-C)=\tan C=\frac{1}{\cot C}$

$\frac{\cot A\cot B-1}{\cot A+\cot B}=\frac{1}{\cot C}$

$\cot A\cot B\cot C-\cot C=\cot A+\cot B$

$\cot A\cot B\cot C=\cot A+\cot B+\cot C$

$\therefore \cot A+\cot B+\cot C=\cot A\cot B\cot C$

17. If A+B+C = $\frac{\pi}{2}$, then prove that $\tan A\tan B+\tan B\tan C+\tan C\tan A=1$

Sol: A+B+C = $\frac{\pi}{2}$

A+B = $\frac{\pi}{2}-C$

$\tan(A+B)=\tan(\frac{\pi}{2}-C)=\cot C=\frac{1}{\tan C}$

$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{1}{\tan C}$

$\tan A.\tan C+\tan B.\tan C=1-\tan A\tan B$

$\therefore \tan A\tan B+\tan B\tan C+\tan C\tan A=1$

18. If A+B+C = $180°$, then prove that $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

Sol: A+B+C $=180°$

A+B = $180°-C$

$\tan(A+B)=\tan(180°-C)=-\tan C$

$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$

$\tan A+\tan B=-\tan C+\tan A\tan B\tan C$

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$

$\therefore \tan A+\tan B+\tan C=\tan A\tan B\tan C$

19. If A+B+C = $180°$, then prove that $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$

Sol: A+B+C $=180°$

A+B = $180°-C$

$\cot(A+B)=\cot(180°-C)=-\cot C$

$\frac{\cot B\cot A-1}{\cot B+\cot A}=-\cot C$

$\cot B\cot A-1=-\cot B\cot C+\cot C\cot A$

$\therefore \cot A\cot B+\cot B\cot C+\cot C\cot A=1$

20. Find the expansion of (i) Sin (*A+ B −C*) (ii) cos(*A− B −C*) .

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21. If $\sin(A+B)=\frac{24}{25}$ and $\cos(A-B)=\frac{4}{5}$ where $0<A<B<\frac{\pi}{4}$, then find tan 2A.

Sol: $\sin(A+B)=\frac{24}{25}\Rightarrow\tan(A+B)=\frac{24}{7}$

$\cos(A-B)=\frac{4}{5}\Rightarrow\tan(A-B)=\frac{-3}{4}$ since A<B $\Rightarrow$ $A-B<0$

$\tan(2A)=\tan[(A+B)+(A-B)]$

$\tan 2A=\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}=\frac{\frac{24}{7}+\frac{-3}{4}}{1-\frac{24}{7}(\frac{-3}{4})}=\frac{24x4-3x7}{28+72}$

$=\frac{96-21}{100}=\frac{75}{100}=\frac{3}{4}$

$\therefore \tan 2A=\frac{3}{4}$

Exercise problems

1(i) Find the value of $\cos^2 52\frac{1}{2}^\circ-\sin^2 22\frac{1}{2}^\circ$

Sol: $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$ ∵ $\cos^2 A - \sin^2 B$ $= \cos(A+B)\cos(A-B)$

$\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

$= \cos(52\frac{1}{2}^\circ + 22\frac{1}{2}^\circ)\cos(52\frac{1}{2}^\circ - 22\frac{1}{2}^\circ)$

$= \cos 75^\circ \cos 30^\circ = \cos(45^\circ + 30^\circ)\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}[\sin 45^\circ.\cos 30^\circ + \cos 45^\circ.\sin 30^\circ]$

$= \frac{\sqrt{3}}{2}[\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}] = \frac{\sqrt{3}}{2}[\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}] = \frac{(3-\sqrt{3})}{4\sqrt{2}}$

1(ii) Find the value of $\cos^2 22\frac{1}{2}^\circ - \cos^2 82\frac{1}{2}^\circ$

Sol: $\cos^2 22\frac{1}{2}^\circ - \cos^2 82\frac{1}{2}^\circ$ ∵ $\cos^2 A - \cos^2 B$ $= -\sin(A+B)\sin(A-B)$

$\cos^2 22\frac{1}{2}^\circ - \cos^2 82\frac{1}{2}^\circ$

$= -\sin(22\frac{1}{2}^\circ + 82\frac{1}{2}^\circ)\sin(82\frac{1}{2}^\circ - 22\frac{1}{2}^\circ)$

$= -\sin 105^\circ \sin 60^\circ = -\sin(60^\circ + 45^\circ)\frac{\sqrt{3}}{2}$

$= -\frac{\sqrt{3}}{2}[\sin 60^\circ.\cos 45^\circ + \cos 60^\circ.\sin 45^\circ]$

$= -\frac{\sqrt{3}}{2}[\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}. + \frac{1}{2}.\frac{1}{\sqrt{2}}.] = -\frac{\sqrt{3}}{2}[\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}] = -\frac{(3+\sqrt{3})}{4\sqrt{2}}$

1(iii) Find the value of $\tan(\frac{\pi}{4} + \theta).\tan(\frac{\pi}{4} - \theta)$

Sol: $\tan(\frac{\pi}{4} + \theta).\tan(\frac{\pi}{4} - \theta) = \{\frac{1+\tan\theta}{1-\tan\theta}\}\{\frac{1-\tan\theta}{1+\tan\theta}\} = 1$

1(iv) Find the value of $\frac{\cot 55^\circ \cot 35^\circ - 1}{\cot 55^\circ + \cot 35^\circ}$

Sol: $\frac{\cot 55^\circ \cot 35^\circ - 1}{\cot 55^\circ + \cot 35^\circ} = \cot(55^\circ + 35^\circ) = \cot 90^\circ = 0$

1(v) Find the value of

sin1140° cos390°- cos780° sin750°

Sol: Given sin1140° cos390°- cos780° sin750°

= sin(3.360°+60°) cos(360°+30°)- cos(2.360°+60°) sin(2.360°+30°)

= sin60°.cos30°- cos60°.sin30°= $\sin(60^\circ - 30^\circ)$

= sin30°= $\frac{1}{2}$

2(i) Prove that cos35°+ cos85°+ cos155°=0

Sol: Given LHS=cos35°+ cos85°+ cos155°

= cos(60° - 25°)+cos(60°+ 25°)+cos(180° - 25°)

= 2 cos60° cos25°- cos25°

= 2.$\frac{1}{2}$ cos25°- cos25°=0= RHS

∴LHS=RHS

2(ii) Prove that sin750°cos480°+ cos120° cos60°= - $\frac{1}{2}$

Sol: LHS= sin750°cos480°+ cos120° cos60°

= sin(2.360°+30°).cos(360°+120°)+ cos(90°+30°) cos60°

= sin30° cos120°- sin30° cos60°

= sin30° cos(90°+30°) - sin30° cos60°

= sin30° (- sin30°) - sin30° cos60°=

$-\frac{1}{2}.\frac{1}{2} - \frac{1}{2}.\frac{1}{2} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$ = RHS

∴LHS=RHS

2(iii) Prove that $\cos\theta + \cos(\frac{4\pi}{3} + \theta) + \cos(\frac{4\pi}{3} - \theta) = 0$

Sol: LHS $= \cos\theta + \cos(\frac{4\pi}{3} + \theta) + \cos(\frac{4\pi}{3} - \theta)$

$= \cos\theta + \cos[\pi + (\frac{\pi}{3} + \theta)] + \cos[\pi + (\frac{\pi}{3} - \theta)$

$= \cos\theta - \cos(\frac{\pi}{3} + \theta) - \cos(\frac{\pi}{3} - \theta)$

$= \cos\theta - 2\cos\frac{\pi}{3}\cos\theta = \cos\theta - 2.\frac{1}{2}\cos\theta = 0 =$ RHS

∴LHS=RHS

2(iv) Prove that $\cos^2\theta + \cos^2(\frac{2\pi}{3} + \theta) + \cos^2(\frac{2\pi}{3} - \theta) = \frac{3}{2}$

Sol: LHS$= \cos^2\theta + \cos^2(\frac{2\pi}{3} + \theta) + \cos^2(\frac{2\pi}{3} - \theta)$

$= \frac{1+\cos 2\theta}{2} + \frac{1+\cos(2\theta + \frac{4\pi}{3})}{2} + \frac{1+\cos(2\theta - \frac{4\pi}{3})}{2}$

$= \frac{1}{2}[3 + \cos 2\theta + \cos(2\theta + \frac{4\pi}{3}) + \cos(2\theta - \frac{4\pi}{3})]$

$= \frac{1}{2}[3 + \cos 2\theta + 2\cos 2\theta.\cos\frac{4\pi}{3}]$ ∵cos(A+B)+cos(A-B)=2cosA.cosB

$= \frac{1}{2}[3 + \cos 2\theta + 2\cos 2\theta.\cos(\pi + \frac{\pi}{3})$

$= \frac{1}{2}[3 + \cos 2\theta + 2\cos 2\theta. - \cos\frac{\pi}{3}]$

$= \frac{1}{2}[3 + \cos 2\theta - 2\cos 2\theta.\frac{1}{2}] = \frac{1}{2}[3 + \cos 2\theta - \cos 2\theta] = \frac{3}{2}$ = RHS

∴LHS=RHS

2(v) Prove that $\sin^2\theta + \sin^2(\theta + \frac{\pi}{3}) + \sin^2(\theta - \frac{\pi}{3}) = \frac{3}{2}$

Sol: LHS$= \sin^2\theta + \sin^2(\theta + \frac{\pi}{3}) + \sin^2(\theta - \frac{\pi}{3})$

$= = \frac{1-\cos 2\theta}{2} + \frac{1-\cos(2\theta + \frac{2\pi}{3})}{2} + \frac{1-\cos(2\theta - \frac{2\pi}{3})}{2}$

$= \frac{1}{2}[3 - \cos 2\theta - [\cos(2\theta + \frac{2\pi}{3}) + \cos(2\theta - \frac{2\pi}{3})]$

$= \frac{1}{2}[3 - \cos 2\theta - [2\cos 2\theta.\cos\frac{2\pi}{3}]$ ∵cos(A+B)+cos(A-B)=2cosA.cosB

$= \frac{1}{2}[3 - \cos 2\theta - 2\cos 2\theta.\cos(\pi - \frac{\pi}{3})$

$= \frac{1}{2}[3 - \cos 2\theta + 2\cos 2\theta. - \cos\frac{\pi}{3}]$

$= \frac{1}{2}[3 - \cos 2\theta + 2\cos 2\theta.\frac{1}{2}] = \frac{1}{2}[3 - \cos 2\theta + \cos 2\theta] = \frac{3}{2}$ = RHS

∴LHS=RHS

3. If $\sin\alpha = \frac{12}{13}$ and $\cos\beta = \frac{3}{5}$ and neither α nor β lie in the first quadrant, then find the quadrant in which $\alpha+\beta$ lies.

Sol: From hypothesis, sin α is positive.⇒ α lies in Q_1or Q_2. But $\alpha \notin Q_1$ ∴ $\alpha \in Q_2$

Also, $\cos\beta$ is positive.⇒ β lies in Q_1or Q_4. But $\beta \notin Q_1$ ∴ $\beta \in Q_2$

Hence, $2n\pi + \frac{\pi}{2} < \alpha < 2n\pi + \pi$ and $2m\pi + \frac{3\pi}{2} < \beta < (2m+2)\pi$ for some integers m,n

⇒ $2k\pi < \alpha + \beta < 2k\pi + \pi$ where k=m+n+1

∴ $\alpha + \beta$ lies either in first or in second quadrant.

4. If $\cos\alpha = \frac{-3}{5}$ and $\sin\beta = \frac{7}{25}$, where $\frac{\pi}{2} < \alpha < \pi$ and $0 < \beta < \frac{\pi}{2}$, then find the values of $\tan(\alpha+\beta)$ and $\sin(\alpha+\beta)$.

Sol: Given $\cos\alpha = \frac{-3}{5}$; $\frac{\pi}{2} < \alpha < \pi$ $\alpha \in Q_2$ and

$\sin\beta = \frac{7}{25}$; $0 < \beta < \frac{\pi}{2}$ $\beta \in Q_1$

$\sin\alpha = \frac{4}{5}$; $\cos\beta = \frac{24}{25}$:;

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{4}{5}}{\frac{-3}{5}} = \frac{-4}{3}$; $\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{7}{25}}{\frac{24}{25}} = \frac{7}{24}$:

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{4}{5}.\frac{24}{25} + \frac{-3}{5}.\frac{7}{25} = \frac{96-21}{25} = \frac{75}{25} = 3$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\frac{-4}{3} + \frac{7}{24}}{1 - \frac{-4}{3}.\frac{7}{24}} = \frac{\frac{-32+7}{24}}{1 - \frac{-28}{72}.} = \frac{\frac{-15}{24}}{\frac{100}{72}.} = \frac{\frac{-45}{72}}{\frac{100}{72}.} = \frac{-9}{20}$

5. Prove that $\frac{\cos 9°+\sin 9°}{\cos 9°-\sin 9°}$ $=\cot 36°$

Sol: LHS=$\frac{\cos 9°+\sin 9°}{\cos 9°-\sin 9°}$; dividing numerator and denominator by sin 9°

$=\frac{\frac{\cos 9°}{\sin 9°}+\frac{\sin 9°}{\sin 9°}}{\frac{\cos 9°}{\sin 9°}-\frac{\sin 9°}{\sin 9°}}=\frac{\cot 9° + 1}{\cot 9° - 1}=\frac{\cot 9° + \cot 45°}{\cot 45° \cot 9° - 1}=\cot(45°-9°)$

$= \cot 36°$=RHS

∴LHS=RHS

MULTIPLE SUB MULTIPLE ANGLES

1.Prove that $\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta} = \tan\frac{\theta}{2}$

Sol: LHS=$\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}=\frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$=\frac{2\sin\frac{\theta}{2}(\sin\frac{\theta}{2}+\cos\frac{\theta}{2})}{2\cos\frac{\theta}{2}(\sin\frac{\theta}{2}+\cos\frac{\theta}{2})}=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$ = RHS

∴ $\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}=\tan\frac{\theta}{2}$

2. Prove that $\frac{\sin 4\theta}{\sin\theta}=8\cos^3\theta-4\cos\theta$

Sol: LHS=$\frac{\sin 4\theta}{\sin\theta}=\frac{\sin 2.2\theta}{\sin\theta}=\frac{2\sin 2\theta\cos 2\theta}{\sin\theta}$

$=\frac{2.2\sin\theta\cos\theta\cos 2\theta}{\sin\theta}= 4\cos\theta\cos 2\theta$

$=4\cos\theta(\cos^2\theta-\sin^2\theta)$

$=4\cos\theta[\cos^2\theta-(1-\cos^2\theta)]$

$=4\cos\theta(2\cos^2\theta-1)= 8\cos^3\theta-4\cos\theta$=RHS

LHS=RHS

3. Prove that $\cos^6 A+\sin^6 A = 1-\frac{3}{4}\sin^2 2A$

Sol: LHS=$\cos^6 A+\sin^6 A=(\cos^2 A)^3+(\sin^2 A)^3$

$=(\cos^2 A+\sin^2 A)^3-3\cos^2 A\sin^2 A(\cos^2 A+\sin^2 A)$

$=1-3\cos^2 A\sin^2 A=1-\frac{3}{4}(4\cos^2 A\sin^2 A)$

$=1-\frac{3}{4}(2\sin A\cos A)^2=1-\frac{3}{4}\sin^2 2A$ =RHS

∴LHS = RHS

4. Prove that $\frac{\sin 3\theta}{1+2\cos 2\theta}= \sin\theta$ and hence find the value of $\sin 15°$

Sol: LHS =$\frac{\sin 3\theta}{1+2\cos 2\theta}=\frac{3\sin\theta-4\sin^3\theta}{1+2(1-2\sin^3\theta)}$

$=\frac{\sin\theta(3-4\sin^2\theta)}{1+2-4\sin^3\theta}=\frac{\sin\theta(3-4\sin^2\theta)}{3-4\sin^3\theta}=\sin\theta$=RHS

Let θ =15°

$\sin 15°=\frac{\sin 3(15°)}{1+2\cos 2(15°)}=\frac{\sin 45°}{1+2\cos 30°}=\frac{\frac{1}{\sqrt{2}}}{1+2\frac{\sqrt{3}}{2}}$

$=\frac{1}{\sqrt{2}(1+\sqrt{3})}=\frac{1}{(\sqrt{2}+\sqrt{6})}=\frac{1}{(\sqrt{6}+\sqrt{2})}\times\frac{(\sqrt{6}-\sqrt{2})}{(\sqrt{6}-\sqrt{2})}$

$=\frac{(\sqrt{6}-\sqrt{2})}{6-2}=\frac{(\sqrt{6}-\sqrt{2})}{4}$

5. Find the value of $\sin^2 42°-\sin^2 12°$.

Sol: $\sin^2 42°-\sin^2 12°=\sin(42°+12°)\sin(42°-12°)$

$=\sin 54°\ \sin 30°$

$=\frac{\sqrt{5}+1}{4}\times\frac{1}{2}=\frac{\sqrt{5}+1}{8}$

∴ $\sin^2 42°-\sin^2 12°=\frac{\sqrt{5}+1}{8}$

6. If $\tan\frac{A}{2}=\frac{5}{6}$ and $\tan\frac{B}{2}=\frac{20}{37}$. Then show that $\tan\frac{C}{2}=\frac{2}{5}$

Sol: A+B+C= 180°

A+B = 180°-C

$\frac{A+B}{2}=\frac{180°-C}{2}=90°-\frac{C}{2}$

$\tan(\frac{A+B}{2})=\tan(90°-\frac{C}{2})$

$=\tan(\frac{A}{2}+\frac{B}{2})=\cot\frac{C}{2}$

$=\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\cot\frac{C}{2}$

$=\frac{\frac{5}{6}+\frac{20}{37}}{1-\frac{5}{6}\times\frac{20}{37}}=\cot\frac{C}{2}$

$=\frac{\frac{185+120}{222}}{\frac{222-100}{222}}=\cot\frac{C}{2}$

$=\frac{305}{122}=\frac{1}{\tan\frac{C}{2}}$

∴ $\tan\frac{C}{2}=\frac{122}{305}=\frac{2X61}{5X61}=\frac{2}{5}$

7. Prove that $\frac{\sin 2\theta}{1+\cos 2\theta}= \tan\theta$

Sol: $\frac{\sin 2\theta}{1+\cos 2\theta}=\frac{2\sin\theta\cos\theta}{2\cos^2\theta}=\frac{\sin\theta}{\cos\theta}=\tan\theta$ =RHS

8. Prove that $\frac{1-\cos 2\theta}{\sin 2\theta}=\tan\theta$. Simplify $\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta}$.

Sol: $\frac{1-\cos 2\theta}{\sin 2\theta}=\frac{2\sin^2\theta}{2\sin\theta\cos\theta}=\frac{\sin\theta}{\cos\theta}=\tan\theta$

$\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta}=\frac{3\cos\theta+4\cos^3\theta-3\cos\theta}{3\sin\theta-3\sin\theta+4\sin^3\theta}=\frac{4\cos^3\theta}{4\sin^3\theta}=\cot^3\theta$

9. Prove that $\frac{\cos 3A+\sin 3A}{\cos A-\sin A}= 1+\sin 2A$

Sol: LHS = $\frac{\cos 3A+\sin 3A}{\cos A-\sin A}$

$=\frac{(4\cos^3 A-3\cos A)+(3\sin A-4\sin^3 A)}{\cos A-\sin A}$

$=\frac{4(\cos^3 A-\sin^3 A)-3(\cos A-\sin A)}{\cos A-\sin A}$

$=\frac{(\cos A-\sin A)[4(\cos^2 A+\sin^2 A+\cos A\sin A)-3]}{\cos A-\sin A}$

$=4(\cos^2 A+\sin^2 A+\cos A\sin A)-3$

$=4+4\cos A\sin A-3=1+\sin 2A$=RHS

∴LHS=RHS

10. If $\frac{\sin\alpha}{a}=\frac{\cos\alpha}{b}$ then prove that $a\sin 2\alpha+b\cos 2\alpha = b$

Sol: $\frac{\sin\alpha}{a}=\frac{\cos\alpha}{b}$

$b\sin\alpha = a\cos\alpha$

LHS= $a\sin 2\alpha+b\cos 2\alpha$

$= a.2\sin\alpha\cos\alpha+b(1-2\sin^2\alpha)$

$= 2\sin\alpha(a\cos\alpha)+b-2b\sin^2\alpha$

$= 2\sin\alpha(b\sin\alpha)+b-2b\sin^2\alpha$

$= 2b\sin^2\alpha+b-2b\sin^2\alpha=b$=RHS

∴LHS=RHS

11. Prove that $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}= 4$

Sol: LHS = $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}$

$= \frac{\cos 10° - \sqrt{3}\ \sin 10°}{\sin 10° \cos 10°} = \frac{2(\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°)}{\sin 10° \cos 10°}$

$= \frac{2(\sin 30° \cos 10° - \cos 30° \sin 10°)}{\sin 10° \cos 10°}$

$= \frac{2\sin(30° - 10°)}{\sin 10° \cos 10°} = \frac{2\sin 20°}{\sin 10° \cos 10°}$

$= \frac{2.2 \sin 10° \cos 10°}{\sin 10° \cos 10°} = 4$ =RHS

∴LHS=RHS

12. Prove that $\frac{\sin 2A}{1-\cos 2A}\frac{1-\cos A}{\cos A} = \tan\frac{A}{2}$

Sol: LHS = $\frac{\sin 2A}{1-\cos 2A}\frac{1-\cos A}{\cos A}$

$= \frac{2\sin A\cos A}{2\sin^2 A}\frac{1-\cos A}{\cos A}$

$= \frac{1-\cos A}{\sin A} = \frac{2\sin^2\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \tan\frac{A}{2}$ = RHS

∴LHS=RHS

13. Prove that $\frac{\cos^3\theta - \cos 3\theta}{\cos\theta} + \frac{\sin^3\theta + \sin 3\theta}{\sin\theta} = 3$

Sol: LHS=$\frac{\cos^3\theta - \cos 3\theta}{\cos\theta} + \frac{\sin^3\theta + \sin 3\theta}{\sin\theta}$

$= \frac{\cos^3\theta - (4\cos^3\theta - 3\cos\theta)}{\cos\theta} + \frac{\sin^3\theta + (3\sin\theta - 4\sin^3\theta)}{\sin\theta}$

$= \frac{3\cos\theta - 3\cos^3\theta}{\cos\theta} + \frac{3\sin\theta - 3\sin^3\theta}{\sin\theta}$

$= 3(1-\cos^2\theta) + 3(1-\sin^2\theta) = 3+3-3(\cos^2\theta + \sin^2\theta)$

= 3+3-3=3=RHS

∴LHS=RHS

14. Prove that $\sin A \sin(60°+A)\sin(60°-A) = \frac{1}{4}\sin 3A$

Sol: LHS=$\sin A\sin(60°+A)\sin(60°-A)$

$= \sin A(\sin 60°\cos A + \cos 60°\sin A)(\sin 60°\cos A - \cos 60°\sin A)$

$= \sin A(\frac{\sqrt{3}}{2}\cos A + \frac{1}{2}\sin A)(\frac{\sqrt{3}}{2}\cos A - \frac{1}{2}\sin A)$

$= \sin A[(\frac{\sqrt{3}}{2})^2\cos^2 A - (\frac{1}{2})^2\sin^2 A]$

$= \sin A[\frac{3}{4}\cos^2 A - \frac{1}{4}\sin^2 A] = \frac{1}{4}\sin A[3\cos^2 A - \sin^2 A]$

$= \frac{1}{4}\sin A[3(1-\sin^2 A) - \sin^2 A]$

$= \frac{1}{4}\sin A[3 - 3\sin^2 A - \sin^2 A] = \frac{1}{4}\sin A[3 - 4\sin^2 A]$

$= \frac{1}{4}[3\sin A - 4\sin^3 A] = \frac{1}{4}\sin 3A$ = RHS

∴LHS=RHS

15. Prove that $\cos A\cos(60°+A)\cos(60°-A) = \frac{1}{4}\cos 3A$

Sol: LHS=$\cos A\cos(60°+A)\cos(60°-A)$

$= \cos A(\cos 60°\cos A - \sin 60°\sin A)(\cos 60°\cos A + \sin 60°\sin A)$

$= \cos A(\frac{1}{2}\cos A - \frac{\sqrt{3}}{2}\sin A)(\frac{1}{2}\cos A + \frac{\sqrt{3}}{2}\sin A)$

$= \cos A[(\frac{1}{2})^2\cos^2 A - (\frac{\sqrt{3}}{2})^2\sin^2 A]$

$= \cos A[\frac{1}{4}\cos^2 A - \frac{3}{4}\sin^2 A] = \frac{1}{4}\sin A[\cos^2 A - 3\sin^2 A]$

$= \frac{1}{4}\cos A[\cos^2 A - 3(1-\cos^2 A)]$

$= \frac{1}{4}\cos A[\cos^2 A - 3 + 3\cos^2 A]$

$= \frac{1}{4}\cos A[4\cos^2 A - 3]$

$= \frac{1}{4}[4\cos^3 A - 3\cos A] = \frac{1}{4}\cos 3A$ = RHS

∴LHS=RHS

16. Prove that $\tan A\tan(60°+A)\tan(60°-A) = \tan 3A$

Sol: LHS=$\tan A\tan(60°+A)\tan(60°-A)$

$= \tan A\frac{\tan 60° + \tan A}{1-\tan 60°\tan A}\frac{\tan 60° - \tan A}{1+\tan 60°\tan A}$

$= \tan A\frac{\sqrt{3}+\tan A}{1-\sqrt{3}\tan A}\frac{\sqrt{3}-\tan A}{1+\sqrt{3}\tan A}$

$= \tan A\frac{(\sqrt{3})^2 - (\tan A)^2}{(1)^2 - (\sqrt{3}\tan A)^2} = \tan A\frac{3-\tan^2 A}{1-3\tan^2 A}$

$= \frac{3\tan A - \tan^3 A}{1-3\tan^2 A} = \tan 3A$ =RHS

∴LHS=RHS

17. Prove that

$(1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1+\cos\frac{7\pi}{10})(1+\cos\frac{9\pi}{10}) = \frac{1}{16}$

Sol: LHS =

$(1+\cos\frac{\pi}{10})(1+\cos\frac{3\pi}{10})(1+\cos\frac{7\pi}{10})(1+\cos\frac{9\pi}{10})$

$= (1+\cos\frac{180°}{10})(1+\cos\frac{3x180}{10})(1+\cos\frac{7x180}{10})(1+\cos\frac{9x180}{10})$

= (1+cos 18°) (1+cos 54°) (1+cos 126°) (1+cos 162°)

= (1+cos 18°) (1+cos 54°) (1-cos 54°) (1-cos 18°)

$= (1-\cos^2 18°)(1-\cos^2 54°) = \sin^2 18°\sin^2 54°$

$= (\frac{\sqrt{5}-1}{4})^2(\frac{\sqrt{5}+1}{4})^2 = (\frac{5-1}{16})^2 = (\frac{4}{16})^2 = (\frac{1}{4})^2 = \frac{1}{16}$ = RHS

∴LHS=RHS

18. Prove that $\cos^2\frac{\pi}{10} + \cos^2\frac{2\pi}{5} + \cos^2\frac{3\pi}{5} + \cos^2\frac{9\pi}{10} = 2$

Sol: LHS = $\cos^2\frac{\pi}{10} + \cos^2\frac{2\pi}{5} + \cos^2\frac{3\pi}{5} + \cos^2\frac{9\pi}{10}$

$= \cos^2\frac{\pi}{10} + \cos^2(\frac{5\pi-\pi}{10}) + \cos^2(\frac{5\pi-\pi}{10}) + \cos^2(\frac{10\pi-\pi}{10})$

$= \cos^2\frac{\pi}{10} + \cos^2(\frac{\pi}{2} - \frac{\pi}{10}) + \cos^2(\frac{\pi}{2} + \frac{\pi}{10}) + \cos^2(\pi - \frac{\pi}{10})$

$= \cos^2\frac{\pi}{10} + \sin^2\frac{\pi}{10} + \cos^2\frac{\pi}{10} + \sin^2\frac{\pi}{10}$

$= 2(\sin^2\frac{\pi}{10} + \cos^2\frac{\pi}{10}) = 2(1)$ =2=RHS

∴LHS=RHS **(model)**

19. Prove that $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7} = \frac{1}{8}$

Sol: LHS=$\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}$

$= \frac{1}{2\sin\frac{2\pi}{7}}(2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7})\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}$

$= \frac{1}{2\sin\frac{2\pi}{7}}\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}$

$= \frac{1}{4\sin\frac{2\pi}{7}}(2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7})\cos\frac{6\pi}{7}$

$= \frac{1}{4\sin\frac{2\pi}{7}}\sin\frac{8\pi}{7}\cos\frac{6\pi}{7}$

$= \frac{1}{4\sin\frac{2\pi}{7}}\sin(\pi + \frac{\pi}{7})\cos(\pi - \frac{\pi}{7})$

$= \frac{1}{4\sin\frac{2\pi}{7}}\sin\frac{\pi}{7}\cos\frac{\pi}{7} = \frac{1}{8\sin\frac{2\pi}{7}}(2\sin\frac{\pi}{7}\cos\frac{\pi}{7})$

$= \frac{1}{8\sin\frac{2\pi}{7}}\sin\frac{2\pi}{7} = \frac{1}{8}$ =RHS

∴LHS=RHS

20. Prove that $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11} = \frac{1}{32}$

Sol: C=$\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$

S=$\sin\frac{\pi}{11}\sin\frac{2\pi}{11}\sin\frac{3\pi}{11}\sin\frac{4\pi}{11}\sin\frac{5\pi}{11}$

CS=$(\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11})$X

$(\sin\frac{\pi}{11}\sin\frac{2\pi}{11}\sin\frac{3\pi}{11}\sin\frac{4\pi}{11}\sin\frac{5\pi}{11})$

CS=

$(\sin\frac{\pi}{11}\cos\frac{\pi}{11})(\sin\frac{2\pi}{11}\cos\frac{2\pi}{11})\left(\sin\frac{3\pi}{11}\cos\frac{3\pi}{11}\right)\left(\sin\frac{4\pi}{11}\cos\frac{4\pi}{11}\right)(\sin\frac{5\pi}{11}\cos\frac{5\pi}{11})$

CS= $\frac{1}{2^5}(2\sin\frac{\pi}{11}\cos\frac{\pi}{11})(2\sin\frac{2\pi}{11}\cos\frac{2\pi}{11})\ \left(2\sin\frac{3\pi}{11}\cos\frac{3\pi}{11}\right)\left(2\sin\frac{4\pi}{11}\cos\frac{4\pi}{11}\right)(2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11})$

CS=$\frac{1}{32}\sin\frac{2\pi}{11}\sin\frac{4\pi}{11}\sin\frac{6\pi}{11}\sin\frac{8\pi}{11}\sin\frac{10\pi}{11}$

CS = $\frac{1}{32}[\sin\frac{2\pi}{11}\sin\frac{4\pi}{11}\sin(\frac{11\pi-5\pi}{11})\sin(\frac{11\pi-3\pi}{11})\sin(\frac{11\pi-\pi}{11})$

CS = $\frac{1}{32}[\sin\frac{2\pi}{11}\sin\frac{4\pi}{11}\sin(\pi-\frac{5\pi}{11})\sin(\pi-\frac{3\pi}{11})\sin(\pi-\frac{\pi}{11})$

CS = $\frac{1}{32}[\sin\frac{2\pi}{11}\sin\frac{4\pi}{11}\sin\frac{5\pi}{11}\sin\frac{3\pi}{11}\sin\frac{\pi}{11}]$

CS = $\frac{1}{32}[\sin\frac{\pi}{11}\sin\frac{2\pi}{11}\sin\frac{3\pi}{11}\sin\frac{4\pi}{11}\sin\frac{5\pi}{11}]$ =$\frac{1}{32}$S

⇒C=$\frac{1}{32}$ =RHS

∴LHS=RHS

&&&

Exercise problems

1(i).Prove the $\cos 10° \cos 30° \cos 50° \cos 70°=\frac{3}{16}$

Sol: LHS= $\cos 10° \cos 30° \cos 50° \cos 70°$

= $\cos 10° \cos 30° \cos(60°-10°) \cos(60°+10°)$

= $\cos 10° \cos 30° [\cos^2 60° - \sin^2 10°]$

= $\cos 10° \frac{\sqrt{3}}{2}[(\frac{1}{2})^2 - \sin^2 10°]$

= $\cos 10° \frac{\sqrt{3}}{2}[\frac{1}{4} - \sin^2 10°]$=$\frac{\sqrt{3}}{8}\cos 10°\ [1-4\sin^2 10°]$

= $\frac{\sqrt{3}}{8}\cos 10°\ [1-4(1-\cos^2 10°]$

= $\frac{\sqrt{3}}{8}\cos 10°\ [1-4+4\cos^2 10°]$

= $\frac{\sqrt{3}}{8}\cos 10°$ [4$\cos^2 10°$ -3]= $\frac{\sqrt{3}}{8}$ [4$\cos^3 10°$ -3cos 10°]

=$\frac{\sqrt{3}}{8}\cos 3.10°$ =$\frac{\sqrt{3}}{8}\cos 30°$=$\frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2}$ =$\frac{3}{16}$ =RHS

∴LHS=RHS

1(ii).Prove the $\cos 24° \cos 48° \cos 96° \cos 192°=\frac{1}{16}$

Sol: LHS=$\cos 24° \cos 48° \cos 96° \cos 192°$

Divide and multiply with 2sin 24°

=$\frac{1}{2\sin 24°}2\sin 24° \cos 24° \cos 48° \cos 96° \cos 192°$

=$\frac{1}{2\sin 24°}\sin 48° \cos 48° \cos 96° \cos 192°$

=$\frac{1}{2.2\sin 24°}2\sin 48° \cos 48° \cos 96° \cos 192°$

=$\frac{1}{2.2\sin 24°}\sin 96°\ \cos 96° \cos 192°$

=$\frac{1}{2.2.2\sin 24°}$2sin 96° $\cos 96° \cos 192°$

=$\frac{1}{2.2.2\sin 24°}\sin 192°\ \ \cos 192°$

=$\frac{1}{2.2.2.2\sin 24°}2\sin 192°\ \ \cos 192°$ = $\frac{1}{2.2.2.2\sin 24°}\sin 384°$

= $\frac{1}{16\sin 24°}\sin(360°+24°)$=$\frac{1}{16\sin 24°}\sin 24°$=$\frac{1}{16}$ = RHS

∴LHS=RHS

1(iii).Prove the $\tan 6° \tan 42° \tan 66° \tan 78°$=1

Sol: LHS= $\tan 6° \tan 42° \tan 66° \tan 78°$

=$\tan 6°\ \tan 66° \tan 42° \tan 78°$

= $\frac{(2sin6°sin66°)(2sin42°sin78°)}{(2cos6°cos66°)(2cos42°cos78°)}$

= $\frac{(cos60°-cos72°)(cos36°-cos120°)}{(cos60°+cos72°)(cos36°+cos120°)}$

= $\frac{(\frac{1}{2}-sin18°)(cos36°+\frac{1}{2})}{(\frac{1}{2}+sin18°)(cos36°-\frac{1}{2})}$ ∵cos 72° = cos (90° - 18°) = sin 18°

∵cos 120°=cos (180°- 60°) = - cos 60° = -1/2

= $\frac{(\frac{1}{2}-\frac{\sqrt{5}-1}{4})(\frac{\sqrt{5}-1}{4}+\frac{1}{2})}{(\frac{1}{2}+\frac{\sqrt{5}-1}{4})(\frac{\sqrt{5}-1}{4}-\frac{1}{2})}$ = $\frac{(3-\sqrt{5})(3+\sqrt{5})}{(\sqrt{5}+1)(\sqrt{5}-1)}$ =$\frac{9-5}{5-1}$=$\frac{4}{4}$= 1=RHS

∴LHS=RHS

1(iv).Prove the $\sin 20° \sin 40° \sin 60° \sin 80°=\frac{3}{16}$

Sol: LHS=$\sin 20° \sin 40° \sin 60° \sin 80°$

=$\sin 20° \sin 60° \sin 40° \sin 80°$

=$\sin 20° \sin 60° \sin(60°-20°) \sin(60°+20°)$

=$\sin 20° \sin 60°[\sin 60° \cos 20°-\cos 60° \sin 20°]$ x

$[\sin 60° \cos 20°+\cos 60° \sin 20°]$

=$\sin 20° \frac{\sqrt{3}}{2}[\frac{\sqrt{3}}{2}\cos 20°-\frac{1}{2}\sin 20°]$ x $[\frac{\sqrt{3}}{2}\cos 20°+\frac{1}{2}\sin 20°]$

=$\frac{\sqrt{3}}{2}\sin 20°[(\frac{\sqrt{3}}{2}\cos 20°)^2-(\frac{1}{2}\sin 20°)^2]$

= $\frac{\sqrt{3}}{2}\sin 20°[\frac{3}{4}\cos^2 20°$ - $\frac{1}{4}\sin^2 20°]$

= $\frac{\sqrt{3}}{8}\sin 20°[3\cos^2 20°$ - $\sin^2 20°]$

= $\frac{\sqrt{3}}{8}\sin 20°[3(1-\sin^2 20°)$ - $\sin^2 20°]$

= $\frac{\sqrt{3}}{8}\sin 20°[3-3\sin^2 20°$ - $\sin^2 20°]$

= $\frac{\sqrt{3}}{8}\sin 20°[3-4\sin^2 20°]$=$\frac{\sqrt{3}}{8}[3\sin 20°-4\sin^3 20°]$

=$\frac{\sqrt{3}}{8}\sin 3.20°$ =$\frac{\sqrt{3}}{8}\sin 60°$=$\frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2}$=$\frac{3}{16}$ =RHS

∴LHS=RHS

2(i) Prove the $\sin^2\theta+\sin^2(60°-\theta)+\sin^2(60°+\theta)=\frac{3}{2}$

Sol: LHS= $\sin^2\theta+\sin^2(60°-\theta)+\sin^2(60°+\theta)$

=$\sin^2\theta+[\sin 60°\cos\theta - \cos 60°\sin\theta\}^2$ x

$[\sin 60°\cos\theta + \cos 60°\sin\theta\}^2$

= $\sin^2\theta+2\{\sin^2 60°.\cos^2\theta + \cos^2 60°.\sin^2\theta\}$

= $\sin^2\theta+2\{(\frac{\sqrt{3}}{2})^2\cos^2\theta + (\frac{1}{2})^2.\sin^2\theta\}$

= $\sin^2\theta+2\{\frac{3}{4}\cos^2\theta + \frac{1}{4}.\sin^2\theta\}$

= $\sin^2\theta+\frac{3}{2}\cos^2\theta + \frac{1}{2}.\sin^2\theta = \sin^2\theta + \frac{1}{2}.\sin^2\theta + \frac{3}{2}\cos^2\theta$

= $\frac{3}{2}\sin^2\theta + \frac{3}{2}\cos^2\theta = \frac{3}{2}[\sin^2\theta + \cos^2\theta] = \frac{3}{2}$ =RHS

∴LHS=RHS

2(ii) Prove the $\cos^2\theta+\cos^2(120°+\theta)+\cos^2(120°-\theta)=\frac{3}{2}$

Sol: LHS =$\cos^2\theta+\cos^2(120°+\theta)+\cos^2(120°-\theta)$

= $\cos^2\theta+\cos^2[90°+(30°+\theta)]+\cos^2[90°+(30°-\theta)]$

= $\cos^2\theta$ - $\sin^2[(30°+\theta)]$ - $\sin^2[(30°-\theta)]$

= $\cos^2\theta$ - $[\sin 30°\cos\theta - \cos 30°\sin\theta\}^2$ x

$[\sin 30°\cos\theta + \cos 30°\sin\theta\}^2$

= $\cos^2\theta$ - $2\{\sin^2 30°.\cos^2\theta + \cos^2 30°.\sin^2\theta\}$

= $\cos^2\theta$ -$2\{(\frac{1}{2})^2\cos^2\theta + (\frac{\sqrt{3}}{2})^2.\sin^2\theta\}$

= $\cos^2\theta$ -$2\{\frac{1}{4}\cos^2\theta + \frac{3}{4}.\sin^2\theta\}$

= $\cos^2\theta + \frac{1}{2}\cos^2\theta + \frac{3}{2}.\sin^2\theta$

= $\cos^2\theta + \frac{1}{2}.\cos^2\theta + \frac{3}{2}\sin^2\theta$

= $\frac{3}{2}\sin^2\theta + \frac{3}{2}\cos^2\theta = \frac{3}{2}[\sin^2\theta + \cos^2\theta] = \frac{3}{2}$ =RHS

∴LHS=RHS

2(iii).Prove the$(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})=\frac{1}{8}$

Sol: Let $\cos\frac{5\pi}{8} = \cos(\frac{8\pi-3\pi}{8}) = \cos(\pi-\frac{3\pi}{8}) = -\cos\frac{3\pi}{8}$

$\cos\frac{7\pi}{8} = \cos(\frac{8\pi-\pi}{8}) = \cos(\pi - \frac{\pi}{8}) = -\cos\frac{\pi}{8}$

LHS= $(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})$

$= (1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1-\cos\frac{3\pi}{8})(1-\cos\frac{\pi}{8})$

$= (1-\cos^2\frac{\pi}{8})(1-\cos^2\frac{3\pi}{8}) = \sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$

$= \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$ $\because \sin\frac{3\pi}{8} = \sin(\frac{4\pi-\pi}{8}) = \sin(\frac{\pi}{2}-\frac{\pi}{8}) = \cos\frac{\pi}{8}$

$= \frac{1}{4}(4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}) = \frac{1}{4}(2\sin\frac{\pi}{8}\cos\frac{\pi}{8})^2 = \frac{1}{4}(\sin\frac{2\pi}{8})^2$

$= \frac{1}{4}\sin^2\frac{\pi}{4} = \frac{1}{4}(\frac{1}{\sqrt{2}})^2 = \frac{1}{8}$ =RHS

∴LHS=RHS

2(iv).Prove that $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$

Sol: Let $\sin\frac{3\pi}{8} = \sin(\frac{4\pi-\pi}{8}) = \sin(\frac{\pi}{2}-\frac{\pi}{8}) = \cos\frac{\pi}{8}$

$\sin\frac{5\pi}{8} = \sin(\frac{4\pi+\pi}{8}) = \sin(\frac{\pi}{2}+\frac{\pi}{8}) = \cos\frac{\pi}{8}$

$\sin\frac{7\pi}{8} = \sin(\frac{8\pi-\pi}{8}) = \sin(\pi-\frac{\pi}{8}) = \sin\frac{\pi}{8}$

LHS= $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

$= \sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}$

$= 2[\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\}$ $\because a^2+b^2 = (a+b)^2 - 2ab$

$= 2[(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8})^2 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}]$

$= 2[1 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}] = 2 - 4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$

$= 2 - (\sin\frac{2\pi}{8})^2 = 2 - (\sin\frac{\pi}{4})^2 = 2 - (\frac{1}{\sqrt{2}})^2 = 2 - \frac{1}{2} = \frac{3}{2}$ = RHS

∴LHS=RHS

2(v).Prove that $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{16}$

Sol: Let C= $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}$

And S= $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{3\pi}{9}\sin\frac{4\pi}{9}$

$\Rightarrow$CS= $(\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9})(\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{3\pi}{9}\sin\frac{4\pi}{9})$

$= (\sin\frac{\pi}{9}\cos\frac{\pi}{9})(\sin\frac{2\pi}{9}\cos\frac{2\pi}{9})(\sin\frac{3\pi}{9}\cos\frac{3\pi}{9})(\sin\frac{4\pi}{9}\cos\frac{4\pi}{9})$

$= \frac{1}{2^4}(2\sin\frac{\pi}{9}\cos\frac{\pi}{9})(2\sin\frac{2\pi}{9}\cos\frac{2\pi}{9})(2\sin\frac{3\pi}{9}\cos\frac{3\pi}{9})(2\sin\frac{4\pi}{9}\cos\frac{4\pi}{9})$

$= \frac{1}{16}(\sin\frac{2\pi}{9}\sin\frac{4\pi}{9}\sin\frac{6\pi}{9}\sin\frac{8\pi}{9})$

$= \frac{1}{16}(\sin\frac{2\pi}{9}\sin\frac{4\pi}{9}\sin(\frac{9\pi-3\pi}{9})\sin(\frac{9\pi-8\pi}{9}))$

$= \frac{1}{16}(\sin\frac{2\pi}{9}\sin\frac{4\pi}{9}\sin(\pi-\frac{3\pi}{9})\sin(\pi-\frac{\pi}{9}))$

$= \frac{1}{16}(\sin\frac{2\pi}{9}\sin\frac{4\pi}{9}\sin\frac{3\pi}{9}\sin\frac{\pi}{9})$

$= \frac{1}{16}(\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{3\pi}{9}\sin\frac{4\pi}{9})$

∴CS= $\frac{1}{16}$S $\Rightarrow$ C= $\frac{1}{16}$ = $\Rightarrow \cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{16}$

3. Prove that $\frac{\cos 3A}{2\cos 2A - 1} = \cos A$

Sol: LHS= $\frac{\cos 3A}{2\cos 2A-1} = \frac{4\cos^3 A - 3\cos A}{2(2\cos^3 A - 1) - 1} = \frac{4\cos^3 A - 3\cos A}{(4\cos^3 A - 2) - 1}$

$= \frac{\cos A(4\cos^2 A - 3)}{(4\cos^3 A - 3)} = \cos A$ RHS

∴LHS=RHS

6. TRANSFORMATIONS

1. Prove that $\sin 34° + \cos 64° - \cos 4° = 0$

Sol: LHS = $\sin 34° + \cos 64° - \cos 4°$

$= \sin 34° - 2\sin(\frac{64°+4°}{2})\sin(\frac{64°-4°}{2})$

$\because \cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

$= \sin 34° - 2\sin 34° \sin 30°$

$= \sin 34° - 2\sin 34°.\frac{1}{2} = \sin 34° - \sin 34°$

=0 = RHS

∴LHS=RHS

2. Prove that $\cos 55° + \cos 65° + \cos 175° = 0$

Sol: LHS= $\cos 55° + \cos 65° + \cos 175°$

$= \cos 55° + 2\cos(\frac{65°+175°}{2})\cos(\frac{175°-65°}{2})$

$\because \cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$= \cos 55° + 2\cos 120° \cos 55°$

$= \cos 55° + 2\cos 55° \cos(90° + 30°)$

$= \cos 55° + 2\cos 55° . -\sin 30° = \cos 55° + 2\cos 55°.(-\frac{1}{2})$

$= \cos 55° - \cos 55° = 0$ = RHS

∴LHS=RHS

3. Prove that $\cos 35° + \cos 85° + \cos 155° = 0$

Sol: LHS= $\cos 35° + \cos 85° + \cos 155°$

$= \cos 35° + 2\cos(\frac{85°+155°}{2})\cos(\frac{155°-85°}{2})$

$\because \cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$= \cos 35° + 2\cos 120° \cos 35°$

$= \cos 35° + 2\cos 35° \cos(90° + 30°)$

$= \cos 35° + 2\cos 35° . -\sin 30° = \cos 35° + 2\cos 35°.(-\frac{1}{2})$

$= \cos 35° - \cos 35° = 0$ = RHS

∴LHS=RHS

4. Prove that $\frac{\sin 70° - \cos 40°}{\cos 50° - \sin 20°} = \frac{1}{\sqrt{3}}$

Sol: LHS = $\frac{\sin 70° - \cos 40°}{\cos 50° - \sin 20°} = \frac{\sin 70° - \cos(90° - 50°)}{\cos(90° - 50°) - \sin 20°}$

$= \frac{\sin 70° - \sin 50°}{\sin 40° - \sin 20°}$

$\because \sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$

$= \frac{2\cos(\frac{70°+50°}{2})\sin(\frac{70°-50°}{2})}{2\cos(\frac{40°+20°}{2})\sin(\frac{40°-20°}{2})} = \frac{2\cos(\frac{120°}{2})\sin(\frac{20°}{2})}{2\cos(\frac{60°}{2})\sin(\frac{20°}{2})} = \frac{2\cos 60° \sin 10°}{2\cos 30° \sin 10°}$

$= \frac{\cos 60°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$ = RHS

∴LHS=RHS

5. Prove that $4(\sin 24° + \cos 6°) = \sqrt{15} + \sqrt{13}$

Sol: LHS = $4(\sin 24° + \cos 6°)$

$= 4[\sin(90° - 66°) + \cos 6°]$

$= 4[\cos 66° + \cos 6°]$

$= 4[2\cos(\frac{66°+6°}{2})\cos(\frac{66°-6°}{2})]$

$\because \cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$= 4.2\cos(\frac{72°}{2})\cos(\frac{60°}{2})]$

$= 4.2\cos 36° \cos 30°$

$= 4.2(\frac{\sqrt{5}+1}{4})\frac{\sqrt{3}}{2} = (\sqrt{5}+1)\sqrt{3}$

$= \sqrt{15} + \sqrt{3}$ = RHS

∴LHS=RHS

6. Prove that $\cos^2 76° + \cos^2 16° - \cos 76° \cos 16° = \frac{3}{4}$

Sol: LHS= $\cos^2 76° + \cos^2 16° - \cos 76° \cos 16°$

$= \cos^2 76° + (1 - \sin^2 16°) - \frac{1}{2}$

$(2\cos 76° \cos 16°)$

$= 1 + (\cos^2 76° - \sin^2 16°) - \frac{1}{2}(\cos(76°+16°) + \cos(76°-16°))$

$= 1 + (\cos^2 76° - \sin^2 16°) - \frac{1}{2}[\cos 92° + \cos 60°]$

$= 1 + \cos 92°.\frac{1}{2} - \frac{1}{2}[\cos 92° + \frac{1}{2}$

$= 1 + \frac{1}{2}\cos 92° - \frac{1}{2}\cos 92° - \frac{1}{4} = \frac{3}{4}$ =RHS

∴LHS=RHS

7. Prove that $\sin 10° + \sin 20° + \sin 40° + \sin 50° = \sin 70° + \sin 80°$

Sol: LHS $= \sin 10° + \sin 20° + \sin 40° + \sin 50°$

$= 2\sin(\frac{10°+20°}{2})\cos(\frac{20°-10°}{2}) + 2\sin(\frac{40°+50°}{2})\cos(\frac{50°-40°}{2})$

$= 2\sin 15° \cos 5° + 2\sin 45° \cos 5°$

$= 2\cos 5° [\sin 15° + \sin 45°]$

$= 2\cos 5° [2\sin(\frac{15°+45°}{2})\cos(\frac{45°-15°}{2})]$

$= 2\cos 5° [2\sin 30°. \cos 15°]$

$= 2\cos 5° [2.\frac{1}{2}. \cos 15°] = 2\cos 15° \cos 5°$

$= \cos(15° + 5°) + \cos(15° - 5°)$

$= \cos 20° \cos 10°$

$= \cos(90° - 70°) + \cos(90° - 80°)$

$= \sin 70° + \sin 80°$ =RHS

∴LHS=RHS

8. Prove that $\sin 50° - \sin 70° + \sin 10° = 0$

Sol: LHS $= \sin 50° - \sin 70° + \sin 10°$

$= [2\cos(\frac{50°+70°}{2})\sin(\frac{50°-70°}{2})] + \sin 10°$

$= 2\cos 60° (-\sin 10°) + \sin 10°$

$= 2.\frac{1}{2}(-\sin 10°) + \sin 10° = 0$=RHS

∴LHS=RHS

9. Prove that $\cos 48° \cos 12° = \frac{3+\sqrt{5}}{8}$

Sol: LHS $= \cos 48° \cos 12° = \frac{1}{2}(2\cos 48° \cos 12°)$

$= \frac{1}{2}(\cos(48° + 12°) + \cos(48° - 12°))$

$= \frac{1}{2}[\cos 60° + \cos 36°]$

$= \frac{1}{2}[\frac{1}{2} + \frac{\sqrt{5}+1}{4}] = \frac{1}{2}[\frac{2+\sqrt{5}+1}{4}] = \frac{3+\sqrt{5}}{8}$ = RHS

∴LHS=RHS

10. Prove that $\sin 78° + \cos 132° = \frac{\sqrt{5}-1}{4}$

Sol: LHS $= \sin 78° + \cos 132°$

$= \sin 78° + \cos(90° + 42°)$

$= \sin 78° - \sin 42°$

$= [2\cos(\frac{78°+42}{2})\sin(\frac{78°-42°}{2})]$

$= [2\cos 60° \sin 18°]$

$= 2.\frac{1}{2}\frac{\sqrt{5}-1}{4} = \frac{\sqrt{5}-1}{4}$ = RHS

∴LHS=RHS

11. Prove that $\cos^2\theta + \cos^2(\frac{2\pi}{3} + \theta) + \cos^2(\frac{2\pi}{3} - \theta) = \frac{3}{2}$

Sol: LHS $= \cos^2\theta + \cos^2(\frac{2\pi}{3} + \theta) + \cos^2(\frac{2\pi}{3} - \theta)$

$= \cos^2\theta + [\cos\frac{2\pi}{3}.\cos\theta - \sin\frac{2\pi}{3}.\sin\theta]^2$

$+ [\cos\frac{2\pi}{3}.\cos\theta + \sin\frac{2\pi}{3}.\sin\theta]^2$

$= \cos^2\theta + [-\frac{1}{2}.\cos\theta - \frac{\sqrt{3}}{2}.\sin\theta]^2 + [-\frac{1}{2}.\cos\theta + \frac{\sqrt{3}}{2}.\sin\theta]^2$

$= \cos^2\theta + [\frac{1}{4}.\cos^2\theta + \frac{\sqrt{3}}{2}.\sin\theta\cos\theta + \frac{3}{4}.\sin^2\theta]$

$+ [\frac{1}{4}.\cos^2\theta - \frac{\sqrt{3}}{2}.\sin\theta\cos\theta + \frac{3}{4}.\sin^2\theta]$

$= \cos^2\theta + \frac{1}{4}.\cos^2\theta + \frac{3}{4}.\sin^2\theta + \frac{1}{4}.\cos^2\theta + \frac{3}{4}.\sin^2\theta$

$= [1 + \frac{1}{4} + \frac{1}{4}]\cos^2\theta + [\frac{3}{4} + \frac{3}{4}]\sin^2\theta$

$= \frac{3}{2}\cos^2\theta + \frac{3}{2}\sin^2\theta = \frac{3}{2}[\sin^2\theta + \cos^2\theta] = \frac{3}{2}$ RHS

∴LHS=RHS

12. Prove that

$\sin^2(\alpha - 45°) + \sin^2(\alpha + 15°) - \sin^2(\alpha - 15°) = \frac{1}{2}$

Sol: LHS $= \sin^2(\alpha - 45°) + \sin^2(\alpha + 15°) - \sin^2(\alpha - 15°)$

$= \sin^2(\alpha - 45°) + \sin(\alpha + 15° + \alpha - 15°) -$

$\sin(\alpha + 15° - \alpha + 15°)$

$= \sin^2(\alpha - 45°) + \sin 2\alpha. \sin 30°$

$= \frac{1-\cos(2\alpha - 90°)}{2} + \sin 2\alpha.\frac{1}{2}$

$= \frac{1 - \sin 2\alpha + \sin 2\alpha}{2} = \frac{1}{2}$ =RHS

∴LHS=RHS

13. Prove that $\frac{\sin(n+1)\alpha - \sin(n-1)\alpha}{\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha} = \tan\frac{\alpha}{2}$

S0l: LHS $= \frac{\sin(n+1)\alpha - \sin(n-1)\alpha}{\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha}$

$= \frac{\sin(n\alpha+\alpha) - \sin(n\alpha-\alpha)}{\cos(n\alpha+\alpha)\alpha + 2\cos n\alpha + \cos(n\alpha-\alpha)}$

$= \frac{2\cos n\alpha\,(\sin\alpha)}{2\cos n\alpha\,(\cos\alpha) + 2\cos n\alpha} = \frac{\sin\alpha}{\cos\alpha + 1} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}}$

$= \tan\frac{\alpha}{2}$ =RHS

∴LHS=RHS

14. If $x + y = \frac{2\pi}{3}$ and $\sin x + \sin y = \frac{3}{2}$ then find x and y.

Sol: Given $x + y = \frac{2\pi}{3}$ and $\sin x + \sin y = \frac{3}{2}$ .....(1)

$\sin x + \sin y = \frac{3}{2}$

$\Rightarrow 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2}) = \frac{3}{2}$

$2\sin\frac{\pi}{3}\cos(\frac{x-y}{2}) = \frac{3}{2}$

$2\frac{\sqrt{3}}{2}\cos(\frac{x-y}{2}) = \frac{3}{2}$

$\cos(\frac{x-y}{2}) = \frac{3}{2}.\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\frac{x-y}{2} = 2n\pi \pm \frac{\pi}{6}$

$x - y = 4n\pi \pm \frac{\pi}{3}$ ....(2)

$x + y = \frac{2\pi}{3}$ ....(3)

add eq(2) and eq(3)

$2x=4n\pi \pm \frac{\pi}{3}+\frac{2\pi}{3}$

$= 4n\pi + \frac{2\pi\pm\pi}{3}$

$= 4n\pi+\pi$ or $4n\pi+\frac{\pi}{3}$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $2n\pi + \frac{\pi}{6}$

$x + y = \frac{2\pi}{3}$

$y = \frac{2\pi}{3} - x = \frac{2\pi}{3} - (2n\pi + \frac{\pi}{2}) = \frac{\pi}{6} - 2n\pi$ or

$y = \frac{2\pi}{3} - (2n\pi + \frac{\pi}{6}) = \frac{\pi}{2} - 2n\pi$

15. If $\cos x+\cos y = \frac{4}{5}$, $\cos x- \cos y = \frac{2}{7}$. Then show that $14\tan\frac{x-y}{2}+5\cot\frac{x+y}{2}=0$

Sol: $\cos x+\cos y = \frac{4}{5}$

$\cos x- \cos y = \frac{2}{7}$

$\frac{\cos x+\cos y}{\cos x-\cos y} = \frac{\frac{4}{5}}{\frac{2}{7}} = \frac{14}{5}$

$\Rightarrow \frac{2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})}{-2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})} = \frac{14}{5}$

$\Rightarrow \frac{\cot(\frac{x+y}{2})}{-\tan(\frac{x-y}{2})} = \frac{14}{5}$

$\Rightarrow 5\cot(\frac{x+y}{2}) = -14\tan(\frac{x-y}{2})$

$\therefore 14\tan\frac{x-y}{2}+5\cot\frac{x+y}{2}=0$

16. If $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{a+b}{a-b}$ then prove that $a\tan\beta = b\tan\alpha$

Sol: $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{a+b}{a-b}$

According to componend dividend rule

$\Rightarrow \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha-\beta)-\sin(\alpha-\beta)} = \frac{(a+b)+(a-b)}{(a-b)-(a-b)}$

$\Rightarrow \frac{2\sin\alpha\cos\beta}{2\cos\alpha\sin\beta} = \frac{a}{b}$

$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{a}{b}$

$\therefore a\tan\beta = b\tan\alpha$

17. If $m\sin B = n\sin(2A+B)$ then show that $(m + n)\tan A = (m - n)\tan(A+B)$

Sol: $m\sin B = n\sin(2A+B)$

$\frac{m}{n} = \frac{\sin(2A+B)}{\sin B}$

By componendo and dividend, we get

$\frac{m+n}{m-n} = \frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B} =$

$\frac{2\sin(A+B)\cos A}{2\cos(A+B)\sin A} = \tan(A+B)\cot A$

$\frac{m+n}{\cot A} = (m-n)\tan(A+B)$

$\therefore (m + n)\tan A = (m - n)\tan(A+B)$

18. If $\tan(A+B) = \lambda\tan(A-B)$ then show that $(\lambda+1)\sin 2B = (\lambda-1)\sin 2A$.

Sol: $\tan(A+B) = \lambda\tan(A-B)$

$\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda}{1}$

$\frac{\sin(A+B)}{\cos(A+B)} X \frac{\cos(A-B)}{\sin(A-B)} = \frac{\lambda}{1}$

Using componendo and dividend

$\frac{\sin(A+B)\cos(A-B)+\cos(A+B)\sin(A-B)}{\sin(A+B)\cos(A-B)-\cos(A+B)\sin(A-B)} = \frac{\lambda+1}{\lambda-1}$

$\frac{\sin 2A}{\sin 2B} = \frac{\lambda+1}{\lambda-1}$

$\therefore (\lambda+1)\sin 2B = (\lambda-1)\sin 2A$.

19. If A+B+C = 180° then prove that $\sin 2A+\sin 2B+\sin 2C = 4\sin A\sin B\sin C$

Sol: Given A+B+C = 180°

LHS = $\sin 2A+\sin 2B+\sin 2C$

$= 2\sin(\frac{2A+2B}{2})\cos(\frac{2A-2B}{2}) + \sin 2C$

$= 2\sin(A+B)\cos(A-B) + \sin 2C$

$= 2\sin(180°-C)\cos(A-B) + \sin 2C$

$= 2\sin C\cos(A-B) + 2\sin C\cos C$

$= 2\sin C[\cos(A-B) + \cos C]$

$= 2\sin C\{\cos(A-B) + \cos[180° - (A+B)]\}$

$= 2\sin C[\cos(A-B) - \cos(A-B)]$

$= 2\sin C[2\sin A\sin B] = 4\sin A\sin B\sin C$ =RHS

∴LHS=RHS

20. If A+B+C = 180° then prove that $\sin 2A-\sin 2B+\sin 2C = 4\cos A\sin B\cos C$ (model)

Sol: Given A+B+C = 180°

LHS = $\sin 2A-\sin 2B+\sin 2C$

$= 2\cos(\frac{2A+2B}{2})\sin(\frac{2A-2B}{2}) + \sin 2C$

$= 2\cos(A+B)\sin(A-B) + \sin 2C$

$= 2\cos(180°-C)\sin(A-B) + \sin 2C$

$= -2\cos C\sin(A-B) + 2\sin C\cos C$

$= 2\cos C[\sin C - \sin(A-B)]$

$= 2\cos C\{\sin[180° - (A+B)] - \sin(A-B)\}$

$= 2\cos C[\sin(A+B) - \sin(A-B)]$

$= 2\cos C[2\cos A\sin B] = 4\cos A\sin B\cos C$ =RHS

∴LHS=RHS

21. If A+B+C = 180° then prove that $\cos 2A+\cos 2B+\cos 2C = -1-4\cos A\cos B\cos C$

Sol: Given A+B+C = 180°

LHS = $\cos 2A+\cos 2B+\cos 2C$

$= 2\cos(\frac{2A+2B}{2})\cos(\frac{2A-2B}{2}) + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + (2\cos^2 C - 1)$

$= 2\cos(180°-C)\cos(A-B) + 2\cos^2 C - 1$

$= -2\cos C\cos(A-B) + 2\cos^2 C - 1$

$= -2\cos C[\cos(A-B) - \cos C] - 1$

$= -2\cos C\{\cos(A-B) - \cos[180° - (A+B)-]\}$

$= -2\cos C[\cos(A-B) - \cos(A+B)] - 1$

$= -2\cos C[2\cos A\sin B] - 1 = -1-4\cos A\cos B\cos C$ =RHS

∴LHS=RHS

22. If A+B+C = $90°$ then prove that

$\cos 2A + \cos 2B + \cos 2C = 1 + 4\sin A \sin B \sin C$

Sol: Given A+B+C = 90°

LHS = $\cos 2A + \cos 2B + \cos 2C$

$= 2\cos(\frac{2A+2B}{2})\cos(\frac{2A-2B}{2}) + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + \cos 2C$

$= 2\cos(\frac{\pi}{2} - C)\ \cos(A-B) + 2\cos 2C$

$= 2\sin C\ \cos(A-B) + (1 - 2\sin^2 C)$

$= 1 + 2\sin C\ [\cos(A-B) - \sin C]$

$= 1 + 2\sin C\ \{\cos(A-B) - \sin[\frac{\pi}{2} - (A+B)]\}$

$= 1 + 2\sin C\ [\cos(A-B) - \cos(A+B)]$

$= 1 + 2\sin C\,[2\sin A \sin B] = 1 + 4\sin A \sin B \sin C$ =RHS

∴LHS=RHS

23. In Δle ABC prove that

$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Sol: A,B,C are Δle angles

A+B+C = 180° ⇒ $\frac{A+B+C}{2} = 90°$

LHS $= \sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2}$

$\because \sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

$= [\frac{1-\cos A}{2} + \frac{1-\cos B}{2} - \frac{1-\cos C}{2}]$

$= \frac{1}{2}[1 - \cos A + 1 - \cos B - 1 - \cos C]$

$= \frac{1}{2}[1 - (\cos A + \cos B) + \cos C]$

$= \frac{1}{2}[1 - (2\cos\frac{A+B}{2}\ \cos\frac{A-B}{2}) + 1 - 2\sin^2\frac{C}{2})]$

$= \frac{1}{2}[2 - 2\cos(90° - \frac{C}{2})\ \cos\frac{A-B}{2}) - 2\sin^2\frac{C}{2})]$

$= \frac{1}{2}[2 - 2\sin\frac{C}{2}\ \cos\frac{A-B}{2}) - 2\sin^2\frac{C}{2})]$

$= \frac{2}{2}[1 - \sin\frac{C}{2}\ \cos\frac{A-B}{2}) - \sin^2\frac{C}{2})]$

$= 1 - \sin\frac{C}{2}[\ \cos\frac{A-B}{2} + \sin\frac{C}{2}]$

$= 1 - \sin\frac{C}{2}[\ \cos\frac{A-B}{2} + \sin(90° - \frac{A+B}{2})]$

$= 1 - \sin\frac{C}{2}[\ \cos\frac{A-B}{2} + \cos\frac{A+B}{2}\ ]$

$= 1 - \sin\frac{C}{2}[2\cos\frac{A}{2}\ \cos\frac{B}{2}\ ]$

$= 1 - 2\cos\frac{A}{2}\ \cos\frac{B}{2}\ \sin\frac{C}{2}$ =RHS

∴LHS=RHS

24. If A+B+C = $0°$ then prove that

$\sin 2A + \sin 2B + \sin 2C = -4\sin A \sin B \sin C$

Sol: Given A+B+C = 0°

LHS = $\sin 2A + \sin 2B + \sin 2C$

$= 2\sin(\frac{2A+2B}{2})\cos(\frac{2A-2B}{2}) + \sin 2C$

$= 2\sin(A+B)\cos(A-B) + \sin 2C$

$= 2\sin(-C)\ \cos(A-B) + \sin 2C$

$= -2\sin C\ \cos(A-B) + 2\sin C\ \cos C$

$= -2\sin C\ [\cos(A-B) - \cos C]$

$= -2\sin C\ [\cos(A-B) - \cos(-(A+B))]$

$= -2\sin C\ [\cos(A-B) - \cos(A+B)]$

$= -2\sin C\,[2\sin A \sin B] = -4\sin A \sin B \sin C$ =RHS

∴LHS=RHS

25. If A+B+C = $270°$ then prove that

$\cos 2A + \cos 2B + \cos 2C = 1 - 4\sin A \sin B \sin C$

Sol: Given A+B+C = 270°

LHS = $\cos 2A + \cos 2B + \cos 2C$

$= 2\cos(\frac{2A+2B}{2})\cos(\frac{2A-2B}{2}) + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + \cos 2C$

$= 2\cos(\frac{3\pi}{2} - C)\ \cos(A-B) + 2\cos 2C$

$= 2(-\sin C)\ \cos(A-B) + (1 - 2\sin^2 C)$

$= 1 - 2\sin C\ [\cos(A-B) + \sin C]$

$= 1 - 2\sin C\ \{\cos(A-B) + \sin[\frac{3\pi}{2} - (A+B)]\}$

$= 1 - 2\sin C\ [\cos(A-B) - \cos(A+B)]$

$= 1 - 2\sin C\,[2\sin A \sin B] = 1 - 4\sin A \sin B \sin C$ =RHS

∴LHS=RHS

26. If A+B+C=2S. Then prove that

$\cos(S-A) + \cos(S-B) + \cos(S-C) = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

Sol: Given A+B+C=2S

LHS $= \cos(S-A) + \cos(S-B) + \cos(S-C) + \cos S$

$= 2\cos(\frac{2S-A-B}{2})\cos(\frac{B-A}{2}) + 2\cos(\frac{2S-C}{2})\ \cos\left(\frac{-C}{2}\right)$

$= 2\cos\left(\frac{C}{2}\right)\ \cos(\frac{B-A}{2}) + 2\cos(\frac{A+B}{2})\ \cos\left(\frac{C}{2}\right)$

$= 2\cos\left(\frac{C}{2}\right)[\ \cos(\frac{A-B}{2}) + \cos(\frac{A+B}{2})\ ]$

$= 2\cos\left(\frac{C}{2}\right)[2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)]$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ = RHS

∴LHS=RHS

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7. TRIGONOMETRIC EQUATION

1. Solve $\tan\theta + 3\cot\theta = 5\sec\theta$

Sol: $\tan\theta + 3\cot\theta = 5\sec\theta$

$\frac{\sin\theta}{\cos\theta}+3\frac{\cos\theta}{\sin\theta}=\frac{5}{\cos\theta}$

$\frac{\sin^2\theta+3\cos^2\theta}{\sin\theta\cos\theta}=\frac{5}{\cos\theta}$

$\sin^2\theta+3\cos^2\theta=5\sin\theta$

$\sin^2\theta+3(1-\sin^2\theta)=5\sin\theta$

$\sin^2\theta+3-3\sin^2\theta=5\sin\theta$

$2\sin^2\theta+5\sin\theta-3=0$

$(2\sin\theta-1)(\sin\theta+3)=0$

$\sin\theta=\frac{1}{2}$; $\alpha=\frac{\pi}{6}$

$\theta=n\pi+(-1)^n\frac{\pi}{6}$

2. Solve $2\cos^2\theta-\sqrt{3}\sin\theta+1=0$

Sol: $2\cos^2\theta-\sqrt{3}\sin\theta+1=0$

$2(1-\sin^2\theta)-\sqrt{3}\sin\theta+1=0$

$2-2\sin^2\theta-\sqrt{3}\sin\theta+1=0$

$2\sin^2\theta+\sqrt{3}\sin\theta-3=0$

$2\sin^2\theta+2\sqrt{3}\sin\theta-\sqrt{3}\sin\theta-(\sqrt{3})^2=0$

$2\sin\theta(\sin\theta+\sqrt{3})-\sqrt{3}(\sin\theta+\sqrt{3})=0$

$(2\sin\theta-\sqrt{3})(\sin\theta+\sqrt{3})=0$

$\sin\theta=\frac{\sqrt{3}}{2}$ or $-\sqrt{3}$; $\alpha=\frac{\pi}{3}$

$\theta=n\pi+(-1)^n\frac{\pi}{3}$

3. Solve $4\cos^2\theta+\sqrt{3}=2(\sqrt{3}+1)\cos\theta$

Sol: $4\cos^2\theta+\sqrt{3}=2(\sqrt{3}+1)\cos\theta$

$4\cos^2\theta-2\sqrt{3}\sin\theta-2\cos\theta-\sqrt{3}=0$

$(2\cos\theta-\sqrt{3})(2\cos\theta-1)=0$

$2\cos\theta-1=0$ or $2\cos\theta-\sqrt{3}=0$

$\cos\theta=\frac{\sqrt{3}}{2}$ or $\frac{1}{2}$

$\cos\theta=\cos\frac{\pi}{3}$ or $\cos\theta=\cos\frac{\pi}{6}$

∴ Principal solution is $\theta=\frac{\pi}{6}$ or $\frac{\pi}{3}$

General equation $\theta=2n\pi\pm\frac{\pi}{6}$

$\theta=2n\pi\pm\frac{\pi}{3}$

4. Solve $7\sin^2\theta+3\cos^2\theta=4$

Sol: $7\sin^2\theta+3\cos^2\theta=4$

$7\sin^2\theta+3(1-\sin^2\theta)=4$

$7\sin^2\theta+3-3\sin^2\theta=4$

$\sin^2\theta=\frac{1}{4}=(\frac{1}{2})^2$

$\sin^2\theta=\sin^2\frac{\pi}{6}\Rightarrow\theta=n\pi\pm\frac{\pi}{6}$

5. Solve $\cot^2\theta-(1+\sqrt{3})\cot\theta+\sqrt{3}=0$

Sol: $\cot^2\theta-(1+\sqrt{3})\cot\theta+\sqrt{3}=0$

$\cot\theta(\cot\theta-\sqrt{3})-(\cot\theta-\sqrt{3})=0$

$(\cot\theta-1)(\cot\theta-\sqrt{3})=0$

$\cot\theta-1=0$ or $\cot\theta-\sqrt{3}=0$

$\cot\theta=1$ or $\cot\theta=\sqrt{3}$

$\tan\theta=1$ or $tan=\frac{1}{\sqrt{3}}$

$\tan\theta=\tan\frac{\pi}{4}$ or $tan\theta=\tan\frac{\pi}{6}$

∴ Principal solution is $\theta=\frac{\pi}{6}$ or $\frac{\pi}{4}$

General equation $\theta=n\pi\pm\frac{\pi}{6}$

$\theta=n\pi\pm\frac{\pi}{4}$

6. Solve $1+\sin^2\theta=3\sin\theta\cos\theta$

Sol: $1+\sin^2\theta=3\sin\theta\cos\theta$

Divide both sides with $\cos^2\theta$

$\frac{1+\sin^2\theta}{\cos^2\theta}=\frac{3\sin\theta\cos\theta}{\cos^2\theta}$

$\frac{1}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{3\sin\theta}{\cos\theta}$

$\sec^2\theta+\tan^2\theta=3\tan\theta$

$(1+\tan^2\theta)+\tan^2\theta=3\tan\theta$

$2\tan^2\theta-3\tan\theta+1=0$

$(2\tan\theta-1)(\tan\theta-1)=0$

$\tan\theta=1$ or $\tan\theta=\frac{1}{2}$; $\theta=\tan^{-1}\frac{1}{2}$

$\tan\theta=\tan\frac{\pi}{4}$ or $tan\,\theta=\tan\frac{\pi}{6}$

∴ Principal solution is $\theta=\frac{\pi}{4}$ or $\tan^{-1}\frac{1}{2}$

General equation $\theta=n\pi\pm\frac{\pi}{4}$

$\theta=n\pi\pm\tan^{-1}\frac{1}{2}$

7. Solve $\sin 5\theta+\sin\theta=\sin 3\theta$

Sol: $\sin 5\theta+\sin\theta=\sin 3\theta$

$(\sin 5\theta+\sin\theta)=\sin 3\theta$

$2\sin(\frac{5\theta+\theta}{2})\cos(\frac{5\theta-\theta}{2})=\sin 3\theta$

$2\sin 3\theta\cos 2\theta=\sin 3\theta$

$\sin 3\theta(2\cos 2\theta-1)=0$

$\sin 3\theta=0$ or $2\cos 2\theta-1=0\Rightarrow\cos 2\theta=-\frac{1}{2}$

$\sin 3\theta=\sin 0$ or $\cos 2\theta=\cos\frac{\pi}{3}$

General equation is $3\theta=n\pi$ or $2\theta=2n\pi\pm\frac{\pi}{3}$

$\Rightarrow\theta=\frac{n\pi}{3}$ or $\theta=n\pi\pm\frac{\pi}{6}$

8. Solve $\cos 8\theta+\cos 2\theta=\cos 5\theta$

Sol: $\cos 8\theta+\cos 2\theta=\cos 5\theta$

$\cos 8\theta+\cos 2\theta-\cos 5\theta=0$

$2\cos(\frac{8\theta+2\theta}{2})\cos(\frac{8\theta-2\theta}{2})-\cos 5\theta=0$

$2\cos 5\theta\cos 3\theta-\cos 5\theta=0$

$\cos 5\theta(2\cos 3\theta-1)=0$

$\cos 5\theta=0$ or $2\cos 3\theta-1=0$

$\cos 5\theta=0$ or $\cos 3\theta=\frac{1}{2}$

$\cos 5\theta=\cos\frac{\pi}{2}$ or $\cos 3\theta=\cos\frac{\pi}{3}$

General equation is $5\theta=2n\pi\pm\frac{\pi}{2}$ or $3\theta=2n\pi\pm\frac{\pi}{3}$

$\theta=\frac{2n\pi}{5}\pm\frac{\pi}{10}$ or $\theta=\frac{2n\pi}{3}\pm\frac{\pi}{9}$

9. Solve $\cos\theta.\cos 2\theta.\cos 3\theta=\frac{1}{4}$

Sol: $\cos\theta.\cos 2\theta.\cos 3\theta=\frac{1}{4}$

$4\cos\theta.\cos 2\theta.\cos 3\theta=1$

$2(2\cos 3\theta.\cos\theta)\cos 2\theta=1$

$2[\cos(3\theta+\theta)+\cos(3\theta-\theta)]\cos 2\theta=1$

$2[\cos 4\theta+\cos 2\theta]\cos 2\theta=1$

$2\cos 4\theta.\cos 2\theta + 2\cos^2 2\theta - 1 = 0$

$2\cos 4\theta.\cos 2\theta + \cos 4\theta = 0$

$\cos 4\theta(2\cos 2\theta + 1) = 0$

$\cos 4\theta = 0$ or $2\cos 2\theta + 1 = 0$

$\cos 4\theta = 0$ or $\cos 2\theta = \frac{-1}{2}$

$\cos 4\theta = \cos\frac{\pi}{2}$ or $\cos 2\theta = \cos\frac{2\pi}{3}$

General equation is $4\theta = 2n\pi \pm \frac{\pi}{2}$ or $2\theta = 2n\pi \pm \frac{\pi}{3}$

$\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$ or $\theta = n\pi \pm \frac{\pi}{6}$

10. Solve $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$

Sol: $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$

Dividing both sides with 2

$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{\sqrt{2}}{2}$

$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{1}{\sqrt{2}}$

$\cos\theta\cos\frac{\pi}{6} + \sin\theta\sin\frac{\pi}{6} = \frac{1}{\sqrt{2}}$

$\cos(\theta - \frac{\pi}{6}) = \cos\frac{\pi}{4}$

General equation is $\theta - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4}$

$\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6} = 2n\pi + \frac{5\pi}{12}$ or $2n\pi - \frac{\pi}{12}$

11. Solve $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$ (model)

Sol: $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

Dividing both sides with 2

$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{\sqrt{2}}{2}$

$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{\sqrt{2}}$

$\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6} = \frac{1}{\sqrt{2}}$

$\sin(\theta - \frac{\pi}{6}) = \frac{1}{\sqrt{2}}$

General equation is $\theta - \frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{4}$

$\theta = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{6}$

12. Solve $\tan\theta + \sec\theta = \sqrt{3}$

Sol: $\tan\theta + \sec\theta = \sqrt{3}$

$\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \sqrt{3}$

$\sin\theta + 1 = \sqrt{3}\cos\theta$

$\sqrt{3}\cos\theta - \sin\theta = 1$

Dividing both sides with 2

$\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = \frac{1}{2}$

$\cos\theta.\cos\frac{\pi}{6} - \sin\theta.\sin\frac{\pi}{6} = \frac{1}{2}$

$\cos(\theta + \frac{\pi}{6}) = \cos\frac{\pi}{3}$

General equation is $\theta + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$\theta = 2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6} = 2n\pi + \frac{\pi}{6}$ or $2n\pi - \frac{\pi}{2}$

13. Solve $1 + |\cos x| + |\cos^2 x| + \ldots\ldots\infty$

Sol: $8^{1+\cos x+\cos^2 x+\cos^3 x+\cdots\infty} = 4^3$ for all $x \in (-\pi, \pi)$

Given $8^{1+\cos x+\cos^2 x+\cos^3 x+\cdots\infty} = 4^3$

For x ⊠⊠0 the given equation has no solution.

For x ⊠⊠0 we have $|\cos x| < 1$

$1 + \cos x + \cos^2 x + \cos^3 x + \cdots.\infty = \frac{1}{1-\cos x}$

Now $8^{1+\cos x+\cos^2 x+\cos^3 x+\cdots\infty} = 4^3$

$\Rightarrow 2^{3(1+\cos x+\cos^2 x+\cos^3 x+\cdots\infty)} = (2^2)^3$

$\Rightarrow 2^{3(\frac{1}{1-\cos x})} = (2)^6$

$\Rightarrow \frac{3}{1-\cos x} = 6$

$1 - \cos x = \frac{1}{2} \Rightarrow \cos x = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}$

14. Solve $4\sin x.\sin 2x.\sin 4x = \sin 3x$

Sol: LHS $= 4\sin x.\sin 2x.\sin 4x$

$= (2\sin x)(2\sin 4x.\sin 2x)$

$= (2\sin x)[\cos(4x - 2x) - \cos(4x+2x)]$

$= 2\sin x(\cos 2x - \cos 6x)$

$= 2\cos 2x\sin x - 2\cos 6x\sin x$

$= \sin(2x + x) - \sin(2x - x) - 2\cos 6x\sin x$

$= \sin 3x - \sin x - 2\cos 6x\sin x$

$\sin 3x - \sin x - 2\cos 6x\sin x = \sin 3x$

$\sin x + 2\cos 6x\sin x = 0$

$\Rightarrow \sin x(1 + 2\cos 6x) = 0$

$\sin x = 0$ or $\cos 6x = -\frac{1}{2}$

$\Rightarrow x = n\pi$

15. Solve $3\operatorname{cosec}\theta = 4\sin\theta$

Sol: $3\operatorname{cosec}\theta = 4\sin\theta$

$\frac{3}{\sin\theta} = 4\sin\theta$

$3 = 4\sin^2\theta$

$\frac{3}{4} = \sin^2\theta$

$(\frac{\sqrt{3}}{2})^2 = \sin\theta$

$\theta = \frac{\pi}{3}$

$\theta = n\pi \pm \frac{\pi}{3}$

16. If $a\cos 2\theta + b\sin 2\theta = c$. Then prove that $\tan\theta_1 + \tan\theta_2 = \frac{2b}{c+a}$ ‘$\tan\theta_1.\tan\theta_2 = \frac{c-a}{c+a}$

Sol: $a\cos 2\theta + b\sin 2\theta = c$

$a[\frac{1-\tan^2\theta}{1+\tan^2\theta}] + b[\frac{2\tan\theta}{1+\tan^2\theta}] = c$

$a(1 - \tan^2\theta) + b(2\tan\theta) = c(1 + \tan^2\theta)$

$a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$

$c\tan^2\theta + a\tan^2\theta - 2b\tan\theta + c - a = 0$

$(a+c)\, a\tan^2\theta - 2b\tan\theta + (c - a) = 0$

This is a quadratic equation in $\tan\theta$ and

$\tan\theta_1$, $\tan\theta_2$ are solutions then we get

$\therefore \tan\theta_1 + \tan\theta_2 = \frac{2b}{c+a}$

$\tan\theta_1.\tan\theta_2 = \frac{c-a}{c+a}$

17. Solve $\sin^{-2}\theta$ - $\cos\theta = \frac{1}{4}$

Sol: $\sin^2\theta$ - $\cos\theta = \frac{1}{4}$

$(1-\cos^2\theta)$ - $\cos\theta = \frac{1}{4}$

$4-4\cos^2\theta - 4\cos\theta = 1$

$4\cos^2\theta + 4\cos\theta - 3 = 0$

$(2\cos\theta + 3)(2\cos\theta - 1) = 0$

$2\cos\theta + 3 = 0$ or $2\cos\theta - 1 = 0$

$\cos\theta = \frac{-3}{2}$ or $\frac{1}{2}$

$\cos\theta = \cos\frac{\pi}{3}$

$\theta = 2n\pi \pm \frac{\pi}{3}$

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Exercise Problems

1(i).Find the principle solution of $2\cos^2 x = 1$

8. HYPERBOLIC FUNCTIONS

1. Prove that

$\sinh(\alpha+\beta) = \sinh\alpha\cosh\beta + \cosh\alpha\sinh\beta$

Sol: RHS = $\sinh\alpha\cosh\beta + \cosh\alpha\sinh\beta$

$= \frac{e^{\alpha}-e^{-\alpha}}{2}\,\frac{e^{\beta}+e^{-\beta}}{2} + \frac{e^{\alpha}+e^{-\alpha}}{2}\,\frac{e^{\beta}-e^{-\beta}}{2}$

$= \frac{1}{4}[(e^{\alpha}-e^{-\alpha})(e^{\beta}+e^{-\beta}) + (e^{\alpha}+e^{-\alpha})(e^{\beta}-e^{-\beta})]$

$= \frac{1}{4}[(e^{\alpha+\beta} + e^{\alpha-\beta} - e^{-\alpha+\beta} - e^{-\alpha-\beta}) + (e^{\alpha+\beta} - e^{\alpha-\beta} + e^{-\alpha+\beta} - e^{-\alpha-\beta})]$

$= \frac{1}{4}2[e^{\alpha+\beta} - e^{-(\alpha+\beta)}] = \frac{e^{\alpha+\beta} - e^{-(\alpha+\beta)}}{2} = \sinh(\alpha+\beta)$ = LHS

$\therefore \sinh(\alpha+\beta) = \sinh\alpha\cosh\beta + \cosh\alpha\sinh\beta$

2. Prove that

$\cosh(\alpha+\beta) = \cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$

Sol: RHS = $\cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$

$= \frac{e^{\alpha}+e^{-\alpha}}{2}\,\frac{e^{\beta}+e^{-\beta}}{2} + \frac{e^{\alpha}-e^{-\alpha}}{2}\,\frac{e^{\beta}-e^{-\beta}}{2}$

$= \frac{1}{4}[(e^{\alpha}-e^{-\alpha})(e^{\beta}+e^{-\beta}) + (e^{\alpha}-e^{-\alpha})(e^{\beta}-e^{-\beta})]$

$= \frac{1}{4}[(e^{\alpha+\beta} + e^{\alpha-\beta} + e^{-\alpha+\beta} + e^{-\alpha-\beta}) + (e^{\alpha+\beta} - e^{\alpha-\beta} - e^{-\alpha+\beta} + e^{-\alpha-\beta})]$

$= \frac{1}{4}2[e^{\alpha+\beta} + e^{-(\alpha+\beta)}] = \frac{e^{\alpha+\beta} + e^{-(\alpha+\beta)}}{2}$

$= \cosh(\alpha+\beta)$ = LHS

$\therefore \cosh(\alpha+\beta) = \cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$

3. Prove that $\tanh(\alpha+\beta) = \frac{\tanh\alpha+\tanh\beta}{1+\tanh\alpha\tanh\beta}$

Sol: LHS = $\tanh(\alpha+\beta) = \frac{\sinh(\alpha+\beta)}{\cosh(\alpha+\beta)}$

$= \frac{\sinh\alpha\cosh\beta+\cosh\alpha\sinh\beta}{\cosh\alpha\cosh\beta+\sinh\alpha\sinh\beta}$

On dividing numerator and denominator by $\cosh\alpha\cosh\beta$, we get

$= \dfrac{\frac{\sinh\alpha\cosh\beta+\cosh\alpha\sinh\beta}{\cosh\alpha\cosh\beta}}{\frac{\cosh\alpha\cosh\beta+\sinh\alpha\sinh\beta}{\cosh\alpha\cosh\beta}} = \dfrac{\frac{\sinh\alpha\cosh\beta}{\cosh\alpha\cosh\beta}+\frac{\cosh\alpha\sinh\beta}{\cosh\alpha\cosh\beta}}{\frac{\cosh\alpha\cosh\beta}{\cosh\alpha\cosh\beta}+\frac{\sinh\alpha\cosh\beta}{\cosh\alpha\cosh\beta}} = \frac{\tanh\alpha+\tanh\beta}{1+\tanh\alpha\tanh\beta}$

$\therefore \tanh(\alpha+\beta) = \frac{\tanh\alpha+\tanh\beta}{1+\tanh\alpha\tanh\beta}$

4. Prove that

$\sinh(\alpha-\beta) = \sinh\alpha\cosh\beta - \cosh\alpha\sinh\beta$

Sol: RHS = $\sinh\alpha\cosh\beta - \cosh\alpha\sinh\beta$

$= \frac{e^{\alpha}-e^{-\alpha}}{2}\,\frac{e^{\beta}+e^{-\beta}}{2} - \frac{e^{\alpha}+e^{-\alpha}}{2}\,\frac{e^{\beta}-e^{-\beta}}{2}$

$= \frac{1}{4}[(e^{\alpha}-e^{-\alpha})(e^{\beta}+e^{-\beta}) - (e^{\alpha}+e^{-\alpha})(e^{\beta}-e^{-\beta})]$

$= \frac{1}{4}[(e^{\alpha+\beta} + e^{\alpha-\beta} - e^{-\alpha+\beta} - e^{-\alpha-\beta}) - (e^{\alpha+\beta} - e^{\alpha-\beta} + e^{-\alpha+\beta} - e^{-\alpha-\beta})]$

$= \frac{1}{4}2[e^{\alpha+\beta} + e^{-(\alpha+\beta)}] = \frac{e^{\alpha+\beta} + e^{-(\alpha+\beta)}}{2} = \sinh(\alpha-\beta)$ = LHS

$\therefore \sinh(\alpha-\beta) = \sinh\alpha\cosh\beta - \cosh\alpha\sinh\beta$
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5. Prove that

$\cosh(\alpha - \beta) = \cosh\alpha\cosh\beta - \sinh\alpha\sinh\beta$

Sol: RHS = $\cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$

$= \frac{e^{\alpha}+e^{-\alpha}}{2}\,\frac{e^{\beta}+e^{-\beta}}{2} - \frac{e^{\alpha}-e^{-\alpha}}{2}\,\frac{e^{\beta}-e^{-\beta}}{2}$

$= \frac{1}{4}[(e^{\alpha}+e^{-\alpha})(e^{\beta}+e^{-\beta}) - (e^{\alpha}-e^{-\alpha})(e^{\beta}-e^{-\beta})]$

$= \frac{1}{4}[(e^{\alpha+\beta} + e^{\alpha-\beta} + e^{-\alpha+\beta} + e^{-\alpha-\beta}) - (e^{\alpha+\beta} - e^{\alpha-\beta} - e^{-\alpha+\beta} + e^{-\alpha-\beta})]$

$= \frac{1}{4}2[e^{\alpha-\beta} + e^{-(\alpha-\beta)}] = \frac{e^{\alpha-\beta}+e^{-(\alpha-\beta)}}{2}$

$= \cosh(\alpha-\beta)$ = LHS

$\therefore \cosh(\alpha - \beta) = \cosh\alpha\cosh\beta - \sinh\alpha\sinh\beta$

6. Prove that $\sinh 2x = 2\sinh x\cosh x$

Sol: RHS = $2\sinh x\cosh x$ =

$2(\frac{e^{x}-e^{-x}}{2})(\frac{e^{x}+e^{-x}}{2}) = \frac{e^{2x}-e^{-2x}}{2} = \sinh 2x$ =LHS

$\therefore \sinh 2x = 2\sinh x\cosh x$

7. Prove that $\cosh 2x = \cosh^2 x + \sinh^2 x$

Sol: RHS= $\cosh^2 x + \sinh^2 x$

$= (\frac{e^{x}+e^{-x}}{2})^2 + (\frac{e^{x}-e^{-x}}{2})^2$

$= \frac{(e^{2x}+e^{-2x}+2)+(e^{2x}+e^{-2x}-2)}{4} = \frac{2e^{2x}+2e^{-2x}}{24}$

$= \frac{e^{2x}+e^{-2x}}{2} = \cosh 2x$

$\therefore \cosh 2x = \cosh^2 x + \sinh^2 x$

8. Prove that $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$

Sol: Let $\tanh(x+y) = \frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$

On taking y+x, we get

$\tanh(x+x) = \frac{\tanh x+\tanh x}{1+\tanh x\tanh x}$

$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$

9. Prove that $\sinh 3x = 3\sinh x + 4\sinh^3 x$

Sol: LHS=sinh 3x

=sinh(2x+x)

= $\sinh 2x\cosh x + \cosh 2x\sinh x$

=$(2\sinh x\cosh x)\cosh x + (1+2\sinh^2 x)\sinh x$

=$(2\sinh x\cosh^2 x) + (1+2\sinh^2 x)\sinh x$

= $2\sinh x(1+\sinh^2 x) + (1+2\sinh^2 x)\sinh x$

= $2\sinh x + 2\sinh^3 x + \sinh x + 2\sinh^3 x$

= $3\sinh x + 4\sinh^3 x$ = RHS

$\therefore \sinh 3x = 3\sinh x + 4\sinh^3 x$

10. Prove that $\cosh 3x = 4\cosh^3 x - 3\cosh x$

Sol: LHS=sinh 3x

=cosh(2x+x)

= $\cosh 2x\cosh x + \sinh 2x\sinh x$

=$(\cosh^2 x + \sinh^2 x)\cosh x + (2\sinh x\cosh x)\sinh x$

=$(\cosh^2 x + \cosh^2 x - 1)\cosh x + (2\sinh^2 x\ \cosh x)$

=$(\cosh^2 x + \cosh^2 x - 1)\cosh x + [(2(\cosh^2 x - 1)\ \cosh x]$

=$(2\cosh^2 x - 1)\cosh x + (2\cosh^2 x - 2))\cosh x$

= $2\cosh^3 x - \cosh x + 2\cosh^3 x - 2\cosh x$

= $4\cosh^3 x - 3\cosh x$ = RHS

$\therefore \cosh 3x = 4\cosh^3 x - 3\cosh x$

11. Prove that $\tanh 3x = \frac{3\tanh x + \tanh^3 x}{1+3\tanh^2 x}$

Sol: LHS = $\tanh 3x = \tanh(2x+x)$

$= \frac{\tanh 2x+\tanh x}{1+\tanh 2x\tanh x} = \frac{\frac{2\tanh x}{1+\tanh^2 x}+\tanh x}{1+\frac{2\tanh x}{1+\tanh^2 x}\tanh x}$

$= \frac{2\tanh x+\tanh x(1+\tanh^2 x)}{1+\tanh^2 x+2\tanh^2 x} =$

$\frac{2\tanh x+\tanh x+\tanh^3 x)}{1+\tanh^2 x+2\tanh^2 x}$

$= \frac{3\tanh x+\tanh^3 x)}{1+3\tanh^2 x}$ = RHS

$\therefore \tanh 3x = \frac{3\tanh x+\tanh^3 x}{1+3\tanh^2 x}$

12. Prove that $\frac{\tanh x}{\operatorname{sech} x-1} + \frac{\tanh x}{\operatorname{sech} x+1} = -2\operatorname{cosech} x$

Sol: LHS = $\frac{\tanh x}{\operatorname{sech} x-1} + \frac{\tanh x}{\operatorname{sech} x+1}$

$= \frac{\tanh x(\operatorname{sech} x+1) + \tanh x(\operatorname{sech} x-1)}{(\operatorname{sech} x-1)(\operatorname{sech} x+1)}$

$= \frac{2\tanh x\operatorname{sech} x}{\operatorname{sech}^2 x-1} = \frac{2\tanh x\operatorname{sech} x}{-\tanh^2 x}$

$= -2\coth x\operatorname{sech} x$

$= -2\frac{\cosh x}{\sinh x}\,\frac{1}{\cosh x} = -2\,\frac{1}{\sinh x} = -$

$2\operatorname{cosech} x$+RHS

$\therefore \frac{\tanh x}{\operatorname{sech} x-1} + \frac{\tanh x}{\operatorname{sech} x+1} = -2\operatorname{cosech} x$

13. Prove that

$[\cosh x + \sinh x]^n = \cosh nx + \sinh nx$

Sol: LHS = $[\cosh x + \sinh x]^n$

$= [\frac{e^{x}+e^{-x}}{2} + \frac{e^{x}-e^{-x}}{2}]^n = [e^{x}]^n = e^{nx}$

RHS = $\cosh(nx) + \sinh(nx)$

$= \frac{e^{nx}+e^{-nx}}{2} + \frac{e^{nx}-e^{-nx}}{2} = e^{nx}$

$\therefore$ LHS = RHS

14. If $\cosh x = \frac{5}{2}$, The prove that

$\cosh 2x = \frac{23}{2}$; $\sinh 2x = \frac{5\sqrt{21}}{2}$

Sol: $\cosh x = \frac{5}{2}$

$\sinh x = \sqrt{\cosh^2 x - 1} = \sqrt{(\frac{5}{2})^2 - 1} = \sqrt{\frac{25}{4} - 1} = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$

$\cosh 2x = \cosh^2 x + \sinh^2 x = (\frac{5}{2})^2 + (\frac{\sqrt{21}}{2})^2 = \frac{25}{4} + \frac{21}{4} = \frac{46}{4} = \frac{23}{2}$

$\cosh 2x = 2\sinh x\cosh x = 2\text{x}\frac{\sqrt{21}}{2}\text{x}\frac{5}{2} = \frac{5\sqrt{21}}{2}$

15. If $u = \log_e(\tan(\frac{\pi}{4} + \frac{\theta}{2}))$ then prove that

$\cosh u = \sec\theta$

Sol: $u = \log_e(\tan(\frac{\pi}{4} + \frac{\theta}{2}))$

$e^{u} = \tan(\frac{\pi}{4} + \frac{\theta}{2})$

$e^{-u} = \cot(\frac{\pi}{4} + \frac{\theta}{2})$

$\cosh u = \frac{e^u+e^{-u}}{2} = \frac{\tan(\frac{\pi}{4}+\frac{\theta}{2})+\cot(\frac{\pi}{4}+\frac{\theta}{2})}{2} = \frac{1}{\sin(\frac{\pi}{4}+\frac{\theta}{2})} = \frac{1}{\cos\theta} = \sec\theta$

$\therefore \cosh u = \sec\theta$

16. If $\sinh x = \frac{3}{4}$ then find $\cosh 2x$ and $\sinh 2x$.

Sol: $\sinh x = \frac{3}{4}$

$\cosh 2x = 2\sinh^2 x + 1 = 2(\frac{3}{4})^2 + 1 = 2.\frac{9}{16} + 1 = \frac{17}{8}$

$\cosh x = \sqrt{\frac{1+\cosh 2x}{2}} = \sqrt{\frac{1+\frac{17}{8}}{2}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

$\sinh 2x = 2\sinh x\cosh x = 2.\frac{3}{4}.\frac{5}{4} = \frac{15}{8}$

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9. LIMITS AND CONTINUITY

Compute the following limits

1. $\lim_{x\to 3}\frac{x^2-8x+15}{x^2-9}$

Sol: $\lim_{x\to 3}\frac{x^2-8x+15}{x^2-9} = \lim_{x\to 3}\frac{(x-5)(x-3)}{(x+3)(x-3)}$

$=\lim_{x\to 3}\frac{(x-5)}{(x+3)} = \frac{3-5}{3+3} = \frac{-2}{6} = \frac{-1}{3}$

2. $\lim_{x\to 0^+}\frac{|x|}{x}$ **;** $\lim_{x\to 0^-}\frac{|x|}{x}$

Sol: Let f(x) $=\lim_{x\to 0^+}\frac{|x|}{x}$, $x\neq 0$

$f(x) = \begin{cases} -x, & x<0 \\ x, & x\geq 0\end{cases}$

$= \begin{cases} -1, & x<0 \\ 1, & x\geq 0\end{cases}$

$\lim_{x\to 0^+}\frac{|x|}{x} = \lim_{x\to 0^+}(+1) = +1$

$\lim_{x\to 0^-}\frac{|x|}{x} = \lim_{x\to 0^-}(-1) = -1$

3. $\lim_{x\to 2^+}([x]+x)$ **;** $\lim_{x\to 2^-}([x]+x)$

Sol: $\lim_{x\to 2^+}([x]+x) =$

$\lim_{x\to 2^+}(x+x) = \lim_{x\to 2^+}(2x) = 2(2) = 4$

$\lim_{x\to 2^-}([x]+x) = \lim_{x\to 2^-}(-x+x) = -2+2 = 0$

4. $\lim_{x\to 0}\frac{\tan x}{x}$

Sol: $\lim_{x\to 0}\frac{\tan x}{x} =$

$\lim_{x\to 0}(\frac{\sin x}{x})(\frac{1}{\cos x}) = \lim_{x\to 0}(\frac{\sin x}{x})\lim_{x\to 0}(\frac{1}{\cos x})$

$=1.1=1$

5. $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

Sol: Given value is $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

By rationalizing the numerator

$= \lim_{x\to 0}\frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} =$

$\lim_{x\to 0}\frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)}$

$= \lim_{x\to 0}\frac{(\sqrt{1+x})^2-1^2}{x(\sqrt{1+x}+1)} = \lim_{x\to 0}\frac{1+x-1}{x(\sqrt{1+x}+1)} =$

$\lim_{x\to 0}\frac{1}{(\sqrt{1+x}+1)}$

By substituting the value x=0, we get $\frac{1}{(\sqrt{1+0}+1)}$

$=\frac{1}{(1+1)} = \frac{1}{2}$

6. $\lim_{x\to 0}[\frac{e^x-1}{\sqrt{1+x}-1}]$

Sol: Given $\lim_{x\to 0}[\frac{e^x-1}{\sqrt{1+x}-1}]$

For $0 < |x| < 1$ by rationalizing the denominator

$= \lim_{x\to 0}[\frac{e^x-1}{\sqrt{1+x}-1}] \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} =$

$\lim_{x\to 0}\frac{(e^x-1)(\sqrt{1+x}+1)}{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}$

$=\lim_{x\to 0}\frac{(e^x-1)(\sqrt{1+x}+1)}{(\sqrt{1+x})^2-1^2} = \lim_{x\to 0}\frac{(e^x-1)(\sqrt{1+x}+1)}{1+x-1}$

$= \lim_{x\to 0}\frac{(e^x-1)(\sqrt{1+x}+1)}{x}$

$=\lim_{x\to 0}\frac{(e^x-1)}{x}.\lim_{x\to 0}(\sqrt{1+x}+1)$

By substituting x=0, we get $\sqrt{1+0}+1$ =1+1=2 ;

$\because \lim_{x\to0}\frac{(e^x-1)}{x}$ =1

=$\lim_{x\to0}\frac{(e^x-1)}{x}.\lim_{x\to0}(\sqrt{1+x}+1)$ =1(2) = 2

7. $\lim_{x\to0}[\frac{a^x-1}{b^x-1}]$ (a>0,b>0, b≠1

Sol: $\lim_{x\to0}[\frac{a^x-1}{b^x-1}]$ for x≠0 by dividing the numerator and denominator with x

$\lim_{x\to0}[\frac{\frac{a^x-1}{x}}{\frac{b^x-1}{x}}]$, we know that $\lim_{x\to0}\frac{a^x-1}{x}=\log_e a$;

similarly $\lim_{x\to0}\frac{b^x-1}{x}=\log_e b$

$\therefore \lim_{x\to0}[\frac{\frac{a^x-1}{x}}{\frac{b^x-1}{x}}]=\frac{\log_e a}{\log_e b}$

$\therefore \lim_{x\to0}[\frac{a^x-1}{b^x-1}]=\frac{\log_e a}{\log_e b}$

8. $\lim_{x\to0}\frac{\sin ax}{\sin bx}$

Sol: $\lim_{x\to0}\frac{\sin ax}{\sin bx}=\lim_{x\to0}\frac{\frac{\sin ax}{a}}{\frac{\sin ax}{a}}=\frac{a}{b}$

$\because \lim_{x\to0}\frac{\sin kx}{x}$ =k

9. $\lim_{x\to0}[\frac{e^{3x}-1}{x}]$

Sol: $\lim_{x\to0}[\frac{e^{3x}-1}{x}]=\lim_{x\to0}[\frac{e^{3x}-1}{3x}]3$= 1.3=3

$\because \lim_{x\to0}\frac{e^x-1}{x}$ =1

10. $\lim_{x\to0}[\frac{e^x-\sin x-1}{x}]$

Sol: $\lim_{x\to0}[\frac{e^x-\sin x-1}{x}]=\lim_{x\to0}[\frac{e^x-1}{x}-\frac{\sin x}{x}]$

= $\lim_{x\to0}\frac{e^x-1}{x}$ - $\lim_{x\to0}\frac{\sin x}{x}$ =1-1=0

11. $\lim_{x\to0}[\frac{e^{3+x}-e^3}{x}]$

Sol: $\lim_{x\to0}[\frac{e^{3+x}-e^3}{x}]=\lim_{x\to0}\frac{e^3(e^x-1)}{x}$

= $e^3\lim_{x\to0}\frac{(e^x-1)}{x}=e^3(1)=e^3$

12. $\lim_{x\to0}[\frac{e^{\sin x}-1}{x}]$

Sol: $\lim_{x\to0}[\frac{e^{\sin x}-1}{x}]=\lim_{x\to0}\frac{e^{\sin x}-1}{\sin x}(\frac{\sin x}{x})$

=$\lim_{x\to0}\frac{e^{\sin x}-1}{\sin x}.\lim_{x\to0}\frac{\sin x}{x}$ = 1.1=1

13. $\lim_{x\to0}[\frac{3^x-1}{\sqrt{1+x}-1}]$

Sol: $\lim_{x\to0}[\frac{3^x-1}{\sqrt{1+x}-1}]=\lim_{x\to0}[\frac{3^x-1}{x}](\frac{x}{\sqrt{1+x}-1})$

= $\lim_{x\to0}[\frac{3^x-1}{x}].\lim_{x\to0}(\frac{x}{\sqrt{1+x}-1})$

= $\log 3.\lim_{x\to0}(\frac{x}{\sqrt{1+x}-1})$

=$\log 3.\lim_{x\to0}(\frac{x}{\sqrt{1+x}-1})(\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1})$

= $\log 3.\lim_{x\to0}(\frac{x(\sqrt{1+x}+1)}{1+x-1})$

=$\log 3.\lim_{x\to0}(\frac{x(\sqrt{1+x}+1)}{x})$

= $\log 3.\lim_{x\to0}((\sqrt{1+x}+1))$

= $\log 3.(\sqrt{1+0}+1)$

= (log 3)(1+1) = 2log 3

14. $\lim_{x\to a}\frac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^2}$

Sol: $\lim_{x\to a}\frac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^2}$

=$\lim_{x\to a}\frac{(x-a)\sin(x-a)}{(x-a)}\{\lim_{x\to a}\frac{\tan(x-a)}{(x-a)}\}^2.\frac{1}{(x+a)^2}$

=$0.1.1\frac{1}{(a+a)^2}$ = 0

15. $\lim_{x\to0}\frac{\cos ax-\cos bx}{x^2}$

Sol: $\lim_{x\to0}\frac{\cos ax-\cos bx}{x^2}$

$\because \cos C-\cos D=2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

= $\lim_{x\to0}\frac{2\sin\frac{ax+bx}{2}\sin\frac{bx-ax}{2}}{x^2}$ =2

$\lim_{x\to0}[\frac{\sin(\frac{a+b}{2})x}{x}][\frac{\sin(\frac{b-a}{2})x}{x}]$

=$2[\lim_{x\to0}[\frac{\sin(\frac{a+b}{2})x}{x}][\lim_{x\to0}[\frac{\sin(\frac{b-a}{2})x}{x}]$

=$2(\frac{a+b}{2})(\frac{b-a}{2})=\frac{b^2-a^2}{2}$

16. $\lim_{x\to a}\frac{x\sin a-a\sin x}{x-a}$

Sol: $\lim_{x\to a}\frac{x\sin a-a\sin x}{x-a}$

=$\lim_{x\to a}\frac{(x\sin a-a\sin a)-(a\sin x-a\sin a)}{x-a}$

=$\lim_{x\to a}\frac{\sin a(x-a)-a(\sin x-\sin a)}{x-a}$

=$\lim_{x\to a}[\frac{(x-a)\sin a}{x-a}]$ -a $\lim_{x\to a}[\frac{(\sin x-\sin a)}{x-a}]$

= $\lim_{x\to a}\sin a$ -a $\lim_{x\to a}[\frac{2\cos(\frac{x+a}{2})2\sin(\frac{x+a}{2})}{x-a}]$=

$\sin a$ -a $2\lim_{x\to a}\cos(\frac{x+a}{2}).\lim_{x\to a}[\frac{\sin(\frac{x+a}{2})}{x-a}]$

= $\sin a$ -a.2 $\cos(\frac{a+a}{2})\frac{1}{2}$ = $\sin a$ -a $\cos a$

17. $\lim_{x\to0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

Sol: $\lim_{x\to0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}=\lim_{x\to0}\frac{(1+x)^{\frac{1}{3}}-(1-x)^{\frac{1}{3}}}{x}$

= $\lim_{x\to0}[\frac{(1+x)^{\frac{1}{3}}-1}{x}-\frac{(1-x)^{\frac{1}{3}}-1}{x}]$

= $\lim_{(1+x)\to1}[\frac{(1+x)^{\frac{1}{3}}-1}{(1+x)-1}+\frac{(1-x)^{\frac{1}{3}}-1}{(1+x)-1}]$

= $\frac{1}{3}(1)^{\frac{1}{3}-1}+\frac{1}{3}(1)^{\frac{1}{3}-1}=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

18. $\lim_{x\to0}\frac{1-\cos 2mx}{\sin^2 nx}$

Sol: $\lim_{x\to0}\frac{1-\cos 2mx}{\sin^2 nx}=\lim_{x\to0}\frac{2\sin^2 mx}{\sin^2 nx}$

=$2\lim_{x\to0}(\frac{\sin mx}{mx})^2(\frac{nx}{\sin nx})^2\frac{m^2x^2}{n^2x^2}$ $2(1)^2(1)^2\frac{m^2}{n^2}$

=$2\frac{m^2}{n^2}$

19. $\lim_{x\to\infty}\frac{x^2+5x+2}{2x^2-5x+1}$

Sol: $\lim_{x\to\infty}\frac{x^2+5x+2}{2x^2-5x+1}=\lim_{x\to\infty}\frac{x^2(1+\frac{5}{x}+\frac{2}{x^2})}{x^2(2-\frac{5}{x}+\frac{1}{x^2})}$

$= \lim_{x\to\infty} \frac{(1+\frac{5}{x}+\frac{2}{x^2})}{(2-\frac{5}{x}+\frac{1}{x^2})} = \frac{1+0+0}{2-0+0} = \frac{1}{2}$

20. $\lim_{x\to\infty} \frac{8|x|+3x}{3|x|-2x}$

Sol: if $x \longrightarrow \infty$ then x>0, Hence $|x|$ = x

$\therefore \lim_{x\to\infty} \frac{8|x|+3x}{3|x|-2x} = \lim_{x\to\infty} \frac{8x+3x}{3x-2x}$

$=\lim_{x\to\infty} \frac{11x}{x} = \lim_{x\to\infty} 11 = 11$

21. $\lim_{x\to\infty}(\sqrt{x+1}-\sqrt{x})$

Sol: $\lim_{x\to\infty}(\sqrt{x+1}-\sqrt{x})=$

$\lim_{x\to\infty} \frac{(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})}{(\sqrt{x+1}+\sqrt{x})}$

$= \lim_{x\to\infty} \frac{x+1-x}{(\sqrt{x+1}+\sqrt{x})}$

$=\lim_{x\to\infty} \frac{1}{(\sqrt{x+1}+\sqrt{x})}$

$= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{(\sqrt{\frac{x}{x}+\frac{1}{x}}+\sqrt{\frac{x}{x}})}$

$= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{(\sqrt{1+\frac{1}{x}}+\sqrt{1})} = \frac{0}{\sqrt{1+0}+1} = \frac{0}{2} = 0$

22. $\lim_{x\to\infty}(\sqrt{x^2+1}-x)$

Sol: $\lim_{x\to\infty}(\sqrt{x^2+1}-x)$

$=\lim_{x\to\infty}(\sqrt{x^2+1}-x)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$

$= \lim_{x\to\infty}(\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}) = \lim_{x\to\infty}(\frac{1}{\sqrt{x^2+1}+x})$

$= \lim_{x\to\infty}(\frac{\frac{1}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}+\frac{x}{x}}) = \lim_{x\to\infty}(\frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}+1})$

$= \frac{0}{\sqrt{1+0}+1} = 0$

23. Evaluate $\lim_{x\to 0}[\frac{\sin(a+bx)-\sin(a-bx)}{x}]$

Sol: $\lim_{x\to 0}[\frac{\sin(a+bx)-\sin(a-bx)}{x}]$

$=\lim_{x\to 0}[\frac{2\cos(\frac{a+bx+a-bx}{2})\sin(\frac{a+bx-a+bx}{2})}{x}]$

$=2\lim_{x\to 0}\cos a \lim_{x\to 0}\frac{\sin bx}{x} = 2\cos a.\ b = 2b\cos a$

24. Evaluate $\log_{x\to 2}\frac{2x^2-7x-4}{(2x-1)(\sqrt{x}-2)}$

Sol: $\log_{x\to 2}\frac{2x^2-7x-4}{(2x-1)(\sqrt{x}-2)} = \log_{x\to 2}\frac{(x-4)(2x+1)}{(2x-1)(\sqrt{x}-2)}$

$=\log_{x\to 2}\frac{(\sqrt{x}-2)(\sqrt{x}+2)(2x+1)}{(2x-1)(\sqrt{x}-2)}$

$=\log_{x\to 2}\frac{(\sqrt{x}+2)(2x+1)}{(2x-1)} = \frac{(\sqrt{2}+2)(2(2)+1)}{(2(2)-1)} = \frac{5(\sqrt{2}+2)}{3}$

25. $\lim_{x\to\infty}\frac{2x^2-x+3}{x^2-2x+5}$

Sol: $\lim_{x\to\infty}\frac{2x^2-x+3}{x^2-2x+5} = \lim_{x\to\infty}\frac{x^2(2-\frac{1}{x}+\frac{3}{x^2})}{x^2(1-\frac{2}{x}+\frac{5}{x^2})} =$

$\lim_{x\to\infty}\frac{(2-\frac{1}{x}+\frac{3}{x^2})}{(1-\frac{2}{x}+\frac{5}{x^2})} = \frac{2-0+0}{1-0+0} = \frac{2}{1} = 2$

26. $\lim_{x\to\infty}\frac{11x^3-3x+4}{13x^3-5x^2-7}$

Sol: $\lim_{x\to\infty}\frac{11x^3-3x+4}{13x^3-5x^2-7} = \lim_{x\to\infty}\frac{x^3(11-\frac{3}{x^2}+\frac{4}{x^3})}{x^3(13-\frac{5}{x}-\frac{7}{x^3})} =$

$\lim_{x\to\infty}\frac{(11-\frac{3}{x^2}+\frac{4}{x^3})}{(13-\frac{5}{x}-\frac{7}{x^3})} = \frac{11-0+0}{13-0-0} = \frac{11}{13}$

27. $\lim_{x\to\infty}\frac{3x^2-4x+5}{2x^3+3x-7}$

Sol: $\lim_{x\to\infty}\frac{3x^2-4x+5}{2x^3+3x-7} = \lim_{x\to\infty}\frac{x^3(\frac{3}{x}-\frac{4}{x^2}+\frac{5}{x^3})}{x^3(2+\frac{5}{x^2}-\frac{7}{x^3})}$

$= \lim_{x\to\infty}\frac{(\frac{3}{x}-\frac{4}{x^2}+\frac{5}{x^3})}{(2+\frac{5}{x^2}-\frac{7}{x^3})} = \frac{0-0+0}{2+0-0} = \frac{0}{2} = 0$

CONTINUITY

1. If f defined by $\mathbf{f(x)} = \begin{cases}\frac{\sin 2x}{x}, & \text{if } x \neq 0\\ 1, & \text{if } x = 0\end{cases}$ **continued at 0**

Sol: $f(x) = \begin{cases}\frac{\sin 2x}{x}, & \text{if } x \neq 0\\ 1, & \text{if } x = 0\end{cases}$ continued at 0

f(0) = 1

$\log_{x\to 0} f(x) = \log_{x\to 0}\frac{\sin 2x}{x} = 2$

$\therefore \log_{x\to 0} f(x) \neq f(0)$

∴x=0 f is continues.

2. If f given by $\mathbf{f(x)} = \begin{cases}K^2x - K & \text{if } x \geq 0\\ 2, & \text{if } x < 1\end{cases}$ **is a continuous function on R then find the value of K.**

Sol: f(x) continuous on R

f(x) at x=1 is continuous

at x=1

$f(1) = K^2x - K = K^2(1) - K = K^2 - K$

LHL = $\lim_{x\to 1-} f(x) = \lim_{x\to 1-} 2 = 2$

At x=1 f(x) is continuous f(1) = LHL

$K^2 - K = 2$

$K^2 - K - 2 = 0$

(K-2)(K+1) = 0

K=2 or -1

3. Show that $\mathbf{f(x)} = \begin{cases}\frac{\cos ax - \cos bx}{x^2}, & x \neq 0\\ \frac{1}{2}(b^2 - a^2), & x = 0\end{cases}$ **where a and b are real constants is continuous at 'a'.**

Sol: $f(x) = \begin{cases}\frac{\cos ax - \cos bx}{x^2}, & x \neq 0\\ \frac{1}{2}(b^2 - a^2), & x = 0\end{cases}$

$f(0) = \frac{1}{2}(b^2 - a^2)$

$\lim_{x\to 0}\frac{\cos ax - \cos bx}{x^2} = \lim_{x\to 0}\frac{2\sin(\frac{ax+bx}{2})\sin(\frac{bx-ax}{2})}{x^2}$

$=2\lim_{x\to 0}\frac{\sin(\frac{a+b}{2})x\ \sin(\frac{b-a}{2})x}{x^2}$

$=2[\lim_{x\to 0}\frac{\sin(\frac{a+b}{2})x}{x}][\lim_{x\to 0}\frac{\sin(\frac{b-a}{2})x}{x}] = 2(\frac{a+b}{2})(\frac{b-a}{2})$

$= \frac{b^2-a^2}{2} \because \lim_{x\to 0}\frac{\sin Kx}{x} = K$

$\therefore \log_{x\to 0} f(x) = f(0)$

So, at x=0 f(x) is continuous.

4. Find real constants a,b. so that the function f given by $\mathbf{f(x)} = \begin{cases} \mathbf{\sin x},\ x \le \mathbf{0} \\ \mathbf{x^2 + a, if\ 0} < x < 1 \\ \mathbf{bx + 3, if\ 1 \le x \le 3} \\ \mathbf{-3},\ x > \mathbf{0} \end{cases}$ **is continuous on R.**

Sol: f(x) continuous on R

f(x) at x=0,3 is continuous

i).At x=0 f(x) is continuous

LHL $=\log_{x\to 0-} f(x)=\log_{x\to 0-} \sin x=\sin(0)=0$

RHL $=\log_{x\to 0+} f(x)=\log_{x\to 0+} x^2 + a=0^2 + a=a$

But x=0, f(x) is continuous

⇒LHL=RHL=a=0

ii) at x=3 f(x) is continuous

LHL$=\log_{x\to 3-} f(x)=\log_{x\to 3-}(bx + 3)$=3b+3

RHL$=\log_{x\to 3+} f(x)=\log_{x\to 3-}(-3)$=-3

But x=3 f(x0 is continuous

LHL=RHL

3b+3 = -3

3b= -6

b=$\frac{-6}{3}$ =-2

@#$

10.DIFFERENTIATION

1. Find the derivative of $\sin(\log x)$ (x>0)

Sol: f'(x) = $\frac{d}{dx}(\sin(\log x)$= $\cos(\log x)\frac{d}{dx}(\log x)$ = $\cos(\log x)\frac{1}{x}$ = $\frac{\cos(\log x)}{x}$

2. Find the derivative of $(x^3 + 6x^2 + 12x - 13)^{100}$

Sol: $\frac{d}{dx}(x^3 + 6x^2 + 12x - 13)^{100}$

$=100(x^3 + 6x^2 + 12x - 13)^{100-1}\frac{d}{dx}(x^3 + 6x^2 + 12x - 13)$

$= 100(x^3 + 6x^2 + 12x - 13)^{99}(3x^2 + 12x + 12)$

3. Find the derivative of $\sin^{-1}\sqrt{x}$

Sol: $\frac{d}{dx}(\sin^{-1}\sqrt{x}) = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}(\sqrt{x}) = \frac{1}{\sqrt{1-x}}\frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2\sqrt{1-x}}\frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x-x^2}}$

4. Find the derivative of $\log(\cosh 2x)$

Sol: $\frac{d}{dx}\log(\cosh 2x) = \frac{1}{\cosh 2x}\frac{d}{dx}(\cosh 2x)$

$= \frac{1}{\cosh 2x}2(\sinh 2x) = 2\tanh 2x$

5. Find the derivative of $(\cot^{-1}x^3)^2$

Sol: $\frac{d}{dx}(\cot^{-1}x^3)^2 = 2\cot^{-1}x^3\frac{d}{dx}(\cot^{-1}x^3) =$

$2\cot^{-1}x^3[\frac{-1}{1+(x^3)^2}]3x^2 = \frac{-6x^2\cot^{-1}x^3}{1+x^6}$

6. Find the derivative of $\log(\sec x + \tan x)$

Sol: $\frac{d}{dx}\log(\sec x + \tan x) = \frac{1}{\sec x+\tan x}\frac{d}{dx}(\sec x + \tan x)$= 1secx+tanx(secxtanx+sec2x)

$= \frac{\sec x(\tan x+\sec x)}{\sec x+\tan x} = \sec x$

7. Find the derivative of $e^{\sin^{-1}x}$

Sol: $\frac{d}{dx}e^{\sin^{-1}x} = e^{\sin^{-1}x}\frac{d}{dx}(\sin^{-1}x) = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

8. Find the derivative of $\sin^{-1}(3x - 4x^3)$

Sol: Let x=sin θ ⇒ $\theta = \sin^{-1}x$

$\therefore \sin^{-1}(3x - 4x^3)$

$=\sin^{-1}(3\sin\theta - 4\sin^3\theta) = \sin^{-1}(\sin 3\theta) = 3\theta$

$=3\sin^{-1}x$

$\therefore \frac{d}{dx}\sin^{-1}(3x - 4x^3 = \frac{d}{dx}(3\sin^{-1}x) = 3\frac{d}{dx}(\sin^{-1}x)$

$= \frac{3}{\sqrt{1-x^2}}$

9. Find the derivative of $\cos^{-1}(4x^3 - 3x)$

Sol: Let x=cos θ ⇒ $\theta = \cos^{-1}x$

$\therefore \cos^{-1}(4x^3 - 3x)$

$=\cos^{-1}(4\cos^3\theta - 3\cos\theta) = \cos^{-1}(\cos 3\theta) = 3\theta$

$=3\cos^{-1}x$

$\therefore \frac{d}{dx}\cos^{-1}(4x^3 - 3x) = \frac{d}{dx}(3\cos^{-1}x)$

$= 3\frac{d}{dx}(\cos^{-1}x) = \frac{-3}{\sqrt{1-x^2}}$

10. Find the derivative of $\tan^{-1}(\frac{2x}{1-x^2})$

Sol: x= tan θ ⇒ $\theta = \tan^{-1}x$

$\therefore \tan^{-1}(\frac{2x}{1-x^2}) = \tan^{-1}(\frac{2\tan\theta}{1-\tan^2\theta}) = \tan^{-1}(\tan 2\theta)$

$=2\theta = 2\tan^{-1}x$

$\therefore \frac{d}{dx}\tan^{-1}(\frac{2x}{1-x^2}) = \frac{d}{dx}(2\tan^{-1}x)$

$= 2\frac{d}{dx}(\tan^{-1}x) = 2(\frac{1}{1+x^2}) = \frac{2}{1+x^2}$

11. Find the derivative of $\tan^{-1}(\frac{a-x}{1+ax})$

Sol: $\frac{d}{dx}\tan^{-1}(\frac{a-x}{1+ax}) = \frac{d}{dx}(\tan^{-1}a - \tan^{-1}x) = \frac{d}{dx}(\tan^{-1}a) - \frac{d}{dx}(\tan^{-1}x) = 0 - \frac{1}{1+x^2} = \frac{-1}{1+x^2}$

12. If y = $\tan^{-1}(\frac{2x}{1-x^2}) + \tan^{-1}(\frac{3x-x^3}{1-3x^2}) - \tan^{-1}(\frac{4x-4x^3}{1-6x^2+x^4})$ then show that $\frac{dy}{dx} = \frac{1}{1+x^2}$

Sol: Let x=tan θ $\Rightarrow \theta = \tan^{-1}x$

$\therefore y = \tan^{-1}(\frac{2x}{1-x^2}) + \tan^{-1}(\frac{3x-x^3}{1-3x^2}) - \tan^{-1}(\frac{4x-4x^3}{1-6x^2+x^4})$

$= \tan^{-1}(\frac{2\tan\theta}{1-\tan^2\theta}) + \tan^{-1}(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}) - \tan^{-1}(\frac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta})$

$= \tan^{-1}(\tan 2\theta) + \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 4\theta)$

$= 2\theta + 3\theta - 4\theta = \theta = \tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

13. If y=$\tan^{-1}[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}]$ for $0< |x| <1$ find $\frac{dy}{dx}$.

Sol: Put $x^2 = \cos 2\theta \Rightarrow 2\theta = \cos^{-1}x^2$

$= \theta = \frac{1}{2}\cos^{-1}x^2$

$y = \tan^{-1}[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}] = \tan^{-1}[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}]$

$= \tan^{-1}[\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}] = \tan^{-1}[\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}]$

$= \tan^{-1}[\frac{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}] = \tan^{-1}[\frac{1+\tan\theta}{1-\tan\theta}]$

$= \tan^{-1}[\tan(\frac{\pi}{4}+\theta)] = \frac{\pi}{4}+\theta$

$\Rightarrow y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

$\frac{dy}{dx} = \frac{d}{dx}[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2] = 0 + \frac{1}{2}[\frac{-1}{\sqrt{1-(x^2)^2}}2x] = \frac{-x}{\sqrt{1-x^4}}$

14. Find the derivative of $\sin^{-1}(\frac{b+a\sin x}{a+b\sin x})$ (a$> 0, b > 0$)

Sol: $\frac{d}{dx}\sin^{-1}(\frac{b+a\sin x}{a+b\sin x}) = \frac{1}{\sqrt{1-(\frac{b+a\sin x}{a+b\sin x})^2}}\frac{d}{dx}(\frac{b+a\sin x}{a+b\sin x})$

$= \frac{(a+b\sin x)}{\sqrt{(a+b\sin x)^2-(b+a\sin x)^2}} X \{\frac{(a+b\sin x)\frac{d}{dx}(b+a\sin x)-(b+a\sin x)\frac{d}{dx}(a+b\sin x)}{(a+b\sin x)^2}\}$

$= \frac{(a+b\sin x)(a\cos x)-(b+a\sin x)(b\cos x)}{\sqrt{(a^2+b^2\sin^2 x+2ab\sin x)-(b^2+a^2\sin^2 x+2ab\sin x)}\ (a+b\sin x)}$

$= \frac{a^2\cos x+ab\sin x\cos x-b^2\cos x-ab\sin x\cos x}{\sqrt{(a^2-b^2)-\sin^2 x(a^2-b^2)}\ (a+b\sin x)}$

$= \frac{(a^2-b^2)\cos x}{\sqrt{(a^2-b^2)(1-\sin^2 x)}\ (a+b\sin x)}$

$= \frac{(a^2-b^2)\cos x}{\sqrt{(a^2-b^2)\cos^2 x}\ (a+b\sin x)} = \frac{\sqrt{a^2-b^2}}{(a+b\sin x)}$

15. Find the derivative of $\cos^{-1}(\frac{b+a\cos x}{a+b\cos x})$ (a$> 0, b > 0$)

Sol: $\frac{d}{dx}\cos^{-1}(\frac{b+a\cos x}{a+b\cos x}) = \frac{-1}{\sqrt{1-(\frac{b+a\cos x}{a+b\cos x})^2}}\frac{d}{dx}(\frac{b+a\cos x}{a+b\cos x})$

$= \frac{-(a+b\cos x)}{\sqrt{(a+b\cos x)^2-(b+a\cos x)^2}} X \{\frac{(a+b\cos x)\frac{d}{dx}(b+a\cos x)-(b+a\cos x)\frac{d}{dx}(a+b\cos x)}{(a+b\cos x)^2}\}$

$= \frac{-(a+b\cos x)(-a\sin x)-(b+a\cos x)(-b\sin x)}{\sqrt{(a^2+b^2\cos^2 x+2ab\cos x)-(b^2+a^2\cos^2 x+2ab\cos x)}\ (a+b\cos x)}$

$= \frac{a^2\sin x+ab\sin x\cos x-b^2\sin x-ab\sin x\cos x}{\sqrt{(a^2-b^2)-\cos^2 x(a^2-b^2)}\ (a+b\cos x)}$

$= \frac{(a^2-b^2)\sin x}{\sqrt{(a^2-b^2)(1-\cos^2 x)}\ (a+b\cos x)}$

$= \frac{(a^2-b^2)\sin x}{\sqrt{(a^2-b^2)\sin^2 x}\ (a+b\cos x)} = \frac{\sqrt{a^2-b^2}}{(a+b\cos x)}$

16. Find the derivative of $\tan(2x)$ from first principle.

Sol: f(x) = $\tan(2x)$

f(x+h) = $\tan 2(x+h) = \tan(2x+2h)$

From first principle

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{\tan(2x+2h)-\tan(2x)}{h}$

$= \lim_{h\to 0}\frac{1}{h}[\frac{\sin(2x+2h)}{\cos(2x+2h)} - \frac{\sin(2x)}{\cos(2x)}]$

$= \lim_{h\to 0}\frac{1}{h}[\frac{\sin(2x+2h)\cos(2x)-\cos(2x+2h)\sin(2x)}{\cos(2x+2h)\cos(2x)}]$

$= \lim_{h\to 0}\frac{1}{h}[\frac{\sin(2x+2h-2x)}{\cos(2x+2h)\cos(2x)}]$

$\because \sin A\cos B - \cos A\sin B = \sin(A-B)$

$= \lim_{h\to 0}\frac{\sin 2h}{h}\frac{1}{\cos(2x+2h)\cos(2x)}$

$= 2\frac{1}{\cos^2(2x)} = 2\sec^2(2x)$

17. Find the derivative of $x\ \sin x$ from first principle.

Sol: f(x) = $x\ \sin x$

f(x+h) = $(x+h)\ \sin(x+h) = x\{\sin(x+h)\} + h\{\sin(x+h)\}$

From first principle

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{x\{\sin(x+h)\}+h\{\sin(x+h)\}-x\ \sin x}{h}$

$= \lim_{h\to 0}\frac{x\{\sin(x+h)-\sin x\}+h\{\sin(x+h)\}}{h}$

$= \lim_{h\to 0}\frac{x}{h}\{\sin(x+h)-\sin x\} + \lim_{h\to 0}\frac{h\{\sin(x+h)\}}{h}$

$= \lim_{h\to 0}\frac{x}{h}[2\cos\frac{(x+h)+x}{2}\ \sin\frac{(x+h)-x}{2}] + \lim_{h\to 0}\ \sin(x+h)$

$= \lim_{h\to 0}\frac{x}{h}[2\cos(\frac{2x}{2}+\frac{h}{2})\ \sin\frac{h}{2}] + \lim_{h\to 0}\ \sin(x+h)$

$= \lim_{h\to 0} 2x\cos(x+\frac{h}{2}).\lim_{h\to 0}\frac{\sin\frac{h}{2}}{h}] + \lim_{h\to 0}\ \sin(x+h)$

$= 2x\cos x.\frac{1}{2} + \sin x = x\cos x + \sin x$

18. Find the derivative of x^2+2 from definition method.

Sol: $f(h)=x^2+2$

$f^1(h)=\frac{f(x+h)-f(h)}{h}$

$=\frac{(x+h)^2+2-(h^2+2)}{h}$

$=\frac{x^2+2xh+h^2+2-x^2-2}{h}$

$=\frac{2xh+h^2}{h}=\frac{h(2x+h)}{h}$

$h+0,$

$f^1(h)=2x.$

19. Find $\frac{d}{dx}[\frac{\cos x}{\cos x+\sin x}]$

Sol: $\frac{d}{dx}[\frac{\cos x}{\cos x+\sin x}]$

$=\frac{(\cos x+\sin x)\frac{d}{dx}(\cos x)-(\cos x)\frac{d}{dx}(\cos x+\sin x)}{(\cos x+\sin x)^2}$

$=\frac{(\cos x+\sin x)(-\sin x)-(\cos x)(-\sin x+\cos x)}{(\cos x+\sin x)^2}$

$=\frac{-\sin x\cos x-\sin^2 x+\sin x\cos x-\cos^2 x}{(\cos x+\sin x)^2}$

$=\frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x+\cos^2 x+2\sin x\cos x}=\frac{-1}{1+\sin 2x}$

20. Find the derivative of a^x using first principles.

Sol : f(x) = a^x

f(x+h) = a^{x+h}

From first principle

f'(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{a^{x+h}-a^x}{h}=a^x\lim_{h\to 0}\frac{a^h-1}{h}=a^x\ln a$

21. Find the derivative of $\cos 2x$ using first principles.

Sol : f(x) = $\cos 2x$

f(x+h) = $\cos 2(x+h)=\cos(2x+2h)$

From first principle

f'(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\cos(2x+2h)-\cos 2x}{h}$

$=\lim_{h\to 0}\frac{1}{h}[-2\sin\frac{(2x+2h+2x)}{2}\sin\frac{(2x+2h)-2x}{2}]$

$=-2\lim_{h\to 0}\frac{1}{h}[\sin\frac{(4x+2h)}{2}\sin\frac{2h}{2}]$

$=-2\lim_{h\to 0}\frac{1}{h}[\sin(2x+h)\sin h]$

$=-2\lim_{h\to 0}[\sin(2x+h)\lim_{h\to 0}\frac{\sin h}{h}]$

$=-2\sin(2x+0).1=-2\sin 2x$

#$%

11. APPLICATION OF DIFFERENTIATION

1. If the increase in the side of a square is 2%. Then the approximate percentage of increase in its area.

Sol: Let x be the side of a square

$\frac{dx}{x}$ X100=2

Area of square A = x^2

$\Rightarrow \log A=\log x^2=2\log x$

$\therefore \log A=2\log x\Rightarrow \frac{1}{A}dA=2.\frac{1}{x}dx$

$\Rightarrow \frac{dA}{A}$X100=2$\frac{dx}{x}$X100=2X2=4

∴ Approximate percentage of increase in area is 4%.

2. Find dy and Δy of y=f(x) = x^2+x at x=10 when Δx=0.1

Sol: y=f(x) = x^2+x ; x=10 ; Δx=0.1

i). Δy=f(x+Δx) – f(x)

$=(x+\Delta x)^2+(x+\Delta x)-(x^2+x)$

$=x^2+2x\Delta x+(\Delta x)^2+x+\Delta x-x^2-x$

= Δx(Δx+2x+1)=0.1(0.1+2(10)+1)=0.1(0.1+21)

=0.1(21.1)=2.11

ii)dy = f'(x) Δx =(2x+1)

Δx=[2(10)+1](0.1)=21(0.1)=2.1

3. Find Δy and dy for the functions y = e^x+x when x=5 and Δx=0.02

Sol: y = f(x) = e^x+x

i) Δy=f(x+Δx) – f(x)

$=e^{x+\Delta x}+(x+\Delta x)-(e^x+x)$

$=e^{5+0.02}+(5+0.02)-(e^5+5)$

$=e^{5.02}+0.02-e^5=e^5(e^{0.02}-1)+0.02$

ii) dy= f'(x) Δx = (e^x+1) Δx= $(e^5+1)(0.02)$

4. Find the equations of the tangent and the normal to the curve y = $5x^4$ at the point(1,5).

Sol: y = $5x^4$

$\frac{dy}{dx}=20x^3$

The slope of the tangent is m=$(\frac{dy}{dx})_p=20(1)^3=20$.

The equation of the tangent at P(1,5) is

y – 5 =20(x-1)

y =20x – 15

The equation of the normal at P(1,5) is

y – 5 =$\frac{-1}{20}$ (x-1)

20y – 100 = -x+1

20y =101 – x

5. Find the slope of the tangent to the curve y=x^3-x+1 at the point whose x coordinate is 2.

Sol: y = x^3-x+1

$\frac{dy}{dx}=3x^2-1$

∴ At x=2 slope of the tangent =$3(2)^2$-1= 12-1=11

6. Find the slope of the tangent to the curve y=$3x^4$-4x at x=4.

Sol: y = $3x^4$- 4x

$\frac{dy}{dx} = 12x^3 - 4$

The slope of the tangent at x=4 is m= $(\frac{dy}{dx})_{x=4}$

$= 12(4)^3 - 4 = 12x64 - 4 = 768 - 4 = 764$

7. Find the lengths of sub-tangent and sub-normal at a point on the curve y = b$\sin\frac{x}{a}$.

Sol: y = b$\sin\frac{x}{a}$

$\frac{dy}{dx} = b.\frac{1}{a}\cos\frac{x}{a} \Rightarrow m = \frac{dy}{dx} = \frac{b}{a}\cos\frac{x}{a}$

Length of sub-tangent = $\left|\frac{y_1}{m}\right| = \frac{b\sin\frac{x}{a}}{\frac{b}{a}\cos\frac{x}{a}} = \left|a\tan\frac{x}{a}\right|$

Length of sub-normal = $|y_1.m| = \left|b\sin\frac{x}{a}.\frac{b}{a}\cos\frac{x}{a}\right|$

$= \left|\frac{b^2}{2a}2\sin\frac{x}{a}.\cos\frac{x}{a}\right| = \left|\frac{b^2}{2a}\sin\frac{2x}{a}\right|$

8. Find the lengths of normal and sub-normal of a point on the curve y = $\frac{a}{2}(e^{\frac{x}{a}} + e^{\frac{-x}{a}})$

Sol: y = $\frac{a}{2}(e^{\frac{x}{a}} + e^{\frac{-x}{a}})$ = a$[\frac{e^{\frac{x}{a}} + e^{\frac{-x}{a}}}{2}]$ = a $\cosh(\frac{x}{a})$

$\frac{dy}{dx} = a\sinh(\frac{x}{a})\frac{1}{a} = \sinh(\frac{x}{a}) = m$

i). Length of normal =

$|y_1.\sqrt{1+m^2}| = \left|a\cosh(\frac{x}{a}).\sqrt{1+\sinh^2(\frac{x}{a})}\right|$

$= \left|a\cosh(\frac{x}{a}).\cosh(\frac{x}{a})\right| = \left|a\cosh^2(\frac{x}{a})\right|$

ii) Length of sub-normal =

$\left|y_1.\frac{dy}{dx}\right| = \left|a\cosh(\frac{x}{a}).\sinh(\frac{x}{a})\right|$

$= \left|\frac{a}{2}2\sinh(\frac{x}{a})\cosh(\frac{x}{a})\right| = \left|\frac{a}{2}\sinh(\frac{2x}{a})\right|$

9. Show that the curves y^2 =4(x+1) and y^2 =36(9-x) intersect orthogonally.

Sol: $y^2 = 4(x+1)$ ....(1)

$y^2 = 36(9-x)$ ....(2)

Solve eq(1) and eq(2)

4(x+1) = 36(9-x)

(x+1) = 9(9-x) = 81 – 9x

10x = 80 ⇒ x=8

Put x=8 in eq(1) y^2 = 4(x+1) = 4(8+1) = 36

⇒ y = ±6

∴ Two curves intersect points P(6,6), Q(8,-6)

i)At P(8,6)

$y^2 = 4(x+1)$

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$ ...(3)

Slope $m_1 = (\frac{dy}{dx})_{P(8,6)} = \frac{2}{6} = \frac{1}{3}$

$y^2 = 36(9-x)$

$2y\frac{dy}{dx} = -36 \Rightarrow \frac{dy}{dx} = \frac{-18}{y}$ ...(4)

Slope $m_2 = (\frac{dy}{dx})_{P(8,6)} = \frac{-18}{6} = -3$

At P(8,6) product of slopes $(m_1)(m_2) = (\frac{1}{3})(-3) = -1$

At P(8,6) curves intersect orthogonally.

ii)At P(8,-6)

$y^2 = 4(x+1)$

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$ ...(3)

Slope $m_1 = (\frac{dy}{dx})_{P(8,-6)} = \frac{2}{-6} = \frac{-1}{3}$

$y^2 = 36(9-x)$

$2y\frac{dy}{dx} = -36 \Rightarrow \frac{dy}{dx} = \frac{-18}{y}$ ...(4)

Slope $m_2 = (\frac{dy}{dx})_{P(8,-6)} = \frac{-18}{-6} = 3$

At P(8,-6) product of slopes $(m_1)(m_2) = (-\frac{1}{3})(3) = -1$

At P(8,-6) curves intersect orthogonally.

10. Show that the curves $6x^2$-5x+2y=0 and $4x^2+8y^2$=3 touch each other at$(\frac{1}{2},\frac{1}{2})$.

Sol: First equation $6x^2$-5x+2y=0 ...(1)

Differentiating w.r.t x

$12x - 5 + 2\frac{dy}{dx} = 0$

$2\frac{dy}{dx} = 5 - 12x$

$\frac{dy}{dx} = \frac{5-12x}{2}$

At P$(\frac{1}{2},\frac{1}{2})$ slope $m_1 = \frac{5-12(\frac{1}{2})}{2} = \frac{5-6}{2} = \frac{-1}{2}$

Second equation $4x^2+8y^2$=3 ...(2)

Differentiating w.r.t x

$8x + 16y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-8x}{16y} = \frac{-x}{2y}$

At P$(\frac{1}{2},\frac{1}{2})$ slope $m_2 = \frac{-\frac{1}{2}}{2\frac{1}{2}} = \frac{-1}{2} = \frac{-1}{2}$

$m_1 = m_2$ at P$(\frac{1}{2},\frac{1}{2})$, slopes are equal.

Substitute P$(\frac{1}{2},\frac{1}{2})$ in eq(1)

$6(\frac{1}{2})^2 - 5(\frac{1}{2}) + 2(\frac{1}{2}) = \frac{6}{4} - \frac{5}{2} + 1 = \frac{6-10+4}{4} = 0$

Substitute P$(\frac{1}{2},\frac{1}{2})$ in eq(2)

$4(\frac{1}{2})^2 + 8(\frac{1}{2})^2 - 3 = \frac{4}{4} + \frac{8}{4} - 3 = 1+2-3 = 0$

∴At P$(\frac{1}{2},\frac{1}{2})$ two curves touch each other.

11. If the tangent at any point on the curve $x^{2/3} + 2^{2/3} = a^{2/3}$ intersects the coordinate axes in A and B, then show that length AB is constant.

Sol: $x^{2/3} + 2^{2/3} = a^{2/3}$ curve at θ point

P($a\cos^3\theta$, $a\sin^3\theta$)

x = $a\cos^3\theta$ and y = $a\sin^3\theta$

∴ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(a\sin^3\theta)}{\frac{d}{d\theta}(a\cos^3\theta)} = \frac{a.3\sin^2\theta(\cos\theta)}{a.3\cos^2\theta(-\sin\theta)} = -\frac{\sin\theta}{\cos\theta}$

At point P($a\cos^3\theta$, $a\sin^3\theta$) slope m = $-\frac{\sin\theta}{\cos\theta}$

∴ Equation for tangent at point P($a\cos^3\theta$, $a\sin^3\theta$), slope $-\frac{\sin\theta}{\cos\theta}$ is

$y - y_1 = m(x - x_1)$

$y - a\sin^3\theta = -\frac{\sin\theta}{\cos\theta}(x - a\cos^3\theta)$

$(y - a\sin^3\theta)\cos\theta = -\sin\theta(x - a\cos^3\theta)$

$(y\cos\theta - a\sin^3\theta\cos\theta) = -x\sin\theta + a\cos^3\theta\sin\theta$

$x\sin\theta + y\cos\theta = a\sin^3\theta\cos\theta + a\cos^3\theta\sin\theta$

$= a\sin\theta\cos\theta(\sin^2\theta + \cos^2\theta)$

$= a\sin\theta\cos\theta$

$\frac{x\sin\theta}{a\sin\theta\cos\theta} + \frac{y\cos\theta}{a\sin\theta\cos\theta} = 1$

$\frac{x}{a\cos\theta} + \frac{y}{a\sin\theta} = 1$

∴A=$(a\cos\theta, 0)$, B=$(0, a\sin\theta)$

∴AB =$\sqrt{(a\cos\theta - 0)^2 + (0 - a\sin\theta)^2}$

=$\sqrt{a^2\cos^2\theta + a^2\sin^2\theta}$ =$\sqrt{a^2(\sin^2\theta + \cos^2\theta)}$

=$\sqrt{a^2}$ = a

∴AB is constant proved.

12. Show that the curves x^2+y^2 =2 and $3x^2+y^2$ =4x have a common tangent at the point (1,1).

Sol: First curve equation $x^2+y^2=2$ ...(1)

Differentiating w.r.t x

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-x}{y}$

At P(1, 1) slope $m_1 = \frac{-1}{1} = -1$

Second curve equation $x^2+y^2=4x$ ...(2)

Differentiating w.r.t x

$6x + 2y\frac{dy}{dx} = 4$

$3x + y\frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2-3x}{y}$

At P(1, 1) slope $m_2 = \frac{2-3(1)}{(1)} = \frac{-1}{1} = -1$

$m_1 = m_2$ at P(1, 1) , slopes are equal.

Substitute P(1, 1) in eq(1)

$x^2+y^2=2 \Rightarrow (1)^2+(1)^2=2$

Substitute P(1, 1) in eq(2)

$(1)^2+(1)^2=4(1)$

∴It is proved that the two curves having common tangent.

13. Find the equation of tangent and normal to the curve y = x^3+4x^2 at (-1,3)

Sol: $y = x^3+4x^2$

Differentiating w.r.t x

$\frac{dy}{dx} = 3x^2+8x$

At P(-1,3) slope m=$3(-1)^2$+8(-1)=3-8=-5

At P(-1,3) slope m=-5 equation of tangent is

$y - y_1 = m(x - x_1)$

y - 3 = -5(x – (-1)) = -5x-5

5x+y+2=0

At P(-1,3) slope equation of normal is

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 3 = -\frac{1}{(-5)}(x - (-1))$

$y - 3 = \frac{1}{5}(x+1)$

5y-15=x+1

x-5y+16=0

14. Show that the length of the sub normal at any point on the curve y^2 =4ax is a constant.

Sol: $y^2=4ax$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$

Length of the sub normal at P = $|y_1.m|$=$|2a|$, a constant.

15. Show that the length of the sub tangent at any point on the curve y^2 = 4ax is a constant.

Sol: $y^2=4ax \Rightarrow y=\sqrt{4ax}$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$

Length of sub-tangent = $\left|\frac{y_1}{m}\right| = \frac{\sqrt{4ax}}{\frac{2a}{y}}$ =

16. A particle is moving in a straight line, so that after t seconds its distance is from a fined point on the line is given by s=f(t)=8t+t^3 find

i) The velocity at time t =2sec

ii) The initial velocity

iii) Acceleration at t=2sec.

Sol: S=f(t) =8t+t^3

Velocity v=$\frac{dS}{dt}$= 8+3t^2

i) The velocity at time t =2sec

V=8+3$(2)^2$ =8+12 =20 m/sec

ii) The initial velocity t=0

V=8+3$(0)^2$ =8+0 =8 m/sec

iii) $\frac{dv}{dt}$ =6t

Acceleration at t=2sec ; 6(2) =12 m/sec^2

17. A particle moving along a straight line has the relation S=t^3+2t+3 connecting the distance"s' described by the particle in time t. Find the velocity and acceleration of the particle at t=4 seconds.

Sol: S=t^3+2t+3

Velocity v=$\frac{dS}{dt}$ =3t^2+2

i)velocity at t=4 sec

v=3t^2+2=3$(4)^2$+2=48+8=56 unit/sec

ii) Acceleration a =$\frac{dv}{dt}$ = 6t

Acceleration at t=4 sec.

=6(4) =24unit/sec^2

18. The distance – time formula for the motion of a particle along a straight line is S = t^3-9t^2+24t-18. Find when and where the velocity is zero.

Sol: S = t^3-9t^2+24t-18

Velocity v=$\frac{dS}{dt}$ =3t^2-18t+24=t^2-6t+8

Velocity is zero ⇒ t^2-6t+8=0

(t-2)(t-4) = 0

t=2; t=4

S at t=2 =$(2)^3$-9$(2)^2$+24(2)-18=8-36+48-18=2 units.

S at t=4 =$(4)^3$-9$(4)^2$+24(4)-18

=64 -144+96 -18=-2 units.

19. Find the equation of tangent and normal to the curve of y=x^4-6x^3+13x^2-10x+5 at (0,5). **(model)**

Sol: y = x^4-6x^3+13x^2-10x+5

$\frac{dy}{dx}$ = 4x^3-18x^2+26x-10

At x=0, slope m=4$(0)^3$-18$(0)^2$+26(0)-10= -10

∴ Equation of tangent y - y_1 = m(x - x_1)

y - 5 = -10(x - 0) =-10x

10x+y-5=0

Slope of normal = - $\frac{1}{m}$ = - $\frac{1}{(-10)}$ =$\frac{1}{10}$

∴ Equation of normal y - y_1 = m_1(x - x_1)

y - 5 = $\frac{1}{10}$ (x - 0)

10y-50=x

X - 10y + 50 = 0

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12. LOCUS

SHORT ANSWER QUESTIONS

1. Find the equation of locus of a point P, if the distance of P from A(3,0) is twice the distance of P from B(-3,0)

Sol: Let P(x,y) be the point in the locus

Given condition is PA =2PB

$\sqrt{(x-3)^2+(y-0)^2}$ =2$\sqrt{(x+3)^2+(y-0)^2}$

x^2+9 – 6x + y^2 =4(x^2+9 + 6x + y^2)= 4x^2+36 + 24x + 4y^2

3x^2+ 3y^2+30x + 27=0

Equation of locus is x^2+ y^2+10x + 9=0

2. Find the equation of a point which is at a distance from A(4,-3).

Sol: Let P(x_1,y_1) be point in the locus

Let A(4,3)

Given condition is PA =$\sqrt{(x_1-4)^2+(y_1+3)^2}$

= ${x_1}^2$-8x_1+16 +${y_1}^2$+6y_1+9

= ${x_1}^2$+${y_1}^2$-8x_1+6y_1+25=0

The equation to the locus of P is

x^2+y^2-8x+6y+25=0

3. Find the equation of locus of a point which equidistant from the points A(-3,2) and B(0,4).

Sol: Let A(-3,2) ; B(0,4).

Let P(x,y) be point in the locus.

Given condition is PA=PB

⇒ PB^2 = $(x+3)^2+(y-2)^2$ = $(x-0)^2+(y-4)^2$

= x^2+6x+9+y^2-4y+4 =x^2+y^2-8y+16

Equation of locus is 6x+4y-3=0

4. **Find the equation of locus of a point P, such that the distance of P from the origin is twice the distance of P from A(1,2).**

Sol: Let P(x,y) be point in the locus, origin O(0,0), A(1,2) be the given point.

Given condition is PO =2PA

PO^2 =4PA^2

$(x-0)^2+(y-0)^2$ =4[$(x-1)^2+(y-2)^2$]

x^2+y^2 = 4[x^2- 2x+1+y^2-4y+4]

3x^2+3y^2-8x-16y+20 = 0

∴The equation to the locus of P is

3x^2+3y^2-8x-16y+20 = 0

5. Find the equation of locus of a point P, the square of whose distance from the origin is 4 times its y-coordinate.

Sol: Let P(x,y) be point in the locus.

Given PO^2 =4y

$(x-0)^2+(y-0)^2$=4y

x^2+y^2 = 4y ⇒ x^2+y^2 - 4y=0

∴The equation to the locus of P is x^2+y^2 - 4y=0

6. Find the equation of locus of a point, such that PA^2+PB^2=2c^2 where A=(a,0), B(-a,0) and 0< $|a|<|c|$

Sol: Let P(x,y) be the point in the locus

Given condition is $PA^2+PB^2=2c^2$

$(x-a)^2+(y-0)^2+(x+a)^2+(y-0)^2=2c^2$

$x^2-2ax+a^2+y^2+x^2+a^2+2ax+y^2=2c^2$

$2x^2+2y^2+2a^2=2c^2$

$x^2+y^2=c^2-a^2$

∴The equation to the locus of P is $x^2+y^2=c^2-a^2$

Essay type questions

7. Find the equation of locus P, if the line segment joining(2,3) and (-1,5) subtends a right angle at P.

Sol: Let P =(x,y) and A(2,3), B(-1,5) be the given points.

Given condition is ∟APB=90°

$PA^2+PB^2=AB^2$

$(x-2)^2+(y-3)^2+(x+1)^2+(y-5)^2=(-1-2)^2+(5-3)^2$

$x^2-4x+4+y^2-6y+9+x^2+1+2x+y^2-10y+25=9+4$

$2x^2+2y^2-2x-16y+26=0$

$x^2+y^2-x-8y+13=0$

∴ The locus of P is $x^2+y^2-x-8y+13=0$

8. Find the equation of the locus of P,if A=(4,0); B=(-4,0) and $|PA-PB|=4$

Sol: Let P=(x,y)

Given condition is $|PA-PB|=4$

$PA=\pm4+PB$

$PA^2=16+PB^2\pm8PB$

$PA^2-PB^2-16=\pm8PB$

$(x-4)^2+(y-0)^2-[(x+4)^2+(y-0)^2]-16=\pm8\sqrt{(x+4)^2+(y-0)^2}$

$x^2-8x+16+y^2-x^2-8x-16-y^2-16=\pm8\sqrt{(x+4)^2+y^2}$

$-16(x+1)=\pm8\sqrt{(x+4)^2+y^2}$

$-2(x+1)=\pm\sqrt{(x+4)^2+y^2}$

$4(x^2+2x+1)=x^2+8x+16+y^2$

$3x^2-y^2=12$

∴ The locus of P is $3x^2-y^2=12$

9. Find the equation of the locus of P, if A =(2,3), B=(2,-3) and $|PA+PB|=8$

Sol: Let P=(x,y)

Given condition is $|PA+PB|=8$

PA= 8-PB

$PA^2=(8-PB)^2$

$PA^2=64+PB^2-16PB$

$16PB-64+PB^2-PA^2$

$16PB-64+[(x-2)^2+(y+3)^2]-[(x-2)^2+(y-3)^2]$

$16PB-64+(y+3)^2-(y-3)^2$

16PB-64+4(3)y =16PB-4(16+3y)

4PB – (16+3y)

Squaring on both sides

$16PB^2=(16+3y)^2$

$16[(x-2)^2+(y+3)^2]=(16+3y)^2$

$16[x^2-4x+4+y^2+6y+9]=256+96y+9y^2$

$16x^2-64x+64+16y^2+96y+144]-256-96y-9y^2=0$

$16x^2+7y^2-64x-48=0$

∴The equation to the locus of P is

$16x^2+7y^2-64x-48=0$

10. A(5,3) and B(3,-2) are two fixed points. Find the equation of the locus of P. So that the area of triangle is 9.

Sol: Let P(x,y)

Given condition is Area of ΔPAB =9

$|(x-5)(y+2)-(y-3)(x-3)|=2(9)$

$|xy+2x-5y-10-(xy-3y-3x+9)|=18$

$|5x-2y-19|=18$

$5x-2y-19=\pm18$

$5x-2y-19=18$ or $5x-2y-19=-18$

5x - 2y -37=0 or 5x-2y-1=0

∴The equation to the locus of P is

(5x - 2y -37)(5x - 2y -1) = 0

11. If the distance from P to the points (2,3) and (2,-3) are in the ratio 2:3, then find the equation of the locus of P.

Sol: Let point P(x,y), A=(2,3) B=(2,-3)

PA:PB=2:3

3PA=2PB

$9PA^2=4PB^2$

$9[(x-2)^2+(y-3)^2]=4[(x-2)^2+(y+3)^2]$

$9[x^2-4x+4+y^2-6y+9]=4[x^2-4x+4+y^2+6y+9]$

∴The equation to the locus of P is

$5x^2+5y^2-20x-78y+65=0$

12. A(1,2), B(2,-3) and C(-2,3) are three points, a point P moves such that $PA^2+PB^2=2PC^2$. Show that the equation of the locus of P is 7x-7y+4=0.

Sol: Points A=(1,2), B=(2,-3) and C=(-2,3)

$PA^2+PB^2=2PC^2$

$[(x-1)^2+(y-2)^2]+(x-2)^2+(y+3)^2]$
$=2[(x+2)^2+(y-3)^2]$

$[x^2-2x+1+y^2-4y+4]+[x^2-4x+4+y^2+6y+9]$
$=2[x^2+4x+4+y^2-6y+9]$

$2x^2+2y^2-6x+2y+18=2x^2+2y^2+8x-12y+26$

14x-14y+8=0

7x-7y+4=0

∴P(x,y) locus of P is 7x-7y+4=0.

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13. TRANSFORMATION OF AXES

Short answer questions

1. When the origin is shifted to (-2,3) by transformation of axes let us find the co-ordinate of (1,2) w.r.t new axes.

Sol: Let (x,y) be the original co-ordinates of (X,Y)
Let (X,Y) be the new co-ordinates of (1,2)
∴ 1 = X-2 ; 2=Y+3
⇒X=3; Y=-1
∴ The new co-ordinates of (1,2) are (3,-1)

2. When the origin is shifted to (2,3) by translation of axes, the co-ordinates of a point P are changed as (4,-3). Find the co-ordinates of P in the original system.

Sol: Let (X,Y) be th eoriginal coordinates of
(x,y) = (4,3)
X=x+h=4+2=6
Y=y+k=-3+3=0
∴ Original coordinates =(6,0)

3. Find the point to which the origin is to be shifted. So that the point(3,0) may change to (2,-3).

Sol: Let P(h,k) be the point to which the origin is to be shifted.
3=2+h; 0=-3+k
H=1 ; k=3
P=(1,3)

4. Find the point to which the origin is to be shifted so as to remove the first degree terms from the equation $4x^2$+9y^2- 8x+36y+4=0

Sol: Comparing the given equation with
$ax^2+by^2+2gx+2fy+c=0$
$4x^2+9y^2-8x+36y+4=0$
We get a=4; b=9; g=-4; f=18; c=4
Required point = $(\frac{-g}{a},\frac{-f}{b}) = (\frac{-(-4)}{4},\frac{-18}{9})$=(1,-2)

5. When the axes are rotated through an angle 30°. Find the new coordinates of (0,5),(-2,4) and (0,0).

Sol: (i) Here (x,y) =(0,5), angle of rotation θ=30°, then

$X = x\cos\theta + y\sin\theta$
$= 0 + 5\sin 30° = 5.\frac{1}{2} = \frac{5}{2}$
$Y = x\sin\theta + y\cos\theta$
$= 0 + 5\cos 30° = 5.\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$
∴ The new coordinates (X,Y) = $(\frac{5}{2}, \frac{5\sqrt{3}}{2})$

(i) Here (x,y) =(-2,4), angle of rotation θ=30°, then
$X = x\cos\theta + y\sin\theta$
$= -2\cos 30° + 4\sin 30° = -2.\frac{\sqrt{3}}{2} + 4.\frac{1}{2} = 2 - \sqrt{3}$
$Y = -x\sin\theta + y\cos\theta$
$= -(-2)\sin 30° + 4\cos 30°$
$= 2.\frac{1}{2} + 2\sqrt{3} = 1 + 2\sqrt{3}$
∴The new coordinates (X,Y) = $(2 - \sqrt{3}, 1 + 2\sqrt{3})$

(iii) Here (x,y) =(0,0), angle of rotation θ=30°, then
$X = x\cos\theta + y\sin\theta$
= 0 +0 =0
$Y = x\sin\theta + y\cos\theta$
= 0 +0 =0
∴ The new coordinates (X,Y) = (0, 0)

6. When the axes are rotated through an angle 60°. Find the original co-ordinates of (3,4),(-7,2) and (2,0).

Sol: i) Here (x,y) =(3,4), angle of rotation θ=60°, then
$X = x\cos\theta - y\sin\theta$
$= 3\cos 60° - 4\sin 60°$
$= 3.\frac{1}{2} - 4.\frac{\sqrt{3}}{2} = \frac{3-4\sqrt{3}}{2}$
$Y = x\sin\theta + y\cos\theta$
$= 3\sin 60° + 4\cos 60°$
$= 3.\frac{\sqrt{3}}{2} + 4.\frac{1}{2} = \frac{3\sqrt{3}+4}{2}$
∴The original coordinates P(X,Y) = $(\frac{3-4\sqrt{3}}{2}, \frac{4+3\sqrt{3}}{2})$

ii) Here (x,y) =(-7,2), angle of rotation θ=60°, then
$X = x\cos\theta - y\sin\theta$
$= -7\cos 60° - 2\sin 60°$
$= -7.\frac{1}{2} - 2.\frac{\sqrt{3}}{2} = \frac{-7-2\sqrt{3}}{2}$
$Y = x\sin\theta + y\cos\theta$
$= -7\sin 60° + 2\cos 60°$
$= -7.\frac{\sqrt{3}}{2} + 2.\frac{1}{2} = \frac{-7\sqrt{3}+2}{2}$
∴The original coordinates Q(X,Y)=$(\frac{-7-2\sqrt{3}}{2}, \frac{2-7\sqrt{3}}{2})$

iii) Here (x,y) =(2,0), angle of rotation θ=60°, then
$X = x\cos\theta - y\sin\theta$
$= 2\cos 60° - 0 = 2.\frac{1}{2} = 1$
$Y = x\sin\theta + y\cos\theta$
$= 2\sin 60° + 0 = 2.\frac{\sqrt{3}}{2} = \sqrt{3}$
∴The original coordinates R(X,Y) = $(1, \sqrt{3})$

7. Find the angle through which the axes are to be rotated so as to remove the xy term in the equation x^2 + 4xy - y^2- 2x +2y – 6 = 0.

Sol: Sol: $x^2 + 4xy - y^2 - 2x + 2y - 6 = 0$.
Comparing the given equation with
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.
We get, a = 1; b = -1; h = 2
Required angle of rotation is
$\Theta = \frac{1}{2}\tan^{-1}(\frac{2h}{a+b}) = \frac{1}{2}\tan^{-1}(\frac{2(2)}{1+(-1)})$
$= \frac{1}{2}\tan^{-1}\infty = \frac{1}{2}.\frac{\pi}{2} = \frac{\pi}{4}$

Essay type questions

8. When the origin is shifted to the point(2,3), the transformed equation of a curve is x^2 + 3xy - 2y^2+ 17x -7y – 11 = 0. Find the original equation of the curve.

Sol: Given transformed equation of a curve is

x^2 + 3xy - 2y^2+ 17x -7y – 11 = 0 ...(1)
Let new point (h,k) =(2,3)
X = x - h ; Y = y – k
= x – 2 ; = y – 3
From eq (1)
$(x-2)^2$ + 3(x-2)(y-3) - 2$(y-3)^2$+ 17(x-2) -7(y-3) – 11 = 0
x^2 +4 -2x+ 3(xy -3x-2y+6)- 2(y^2+9-6y)+ 17x -34 -7y +21– 11=0
x^2+ 4 -2x +3xy -9x -6y +18 -2y^2-18+12y +17x -34 -7y +21– 11=0
∴The original equation of the curve is
x^2 + 3xy - 2y^2+ 4x - y -20 = 0

9. When the origin is shifted to (-1,2) by the translation of axes, find the transformed equation to x^2 + y^2 + 2x - 4y +1 = 0.

Sol: Given equation x^2 + y^2 + 2x - 4y +1 = 0 ...(1)
New point (h,k) = (-1,2)
X = x + h ; Y = y + k
= x + (-1)=x-1 ; = y + 2
From eq (1)
$(x-1)^2$ + $(y+2)^2$ + 2(x-1) – 4(y+2)+1 = 0
x^2 -2x+1+ y^2+4y+4+2x-2-4y-8+1=0
∴The transformed equation of the curve is
x^2 + y^2-4 = 0.

10. When the axes are rotated through an angle 45°. Find the original equation of the curve 17x^2 -16xy + 17y^2 = 225

Sol: Given equation is 17x^2 -16xy + 17y^2 = 225 ...(1)
rotated through an angle 45°

$X = x\cos\theta + y\sin\theta$

$= x\cos 45° + y\sin 45° = x\frac{1}{\sqrt{2}} + y\frac{1}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

$Y = y\cos\theta - x\sin\theta$

$= y\cos 45° - x\sin 45° = y\frac{1}{\sqrt{2}} - x\frac{1}{\sqrt{2}} = \frac{y-x}{\sqrt{2}}$

From eq (1)

$17(\frac{x+y}{\sqrt{2}})^2 - 16(\frac{x+y}{\sqrt{2}})(\frac{y-x}{\sqrt{2}}) + 17(\frac{y-x}{\sqrt{2}})^2 = 225$

$17[\frac{x^2+y^2+2xy}{2}] - 16[\frac{y^2-x^2}{2}] + 17[\frac{x^2+y^2-2xy}{2}] = 225$

$\frac{17x^2+17y^2+34xy-16y^2+16x^2+17x^2+17y^2-34xy}{2}] = 225$

$50x^2 + 18y^2 = 2(225) \Rightarrow 25x^2 + 9y^2 = 225$

11. When the axes are rotated through an angle $\frac{\pi}{4}$. Find the transformed equation 3x^2 +10xy + 3y^2 = 9

Sol: $\theta = \frac{\pi}{4}$ = 45°

$X = x\cos\theta - y\sin\theta$

$= x\cos 45° - y\sin 45° = x\frac{1}{\sqrt{2}} - y\frac{1}{\sqrt{2}} = \frac{x-y}{\sqrt{2}}$

$Y = y\cos\theta + x\sin\theta$

$= y\cos 45° + x\sin 45° = y\frac{1}{\sqrt{2}} + x\frac{1}{\sqrt{2}} = \frac{y+x}{\sqrt{2}}$

∴The transformed equation is

$3(\frac{x-y}{\sqrt{2}})^2 + 10(\frac{x-y}{\sqrt{2}})(\frac{x+y}{\sqrt{2}}) + 3(\frac{x+y}{\sqrt{2}})^2 = 9$

$3[\frac{x^2+y^2-2xy}{2}] + 10[\frac{x^2-y^2}{2}] + 3[\frac{x^2+y^2+2xy}{2}] = 9$

$3x^2 + 3y^2 - 6xy + 10x^2 - 10y^2 + 3x^2 + 3y^2 + 6xy] = 18$

$16x^2 - 4y^2 - 18 = 0$

∴The transformed equation is $8x^2 - 2y^2 - 9 = 0$

14. STRAIGHT LINES

Short answer questions.

1.Find the equation of straight line joining through the point(2,3) and making non-zero intercept on the co-ordinate axes whose sum is zero.

Sol: Let te intercepts be a,-a (a≠0)
∴ Equation to the line is $\frac{x}{a} + \frac{y}{-a}$ =1
x-y = a ....(1)
eq(1) passing through (2,3)
2-3 =a ⇒a = -1
∴ Required line is x-y+1=0

2. Find the value of x, if the slope of the line passing through (2,5) and (x,3) is 2.

Sol: Slope of the line passing through (2,5), (x,3) is

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{3-5}{x-2} = \frac{-2}{x-2}$

Given slope m=2

$\frac{-2}{x-2} = 2$

$x-2 = \frac{-2}{2} = -1$

x = -1+2 = 1

3. Find the value of y if the line joining the points (3,y),(2,7) is parallel to the line joining the points (-1,4) (0,6).

Sol: Slope of line joining points (3,y),(2,7) is

$m_1 = \frac{y-7}{3-2} = y-7$

Slope of line joining points (-1,4) (0,6) is

$m_2 = \frac{4-6}{-1-0} = 2$

Lines are parallel ⇒ $m_1 = m_2$
y -7 = 2 ⇒y = 9

4. Find the equation of straight line which makes an angle of $\frac{\pi}{4}$ with x-axis and passing through the points (0,0)

Sol: Slope of straight line $\theta = \frac{\pi}{4}$

$m = \tan\frac{\pi}{4} = \tan 45° = 1$

Equation of a line is $y-y_1 = m(x-x_1)$
y-0 = 1(x-0)
y = x

5. Show that the points (-5,1)(5,5)(10,7) are collinear

Sol: Let A(-5,1),B(5,5), C(10,7)

Slope of AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{5-(-5)} = \frac{4}{10} = \frac{2}{5}$

Slope of AC = $\frac{y_2-y_1}{x_2-x_1} = \frac{7-1}{10-(-5)} = \frac{6}{15} = \frac{2}{5}$

Slope of AB = Slope of AC
∴A,B,C are collinear.

6. Find the sum of the squares of the intercepts of the line 4x-3y=12 on the coordinate axes.

Sol: Given line is 4x-3y =12

$\frac{4x}{12} - \frac{3y}{12} = 1$

$\frac{x}{3} + \frac{y}{(-4)} = 1$

Sum of the squares of the intercepts is $(3)^2 + (-4)^2$ = 9+16=25

7. Find the equation of straight line which makes an angle of $\alpha = 150°$ with x-axis and passing through (1,2).

Sol: Slope $y - y_1 = m(x-x_1)$

Angle of straight line θ=150°

m = tan θ = tan 150° =tan(180° - 30°)

= -tan 30° = - $\frac{1}{\sqrt{3}}$

Point(1,2), slope = - $\frac{1}{\sqrt{3}}$ passing straight line

y-2 = - $\frac{1}{\sqrt{3}}$(x-1)

$\sqrt{3}$(y-2) = - x+1

x+$\sqrt{3}$y – (2$\sqrt{3}$+1) = 0

8.**Transform the straight line 4x – 3y + 12 = 0 into a) slope – intercept form b) Intercept form c) normal form**

Sol: a) Given equation is 4x – 3y + 12 = 0

3y = 4x +12

y = $\frac{4}{3}$ x +4

Which is slope-intercept form

b) Given equation is 4x – 3y + 12 = 0

$\frac{-4x}{12} + \frac{3y}{12} = 1$

$\frac{x}{-3} + \frac{y}{4} = 1$ which is the intercept form

c) Given equation is 4x – 3y + 12 = 0

-4x+3y=12

$\frac{-4}{\sqrt{4^2+3^2}}$ x + $\frac{3}{\sqrt{4^2+3^2}}$ y = $\frac{12}{\sqrt{4^2+3^2}}$

$\frac{-4}{5}$ x + $\frac{3}{5}$ y = $\frac{12}{5}$ which is perpendicular form.

9. Find the ratios in which i) x-axis and ii) y-axis divide the line segment AB joining A(2,-3) & B(3,-6)

Sol: i)X-axis divides $\overline{AB}$ in the ratio

$-y_1 : y_2$=+3 : -6 = 1 : -2

ii) y-axis divides $\overline{AB}$ in the ratio $-x_1 : x_2$ = -2 : 3

10. Find the value of K if the lines 2x-3y+K=0, 3x-4y+13=0 and 8x-11y+33=0 are concurrent.

Sol: The given lines are

2x-3y+K=0 ...(1),

3x-4y+13=0 ...(2) ,

8x-11y+33=0 ...(3)

If (h,k) be the point of inter section of (2) and (3) then

3h-4k+13=0

8h-11k+33=0

24h-32k+104=0 eq(2)X8

24h-33k+ 99=0 eq(3) x3

K + 5 =0

K=-5

Substitute k=-5 in 3q(2)

3h-4(-5)+13=0

3h+20+13 = 0

h= $\frac{-33}{3}$ = -11

Since three line are concurrent, the point(-11,-5) should satisfy the eq(1)

∴2(-11)-3(-5)+K=0

-22+15+k=0 ⇒ K=7

11. Find the angle between straight line y=4-2x; y=3x+7

Sol: $\cos\theta = \frac{a_1a_2+b_1b_2}{\sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)}} = \frac{2.3+1.(-1)}{\sqrt{(2^2+1^2)(3^2+(-1)^2)}}$

$= \frac{6-1}{\sqrt{5.10}} = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{4}$

12. Find the length of perpendicular drawn from the point (-2,-3) to the straight line 5x-2y+4=0.

Sol: length of perpendicular from the point $P(x_0,y_0)$ to the straight line ax +by + c = 0 *is*

$\left|\frac{ax_0+by_0+c}{\sqrt{a^2+b^2}}\right| = \left|\frac{5(-2)+(-2)(-3)+4}{\sqrt{5^2+(-2)^2}}\right| = \left|\frac{-10+6.+4}{\sqrt{25+4}}\right|$=0

13. Find the distance between parallel lines 3x-4y = 12 and 3x-4y = 7

Sol: Given lines 3x-4y = 12 and 3x-4y = 7

3x-4y -12 =0 and 3x-4y – 7=0

Distance between parallel lines is =

$\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right| = \frac{(-12)-(-7)}{\sqrt{3^2+4^2}} = \frac{-5}{5}$ = -1

14. Find the value of p if the straight lines 3x+7y-1=0 and 7x-py+3=0 are mutually perpendicular.

Sol: Given lines are perpendicular

3(7)+7(-p) = 0

21=7p ⇒p =3

15. Find the foot of the perpendicular drawn from (4, 1) upon the straight line $3x - 4y + 12 = 0$.

Sol: Let foot of the perpendicular drawn from (4,1) is (h,k)

Given point (x_1,y_1) = (4,1)

$3x - 4y + 12 = 0$. Compare with ax+by+c=0

A=3; b=-4; c=12

Equation for foot of perpendicular is

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$

$\frac{h-4}{3} = \frac{k-1}{-4} = \frac{-(3(4)+(-4)(1)+12)}{3^2+(-4)^2}$

$\frac{h-4}{3} = \frac{k-1}{-4} = \frac{-(12-4+12)}{9+16} = \frac{-20}{25} = \frac{-4}{5}$

$\frac{h-4}{3} = \frac{-4}{5}$ ⇒5h -20 = -12

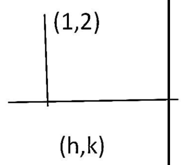
5h=20-12 = 8 ⇒h = $\frac{8}{5}$

$\frac{k-1}{-4} = \frac{-4}{5}$ ⇒5k -5 = 16

5h=16+5 = 21 ⇒h = $\frac{21}{5}$

Foot of the perpendicular (h,k) =$(\frac{8}{5}, \frac{21}{5})$

16. Find the image the point(1,2) is the straight line 3x+4y-1=0.



Sol: Let image is (h,k),

(x_1,y_1) = (1,2)

3x+4y-1=0

3x+4y-1=0

A=3; b=4; c=-1

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$

$\frac{h-1}{3} = \frac{k-2}{4} = \frac{-2(3(1)+(4)(2)-1)}{3^2+(4)^2}$

$\frac{h-4}{3} = \frac{k-2}{4} = \frac{-2(3+8-1)}{9+16} = \frac{-20}{25} = \frac{-4}{5}$

$\frac{h-1}{3} = \frac{-4}{5}$ ⇒5h -5 = -12

5h= -12 +5= -7 ⇒h = $\frac{-7}{5}$

$\frac{k-2}{4} = \frac{-4}{5}$ ⇒5k -10 = -16

5k= -16 +10= -6 ⇒k = $\frac{-6}{5}$

∴Image of point(1,2) is =$(\frac{-7}{5}, \frac{-6}{5})$

17. Find the foot of the perpendicular drawn from the point(3,0) on to the line 4x+12y-41=0

Sol: If (h,k) be the foot of the perpendicular from(3,0) to the line 4x+12y-41=0, then

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$

$\frac{h-3}{4} = \frac{k-0}{12} = \frac{-(4(3)+(12)(0)-41)}{4^2+(12)^2}$

$\frac{h-3}{4} = \frac{k}{12} = \frac{-(12-41)}{16+144} = \frac{29}{160}$

$\frac{h-3}{4} = \frac{29}{160}$ ⇒160h -480 = 116

160h=116+480 = 596 ⇒h = $\frac{149}{40}$

$\frac{k}{12} = \frac{29}{160}$ ⇒160k = 348

⇒k = $\frac{348}{160} = \frac{87}{40}$

Foot of the perpendicular (h,k) = $(\frac{149}{40}, \frac{87}{40})$

18. If the straight lines ax +by +c=0, bx +cy +a=0, cx + ay +b=0 are concurrent, then prove that $a^3+b^3+c^3$=3abc

Sol: The given lines are concurrent

Let P($\alpha+\beta$) be the point of concurrence, then

aα+bβ+c=0 …(1)

bα+cβ+a=0 …(2)

cα+aβ+b=0 …(3)

By solving eq(1) and eq(2)

abα+$b^2\beta$+bc=0 …(1)

abα+acβ+a^2=0 …(2)

(b^2-ac)β =a^2-bc

$\beta = \frac{a^2-bc}{(b^2-ac)}$

bα+cβ+a=0

bα+c$\frac{a^2-bc}{(b^2-ac)}$+a=0

bα= $\frac{bc^2-a^2c}{(b^2-ac)}$ -a

= $\frac{bc^2-a^2c-ab^2+a^2c}{(b^2-ac)} = \frac{bc^2-ab^2}{(b^2-ac)}$

$\alpha = \frac{c^2-ab}{(b^2-ac)}$

α, β values substitute in eq(3)

cα+aβ+b=0

c.$\frac{c^2-ab}{(b^2-ac)}$ +a.$\frac{a^2-bc}{(b^2-ac)}$+b=0

$c^3 - abc + a^3 -$ abc $+b^3 -$ abc=0

∴ $a^3+b^3+c^3$=3abc

19. Find the equation of the line which passes through(0,0) and the point of intersection of the lines x+y+1=0 and 2x-y+5=0.

Sol: If P($\alpha+\beta$) be the point of intersection of the lines 2x-y+5=0; x+y+1=0

2α-β+5=0 α+β+1=0

α β c

-1 5   2   -1

1   1   1   1

By the method of cross multiplication

$\frac{\alpha}{-1-5} = \frac{\beta}{5-2} = \frac{1}{3+2} = \frac{1}{5}$

$\alpha = -\frac{6}{5}$; $\beta = \frac{3}{5}$

∴P($-\frac{6}{5}, \frac{3}{5}$)

20. Show that the distance of the point(6,-2) from the line 4x+3y=12 is half the distance of the point(3,4) from the line (4x-3y=12).

Sol: Equation of AB is 4x + 3y − 12 = 0

PQ = Length of the perpendicular

from $P = \frac{|24-6-12|}{\sqrt{16+9}} = \frac{6}{5}$

Equation of CD is 4x − 3y − 12 = 0

RS = Length of the perpendicular

from $R = \frac{|12-12-12|}{\sqrt{16+9}} = \frac{12}{5}$

∴ PQ = $\frac{1}{2}$RS

21.Transform the equation of the line x+y+2=0 into i)slope-intercept form ii) intercept form iii) normal form

Sol: x+y+2=0

i)Slope intercept form   y = mx+c

y = -x -2

ii) Intercept form $\frac{x}{a}+\frac{y}{b}$ =1

$\frac{x}{-2} + \frac{y}{-2} = 1$

iii)Normal form

Given equation x+y+2=0

X+y=-2

Divide with $\sqrt{a^2+b^2} = \sqrt{2}$ on both sides

$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = -\sqrt{2}$

x cos$\frac{\pi}{4}$ + y sin$\frac{\pi}{4}$ = -$\sqrt{2}$

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15. PAIR OF STRAIGHT LINES

Essay type questions

1.Find the acute angle between the pair of lines represented x^2-7xy+12y^2=0

Sol: Given line x^2-7xy+12y^2=0

Compare with ax^2+2hxy+by^2=0

$\tan\theta = \pm\frac{2\sqrt{h^2-ab}}{a+b}$

A=1; b=12; h= $-\frac{7}{2}$

$\tan\theta = \pm\frac{2\sqrt{(-\frac{7}{2})^2-(1)(12)}}{1+12}$

$= \pm\frac{2\sqrt{\frac{49}{4}-12}}{13} = \pm\frac{\frac{2}{2}\sqrt{49-48}}{13} = \pm\frac{1}{13}$

Acute angle $\theta = \tan^{-1}(\frac{1}{13})$

2. Find the centroid and area of a triangle formed by the lines 2y^2- xy - 6x^2 = 0; x+y+4 = 0

Sol: 2y^2- xy - 6x^2 = 0 ....(1)

x+y+4 = 0 ....(2)

y = -(x+4) ....(3)

substitute y value in eq(1)

$2[-(x+4)]^2$- x[-(x+4)] - 6x^2 = 0

2[x^2+8x+16] +x^2+4x - 6x^2 = 0

2x^2+16x+32 +x^2+4x - 6x^2 = 0

-3x^2+20x+32 = 0

3x^2-20x-32 = 0

3x^2-24x+4x-32 = 0

3x(x-8)+4x-32 = 0

(3x+4)(x-8) =0

x= $-\frac{4}{3}$ or 8

Substitute x= $-\frac{4}{3}$ in eq(3)

y = -(x+4) = -[$-\frac{4}{3}$ + 4] =-[$\frac{-4+12}{3}$] = $-\frac{8}{3}$

∴Point A=[$-\frac{4}{3}$, $-\frac{8}{3}$]

x = 8 substitute in eq(3)

y = -(8+4) = -12

Point B=[8, - 12]

2y^2- xy - 6x^2 = 0 cut at O(0,0)

∴Centroid of ΔOAB = [$\frac{0-\frac{4}{3}+8}{3}$, $\frac{0-\frac{4}{3}-12}{3}$]

=[$\frac{-4+24}{3(3)}$, $\frac{-8-36}{3(3)}$]=($\frac{20}{9}$, $\frac{-44}{9}$)

Area of ΔOAB = $\frac{1}{2}|x_1y_2 - x_2y_1|$ =

$\frac{1}{2}\left|\left(-\frac{4}{3}\right)(-12) - \left(-\frac{8}{3}\right)(8)\right|$

$= \frac{1}{2}\left|\frac{48}{3} + \frac{64}{3}\right|$

$= \frac{1}{2}\left|\frac{112}{3}\right| = \frac{56}{3}$ square units

3. Find the equation of pair of lines intersecting at (2,-1) and perpendicular to the pair of line 6x^2-13xy-5y^2=0.

Sol: Equation to the pair of lines perpendicular to 6x^2-13xy-5y^2=0 and passing through (2,-1) is

-5$(x-2)^2$-13(x-2)(y+1)+6$(y+1)^2$=0

-5[x^2-4x+4]-13[xy+x-2y-2]+6[y^2+2y+1]=0

-5x^2+20x-20-13xy-13x+26y+26+6y^2+12y+6=0

-5x^2+20x-20-13xy-13x+26y+26+6y^2+12y+6=0

-5x^2+13xy+6y^2+33x-14y-40=0

5x^2-13xy- 6y^2-33x+14y+40=0

4. Find the equation of pair of lines intersecting at (2,-1) and perpendicular to the pair of line 6x^2-13xy-5y^2=0.

Sol: Equation to the pair of lines perpendicular to 6x^2-13xy-5y^2=0 and passing through (2,-1) is

X=x-2, Y=y+1

6$(x-2)^2$-13(x-2)(y+1)-5$(y+1)^2$=0

6[x^2-4x+4]-13[xy+x-2y-2] - 5[y^2+2y+1]=0

6x^2-24x+24-13xy-13x+26y+26 -5y^2-10y-5=0

6x^2-13xy-5y^2-37x+16y+45=0

5. Find the combined equation of pair of bisectors of the angle between the pair of straight lines represented by 6x^2-11xy+3y^2=0

Sol: Given 6x^2-11xy+3y^2=0

Comparing with ax^2+2hxy+by^2=0

a=6; b=3; h=$\frac{-11}{2}$

Combined equation of pair of bisectors

h(x^2-y^2)=(a-b)xy

$\frac{-11}{2}$ (x^2-y^2)=(6-3)xy

11x^2+6xy+11y^2=0

6. Show that the equation 2x^2-13xy-7y^2+x+23y-6=0 represents a pair of straight lines and also find the angle between and the coordinates of the point of intersection of lines.

Sol: Given equation is 2x^2-13xy-7y^2+x+23y-6=0

Comparing the given equation with

ax^2+2hxy+by^2+2gx+2fy+c=0

we get a=2; h= $\frac{-13}{2}$; b=-7; g=$\frac{1}{2}$; f=$\frac{23}{2}$; c=-6

Now abc +2fgh -af^2- bg^2- ch^2

=(2)(-7)(-6)+2($\frac{23}{2}$)($\frac{1}{2}$)($\frac{-13}{2}$)-2($\frac{23}{2}$)^2-(-7) ($\frac{1}{2}$)^2-(-6) ($\frac{-13}{2}$)^2

=84-$\frac{299}{4}$ - $\frac{1058}{4}$ +$\frac{7}{4}$ +$\frac{1014}{4}$

=$\frac{336-299-1058+7+1014}{4}$ =0

h^2-ab = ($\frac{-13}{2}$)^2-(2)(7) = $\frac{169}{4}$+14 $> 0 \Rightarrow h^2 >$ ab

g^2-ac = ($\frac{1}{2}$)^2-(2)(-6) = $\frac{1}{4}$ +12 $> 0 \Rightarrow g^2 >$ ac

f^2-bc = ($\frac{23}{2}$)^2-(-7)(-6) = $\frac{529}{4}$ - 42 $> 0 \Rightarrow f^2 >$ bc

Given equation represents a pair of lines

Point of intersection $= \left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$

$= \left(\frac{\left(\frac{-13}{2}\right)\left(\frac{23}{2}\right)-(-7)\left(\frac{1}{2}\right)}{(2)(-7)-\left(\frac{-13}{2}\right)^2}, \frac{\left(\frac{1}{2}\right)\left(\frac{-13}{2}\right)-(2)\left(\frac{23}{2}\right)}{(2)(-7)-\left(\frac{-13}{2}\right)^2}\right)$

$= \left(\frac{\left(\frac{-299}{4}\right)+\left(\frac{7}{2}\right)}{(-14)-\left(\frac{169}{4}\right)}, \frac{\left(\frac{-13}{4}\right)-(23)}{(-14)-\left(\frac{169}{4}\right)}\right)$

$= \left(\frac{-299+14}{-56-169}, \frac{-13-92}{-56-169}\right)$

$= \left(\frac{-285}{-225}, \frac{-105}{-225}\right) = \left(\frac{19}{15}, \frac{7}{15}\right)$

If θ is the acute angle between the lines

$\cos\theta = \frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$

$= \frac{|2-7|}{\sqrt{(2+7)^2+4\left(\frac{-13}{2}\right)^2}} = \frac{5}{\sqrt{81+169}}$

$= \frac{5}{\sqrt{250}} = \frac{5}{5\sqrt{10}} = \frac{1}{\sqrt{10}}$

$\Theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$

7. Show that the equation $8x^2$ - 24xy + $18y^2$- 6x + 9y - 5 = 0 represents a pair of parallel lines and find the distance between them.

Sol: Given equation is $8x^2$ - 24xy + $18y^2$- 6x + 9y - 5 = 0

Comparing with ax^2+2hxy+by^2+2gx+2fy+c=0

we get a=8; h= -12; b= 18; g=-3; f=$\frac{9}{2}$; c=-5

Δ = abc +2fgh -af^2- bg^2- ch^2

= 8(18)(-5) +2($\frac{9}{2}$)(-3)(-12) -8($\frac{9}{2}$)^2 - 18$(-3)^2$ – (-5)$(-12)^2$

= -720+324-162-162+720 = 0

h^2-ab = $(-12)^2$-(8)(18) = 144-144 $= 0 \Rightarrow h^2 =$ ab

g^2-ac = $(-3)^2$-(8)(-5) = 9 +40 $> 0 \Rightarrow g^2 >$ ac

f^2-bc = $(\frac{9}{2})^2$-(18)(-5) = $\frac{81}{4}$ +90 $> 0 \Rightarrow f^2 >$ bc

Since Δ = 0, $h^2 =$ ab, $g^2 >$ ac and $f^2 >$ bc the given equation represents a pair of parallel lines.

The distance between the parallel lines =$2\sqrt{\frac{g^2-ac}{a(a+b)}}$

$=2\sqrt{\frac{(-3)^2-8(-5)}{8(8+18)}} = 2\sqrt{\frac{9+40}{8(26)}}$

$=2\sqrt{\frac{49}{208}} = \frac{2x7}{4\sqrt{13}} = \frac{7}{2\sqrt{13}}$

8. Show that the lines joining the origin to the points of inter section of curve x^2 - xy + y^2+ 3x + 3y - 2 = 0 and the straight line x-y-$\sqrt{2}$ =0 are normally perpendicular.

Sol: Given curve is x^2 - xy + y^2+ 3x + 3y - 2 = 0 ...(1)

Given is x-y-$\sqrt{2}$ =0

$\frac{x-y}{\sqrt{2}}$ = 1 ...(2)

Let A and B be the points of intersection of eq(1) and eq(2).

The combined equation of OA and OB is

x^2 - xy + y^2+ (3x + 3y)($\frac{x-y}{\sqrt{2}}$) - 2($\frac{x-y}{\sqrt{2}}$)^2=0

$\sqrt{2}x^2$ -$\sqrt{2}$ xy + $\sqrt{2}y^2$+ $3x^2$ - $3y^2$ -$\sqrt{2}x^2$ + $\sqrt{2}y^2$+$\sqrt{2}$ xy = 0

$3x^2 - 3y^2 = 0$

In the above equation co-efficient of x^2 + co-efficient of y^2=0

∴ The lines are mutually perpendicular.

9. Find the values of K. If the lines joining the origin to the points of intersection of the curve $2x^2$ -2 xy + $3y^2$+ 2x - y - 1 = 0 and the lines x+2y+K are mutually perpendicular.

Sol: The given equation is x+2y=K ⇒ $\frac{x+2y}{K}$ = 1

Let A,B be the points of intersection of the given line and the given curve.

The combined equation OA and OB is

$2x^2$ -2 xy + $3y^2$+ (2x – y)($\frac{x+2y}{K}$) – ($\frac{x+2y}{K}$)^2 = 0

K^2($2x^2$ -2 xy + $3y^2$)+K($2x^2$ +-3 xy - $2y^2$) – (x^2 +4 xy + $4y^2$) =0

($2K^2$+2K-1)x^2-($2K^2$-3K+4)xy +($3K^2$-2K-4)y^2 = 0

$\llcorner AOB = \frac{\pi}{2}$

co-efficient of x^2 + co-efficient of y^2=0

$2K^2$+2K-1+$3K^2$-2K-4=0

$5K^2$-5=0

K^2 =1⇒K = ± 1

10. Find the angle between the lines joining the origin to the points of intersection of the curve $7x^2$ -4 xy + $8y^2$+ 2x - 4y - 8 = 0 with the straight line 3x-y=2.

Sol: The given line equation is 3x-y=2 ⇒ $\frac{3x-y}{2}$ = 1

Let A,B be the points of intersection of the give line and the given curve.

The combined equation of OA and OB is

$7x^2$ -4 xy + $8y^2$+ 2(x - 2y)($\frac{3x-y}{2}$) – 8($\frac{3x-y}{2}$) ^{2}= 0

$7x^2$ -4 xy + $8y^2$+$3x^2$-xy-6xy+$2y^2$-$18x^2$+12xy-$2y^2$=0

-$8x^2$ + xy + $8y^2$ =0

co-efficient of x^2 + co-efficient of y^2=0

=-8+8=0

$\llcorner AOB = \frac{\pi}{2}$

The required angle θ=$\frac{\pi}{2}$

11. Find the condition for the lines joining the origin to the points of intersection of the circle x^2+y^2=a^2 and the line lx+my=1 to coincide.

Sol: The combined equation of $\overline{OA}$ and OB is

x^2+y^2-$a^2(lx + my)^2$=0

(1-a^2l^2)x^2+(1-a^2m^2)y^2-2a^2lmxy=0

x^2+y^2-$a^2l^2x^2$-$a^2m^2y^2$- 2a^2lmxy=0

Given the lines are mutually perpendicular

co-efficient of x^2 + co-efficient of y^2=0

1-a^2l^2+1-a^2m^2=0

$a^2(l^2+m^2)$ = 2

%^&

16. THREE DIMENSIONAL COORDINATES

Short Answer questions.

1.Find x if the distance between (5,-1,7) and (x,5,1) is 9 units.

Sol: Let A(5,-1,7); B(x,5,1)

Given AB = 9

$(5-x)^2+(-1-5)^2+(7-1)^2 = AB^2 = 9^2$

$25+x^2-10x+36+36 = 81$

$x^2-10x+16=0$

(x-8)(x-2)=0 ⇒x=8 or 2

2. Show that the points (2,3,5) (-1,5,-1) and (4,-3,2) form a straight angled isosceles triangle.

Sol: Lat A(2,3,5); B(-1,5-1); C(4,-3,2)

AB=$\sqrt{(2+1)^2+(3-5)^2+(5+1)^2}$

= $\sqrt{9+4+36}$ = $\sqrt{49}$ = 7

BC=$\sqrt{(-1-4)^2+(5+3)^2+(-1-2)^2}$

=$\sqrt{25+64+9}$ = $\sqrt{98}$ = $7\sqrt{2}$

CA=$\sqrt{(4-2)^2+(-3-3)^2+(2-5)^2}$

=$\sqrt{4+36+9}$ = $\sqrt{49}$ = 7

AB=AC ⇒ ΔABC is isosceles

AB^2+AC^2 =49+49=98=BC^2

ΔABC is right angled

∴ ΔABC is right angled isosceles triangle.

3. Show that the points (1,2,3) (2,3,1) and (3,1,2) form an equilateral triangle.

Sol: Let A=(1,2,3); B=(2,3,1) and C=(3,1,2)

AB=$\sqrt{(1-2)^2+(2-3)^2+(3-1)^2}$ =$\sqrt{1+1+4}$ = $\sqrt{6}$

BC=$\sqrt{(2-3)^2+(3-1)^2+(1-2)^2}$ =$\sqrt{1+4+1}$ = $\sqrt{6}$

CA=$\sqrt{(3-1)^2+(2-1)^2+(2-4)^2}$ =$\sqrt{4+1+1}$ = $\sqrt{6}$

AB=BC=CA=$\sqrt{6}$

∴ ΔABC is an equilateral triangle

4. Show that the points (1,2,3) (7,0,1) and (-2,3,4) are collinear.

Sol: Let A=(1,2,3); B=(7,0,1) and C=(-2,3,4)

AB=$\sqrt{(1-7)^2+(2-0)^2+(3-1)^2}$

=$\sqrt{36+4+4}$ = $\sqrt{44}$ = $2\sqrt{11}$

BC=$\sqrt{(7+2)^2+(0-3)^2+(1-4)^2}$

=$\sqrt{81+9+9}$ = $\sqrt{99}$ = $3\sqrt{11}$

AC=$\sqrt{(1+2)^2+(2-3)^2+(3-4)^2}$ =$\sqrt{9+1+1}$ = $\sqrt{11}$

AB+AC =$2\sqrt{11}$+$\sqrt{11}$ =$3\sqrt{11}$ =BC

∴ A,B,C are collinear.

5. Find the coordinated of vertex C of ΔABC, if its centroid is origin and the vertices A,B are (1,1,1) and (-2,4,1) respectively.

Sol: Let A(1,1,1); B(-2,4,1) C(x,y,z)

Given centroid (0,0,0)

$\left(\frac{1-2+x}{3}, \frac{1+4+y}{3}, \frac{1+1+z}{3}\right)$ =(0,0,0)

∴C(1,-5,-2)

6. If (3,2,-1)(4,1,1) and (6,2,5) are three vertices and (4,2,2) is a centroid of tetrahedron . find the fourth vertex.

Sol: Let (3,2,-1)(4,1,1) (6,2,5) and (x,y,z) be the vertices of the tetrahedron

Centroid = (4,2,2)

$\left(\frac{3+4+6+x}{4}, \frac{2+1+2+y}{4}, \frac{-1+1+5+z}{4}\right)$ =(4,2,2)

$\frac{13+x}{4}$ =4;	$\frac{5+y}{4}$ =2;	$\frac{5+z}{4}$ =2;
13+x=16;	5+y=8;	5+z=8
X=3;	y=3;	z=3

∴Forth vertex = (3,3,3)

7. Find the distance between the midpoint of line segment AB and the point(3,-1,2) where A=(6,3,-4) and B=(-2,-1,2)

Sol: A=(6,3,-4); B=(-2,-1,2); P=(3,-1,2)

midpoint of $\overline{AB}$ is Q =$\left[\frac{6-2}{2},\frac{3-1}{2},\frac{-4+2}{2}\right]$ =(2,1,-1)

Distance between mid Q and P

PQ =$\sqrt{(3-2)^2+(-1-0)^2+(2+1)^2}$

=$\sqrt{1+4+9}$ = $\sqrt{14}$

8. The three consecutive vertices of a parallelogram are given as (2,4,-1)(3,6,-1)(4,5,1). Find the fourth vertex.

Sol: Let the parallelogram is ABCD

A=(2,4,-1); B=(3,6,-1); C=(4,5,1); D=(a,b,c)

Midpoint of AC= Midpoint of BD

$\left(\frac{2+4}{2}, \frac{4+5}{2}, \frac{-1+1}{2}\right) = \left(\frac{3+a}{2}, \frac{6+b}{2}, \frac{-1+c}{2}\right)$

$\frac{3+a}{2}$=3, $\frac{6+b}{2}$=9, $\frac{-1+c}{2}$=0

A=3; b=3; c=1

∴Forth vertex is D= (3,3,1)

&*&

17. DIRECTION COSINES AND DIRECTION RATIOS

Short Answer questions

1.If the line makes angles α,β,γ with the +ve directions of x,y,z axes. What is the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

Sol: $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=1-\cos^2\alpha+1-\cos\beta+1-\cos^2\gamma$

$=3-(\cos^2\alpha+\cos\beta+\cos^2\gamma)=3-1=2$

2. What are the direction cosine of the line joining the points(-4,1,7) and (2,-3,2)

Sol: Let A=(-4,1,7) B=(2,-3,2)

Direction ratios of $\overline{AB}$ are =(2+4:-3-1:2-7)=(6:-4:-5)

= -6:4:5

Direction cosines of $\overline{AB}$ are

$[\frac{-6}{\sqrt{(-6)^2+4^2+5^2}},\frac{4}{\sqrt{(-6)^2+4^2+5^2}},\frac{5}{\sqrt{(-6)^2+4^2+5^2}}]$

$=[\frac{-6}{\sqrt{77}},\frac{4}{\sqrt{77}},\frac{4}{\sqrt{77}}]$

3. If (6,10,10)(1,0,-5)(6,-10,1) are thee vertices of a triangle. Find the direction ratios of its sides. Also show that it is a right angle triangle.

Sol: Let A(6,10,10) B(1,0,-5) C(6,-10,0)

Direction ratio of AB = (1-6,0-10,-5-10)

= (-5,-10,-15)=(1,2,3)

Direction ratio of BC =(6-1,0-10, 05)

=(5,-10,5)=(1,-2,1)

Direction ratio of CA =(6-6,-10-10,0-10)

=(0,-20,-10)=(0,2,1)

(1)(1)+2(-2)+3(1) =1-4+3=0

∟ABC =90° :∴ ΔABC is right angled triangle.

4. Find the ratio in which the XZ-plane divides the line joining A(-2,3,4) and B(1,2,3)

Sol: Lat A(-2,3,4) B(1,2,3)

The ratio in which the XZ plane divides AB

$-y_1:y_2$ i.e., -3:2

∴ A(-2,3,4) B(1,2,3) divides in the ratio -3:2

$=\frac{-3x1+2x-2}{-3+2},\frac{-3x2+2x3}{-3+2},\frac{-3x3+2x4}{-3+2}=(\frac{-7}{-1},\frac{0}{-1},\frac{-1}{-1})=(7,0,1)$

5. Show that the lines PQ and RS are parallel, if P=(2,3,4); Q=(4,7,8) R=(-1,-2,1) S=91,2,5)

Sol: Direction ratio of PQ =(4-2,7-3,8-4)

= (2,4,4)=(1,2,2)

Direction ratio of RS =(1+1,2+2,5-1)

= (2,4,4)=(1,2,2)

∴PQ=RS

PQ and RS are parallel.

Essay questions

6. Find the direction cosines two lines which are connected by its relation l+m+n=0 and mn-2nl-2lm=0

Sol:Let l+m+n=0 ...(1)

2lm-mn+2nl=o

l=-m-n

2m(-m-n)-mn+2n(-m-n)=0

$-2m^2$-2mn-mn-2mn-$2n^2$=0

$-2m^2$-5mn-$2n^2$=0

$2m^2$+5mn+$2n^2$=0

(2m+n)(m+2n) =0 ⇒m= $-\frac{n}{2}$ 0r -2n

If m=$-\frac{n}{2}$ then l= $\frac{n}{2}-n=-\frac{n}{2}$

l:m:n= $-\frac{n}{2}$: $-\frac{n}{2}$:n=1:1:-2

If m=-2n then l=+2n-n=n

l:m:n=n:-2n:n=1:-2:1

∴Direction cosines of the lines are

$[\frac{1}{\sqrt6}:\frac{1}{\sqrt6}:\frac{-2}{\sqrt6}]$ and $[\frac{1}{\sqrt6}:\frac{-2}{\sqrt6}:\frac{1}{\sqrt6}]$

If θ is the acute angle between the lines, then

$\cos\theta=[\frac{1}{2}+\frac{2}{6}-\frac{2}{6}]=\frac{3}{6}=\frac{1}{2}$; ⇒ $\theta=\cos^{-1}\frac{1}{2}$=60°

7. Find the direction cosines of two lines which are connected by the relation l -5m+3n=0 and $7l^2+5m^2-3n^2=0$

Sol: Let l -5m+3n=0 ...(1)and

$7l^2+5m^2-3n^2=0$ ...(2)

l=5m-3n

$7(5m-3n)^2+5m^2-3n^2=0$

$7(25m^2+9n^2-30mn)+5m^2-3n^2=0$

$180m^2-210mn+60n^2=0$

$6m^2-7mn+2n^2=0$

$6m^2-4mn-3mn+2n^2=0$

2m(3m-2n)-n(3m-2n) =0

(3m-2n)(2m-n)=0

n=2m or $\frac{3m}{2}$

If n=$\frac{3m}{2}$ then l=5m-3n=5m-3.$\frac{3m}{2}=\frac{m}{2}$

∴l:m:n=$\frac{m}{2}$:m: $\frac{3m}{2}$=1:2:3

If n=2m then l=5m-6m=-m

∴l:m:n= $-m$:m: $2m$=-1:1:2

∴ Direction cosine of the lines are

$[\frac{1}{\sqrt{1^2+2^2+3^2}},\frac{2}{\sqrt{1^2+2^2+3^2}},\frac{3}{\sqrt{1^2+2^2+3^2}}]=[\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}]$ and

$[\frac{-1}{\sqrt{1^2+1^2+2^2}},\frac{1}{\sqrt{1^2+1^2+2^2}},\frac{2}{\sqrt{1^2+1^2+2^2}}]=[\frac{-1}{\sqrt6},\frac{1}{\sqrt6},\frac{2}{\sqrt6}]$

8. Find the angle between the lines where direction cosines are given by the equations 3l+m+5n=0 and 6mn-2nl+5lm=0

Sol: Let 3l+m+5n=0 ...(1)and

6mn-2nl+5lm=0 ...(2)

m=-3l-5n

6n(-3l-5n)-2nl+5l(-3l-5n)=0

-18ln-$30n^2$-2nl-$15l^2$-25ln=0

-$15l^2$-$30n^2$-45ln=0

l^2+3ln+$2n^2$=0

(l+n)(l+2n)=0

L=-n or -2n

If l=-n then m=3n-5n=-2n

l:m:n = -n:-2n:n= -1:-2:1=1:2:-1

If l=-2n then m=-3(-2n)-5n=n

l:m:n = -2n: n:n= -2:1:1=2:-1:-1

Direction cosines of the two lines are (1,2,-1) and (2,-1,-1)

If θ is the acute angle between the lines, then

$\cos\theta = \frac{(1)(2)+2(-1)+(-1)(-1)}{\sqrt{1+4+1}\sqrt{4+1+1}}$

$= \frac{2-2+1}{\sqrt{6}\sqrt{6}} = \frac{1}{6}$

$\Rightarrow \theta = \cos^{-1}(\frac{1}{6})$

9. Find the angle between the lines where direction cosines satisfy equations l+m+n=0 ; $l^2+m^2-n^2=0$

Sol: Let l+m+n=0 ...(1)

$l^2+m^2-n^2=0$ ...(2)

l= -m-n

$(-m-n)^2+m^2-n^2=0$

$m^2+n^2+2mn+m^2-n^2=0$

$2m^2+2mn=0 \Rightarrow 2m(m+n)=0$

m=0 or -n

If m=0 then l=-n

l:m:n = -n: 0:n= -1:0:1=1:0:-1

If m=-n then l=0

l:m:n = 0:-n:n= 0:-1:1

if θ is the acute angle between the lines, then

$\cos\theta = \frac{(-1)(0)+0(-1)+(1)(1)}{\sqrt{1+0+1}\sqrt{0+1+1}} = \frac{0-0+1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

$\Rightarrow \theta = \cos^{-1}(\frac{1}{2}) = 60°$

∴angle between the lines is 60°or 120°

^^^^

20. Find the expansion of (i) Sin (*A*+ *B* −*C*)
(ii) cos(*A*− *B* −*C*) .

Sol: $\sin(A+B-C) = \sin[(A+B)-C]$

$= \sin(A+B).\cos C - \cos(A+B)\sin C$

$= (\sin A\cos B + \cos A\sin B)\cos C$

$-(\cos A\cos B - \sin A\sin B)\sin C$

$= \sin A\ \cos B\ \cos C + \cos A\ \sin B\ \cos C$

$-\cos A\ \cos B\ \sin C + \sin A\sin B\ \sin C$

cos(A - B - C)

$\cos(A-B-C) = \cos\{(A-B)-C\}$

$= \cos(A-B)\cos C + \sin(A-B)\sin C$

$= (\cos A\cos B + \sin A\sin B)\cos C$

$+(\sin A\ \cos B\ \cos A\ \sin B)\sin C$

$= \cos A\cos B\ \cos C + \sin A\ \sin B\ \cos C$

$+\sin A\ \cos B\ \sin C - \cos A\sin B\ \sin C$

VOCATIONAL BRIDGE COURSE

First Year - Paper – I (w.e.f. 2018-19)

MATHEMATICS SCHEME OF EXIMATION

(WEIGHTAGE)

Total Questions : 15

Time: 3 Hours Max.Marks: 75

Note: In section A – Answer all Questions

In section B – Answer any three Questions

Section – A 10x3=30

Note: i) *Answer all the questions*

ii) Each question carries 3 marks.

1. From Algebra 2. From Algebra
3. From Calculus 4. From Calculus
5. From Co-ordinate Geometry
6. From Co-ordinate Geometry
7. From Co-ordinate Geometry
8. From Trigonometry
9. From Trigonometry
10. From Trigonometry

Section – B 3x15=45

Note: i) *Answer any 3 questions*

ii) Each question carries 15 marks.

11. From Algebra with internal choice
12. From Calculus with internal choice
13. From Co-ordinate Geometry with internal choice
14. From Trigonometry with internal choice
15. I(a) – From Algebra

I(b) – From Calculus

OR

II(a) – from Co-ordinate Geometry

II(b) – from Trigonometry

VOCATIONAL BRIDGE COURSE
MATHEMATICS – First Year (w.e.f. 2018-2019)
MODEL QUESTION PAPER

Time: 3 Hours **Max. Marks: 75**

Section A **10x3=30**

Note:

i) Answer **all** questions

ii) Each question carries **3** marks

1. A function f : $A \to B$ is defined by $f(x) = x^2 + x + 1$. If $A = \{-2, -1, 0, 1, 2\}$, then find B.
2. If the vectors $-3i + 4j + pk$ and $qi + 8j + 6k$ are collinear, then find p and q.
3. $\lim_{x\to 2} \frac{2x^2-7x-4}{2x-1}$
4. Find $\frac{d}{dx}\left(\frac{\cos x}{\cos x + \sin x}\right)$
5. A point P moves such that PA = PB where A = (−3, 2) and B = (0, 4) . Find the equation to the locus of P.
6. Transform the line equation of the line $x + y + 2 = 0$ into
(i) slope – intercept form (ii) intercept form
(iii) normal form.
7. The three consecutive vertices of a parallelogram are given as (2,4, –1), (3,6,–1),(4,5,1). Find the fourth vertex.
8. Simplify: sin 330°.cos120° + cos 210°.sin 300° .
9. Simplify $\frac{3\cos\theta+\cos 2\theta}{\sin\theta-\sin 3\theta}$
10. If sinh x = 3/4, find cosh (2 x)

Section B **3x15 =45**

Note:

i) Answer any **3** questions

ii) Each question carries **15** marks

11.I(a) Prove by mathematical induction.
$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$.

I(b) If $A=\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ find $(A^T)^{-1}$

OR

II(a) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$=(a-b)(b-c)(c-a)

II(b) If $\bar{a}$=(1,-2,1); $\bar{b}$=(2,1,1) and $\bar{c}$=(1,2,-1) then find $|\bar{a} \times (\bar{b} \times \bar{c})|$ and $|(\bar{a} \times \bar{b}) \times \bar{c}|$

12.I(a) Evaluate $\lim_{x\to 0}\left[\frac{\sin(a+bx)-\sin(a-bx)}{x}\right]$

I(b) Find the derivative of x^2+2x from first principles.

OR

II(a) Show that f(x) = $\frac{\cos ax-\cos bx}{x^2}$ for x≠0
= $\frac{b^2-a^2}{2}$ for x=0
where a and b are real constants, is continuous at x = 0

II(b) Find the equations of tangent and normal to the curve of y=x^4-6x^3+13x^2-10x+5 at (0,5).

13 I(a) Find the foot of the perpendicular drawn from the point (3,0) upon the straight line 5x+ 12y- 41=0.

I (b) Find the equation to the straight line which passes through (0, 0) and also the point of intersection of the lines x + y + 1 = 0 and 2 y − y + 5 = 0

OR

II(a) When the axes are rotated through on angle π , find the transformed equation of $3x^2 + 10xy + 3y^2 = 4$.

II(b). If (6,10,10), (1, 0, −5), (6, −10, 0) are the vertices of a triangle, find the direction ratios of its sides. Also, show that it is a right angled triangle.

14.I(a) If $\sin(A+B)=\frac{24}{25}$ and $\cos(A-B)=\frac{4}{5}$ where $0<A<B<\frac{\pi}{4}$, then find sin 2A.

I(b) Solve $\sin^{-2}\theta$ - $\cos\theta = \frac{1}{4}$

OR

II(a) If A+B+C = 180° then prove that
sin 2A -sin 2B+sin 2C =4cos A sin B cos C

II(b) Solve $\sqrt{3}\cos\theta+\sin\theta = \sqrt{2}$

15 I(a). Solve $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$ by matrix inversion method.

I(b) .Find the equations of tangent and normal to the curve of y=x^4-6x^3+13x^2-10x+5 at (0,5).

OR

II(a) Show that the equation $2x^2$-13xy-7y^2+x+23y-6=0 represents a pair of straight lines and also find the angle between and the coordinates of the point of intersection of lines.

II(b) Prove that
$\cos^2\frac{\pi}{10}+\cos^2\frac{2\pi}{5}+\cos^2\frac{3\pi}{5}+\cos^2\frac{9\pi}{10}=2$

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---" HARD WORK IS SECRETE OF SUCCESS"---