VOCATIONAL BRIDGE COURSE

Second Year - Paper – II (w.e.f. 2019-2020) MATHEMATICS(2004)

MODEL QUESTION PAPER

Time: 3 Hours Max. Marks: 75

Section – A 10x3=30

Note: i) Answer all the questions

ii) Each question carries 2 marks.

1.Express $\frac{2+5i}{3-2i}+\frac{2-5i}{3+2i}$ in the form of a+ib

2. Obtain the quadratic equations whose roots are $\frac{m}{n}, \frac{-n}{m}$ (m≠0, n≠0)

3.Find the coefficients of x^{11} in the expansion $(2x^3+\frac{3}{x^3})^{13}$

4.Evaluate $\int\frac{1+\cos 2x}{1-\cos 2x}$ dx

5.Evaluate $\int x^2e^x\,dx$

6. Evaluate $\int_0^1\frac{1}{1+x^2}dx$

7. Find the equations of the circles whose extremities of a diameter are (7,-3)and (3,5)

8. Find the eccentricity and latus rectum of the hyperbola $x^2-4y^2=4$

9. Find the mean deviation about the mean for the data:3,6,10,4,10,9

10. Two dice are rolled. Find the probability that none of the dice shows the number '2'.

Section – B 3x15=45

Note: i) Answer any 3 questions

ii) Each question carries 15 marks.

11.(i)(a) If the coefficient of 10^{10} in the expansion of $(ax^2+\frac{1}{bx})^{11}$ is equal to the coefficient of 10^{-10} in the expansion of $(ax^2-\frac{1}{bx})^{11}$. Find the relation between *a* and *b* where *a* and *b* are real numbers.

11.(i)(b) Resolve $\frac{5x+1}{(x-1)(x+2)}$ into partial fractions.

(OR)

11.(ii)(a) If α,β are the roots of the equation ax^2+b x + c=0, then find the following (i)$\alpha^3+\beta^3$ (ii) $\alpha^2+\beta^2$ (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$.

11.(ii)(b) If $(n+1)P_5$: nP_5=3:2 then find the value of n.

12.(i)(a)Evaluate $\int\frac{\sec^2x}{(1+\tan x)^2}dx$

12.(i)(b) Evaluate $\int_0^1\frac{1}{\sqrt{3-2x}}dx$

(OR)

12.(ii)(a)Evaluate $\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}$

12.(ii)(b) Solve $\frac{dy}{dx}=\frac{xy+y}{xy+x}$

13.(i)(a) Show that the line x+y+1=0 touches towards the circle $x^2+y^2-3x+7y+14=0$ and find its point of contact.

13.(i)(b) Find the equation of the parabola whose focus is (3,5) and vertex is (1,3)

(OR)

13.(ii)(a) Find the equation of the ellipse whose focus is (1,-1), eccentricity $\frac{2}{3}$ and directrix is x+y+2=0.

13.(ii)(b) Find the equation of the hyperbola whose asymptotes are the straight lines x+2y+3=0, 3x+4y+5=0 and which passes through the point (1,-1).

14.(i)(a) Find the probability that a non leap year contains i) 53 Sundays ii) 52 Sundays only.

14. I (b) Find the mean deviation about median of the following frequency distribution.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

(OR)

14.(ii)(a) If A and B are two events with $P(A\cup B)$=0.65 and $P(A\cap B)$=0.15. Then find the value of $P(A^C)+P(B^C)$

14.(ii)(b) Find the variance and standard deviation of the following data 5,12,3,18,6,8,2,10.

15.(i)(a)Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions.

15.(i)(b) Solve$(x^2-y^2)\frac{dy}{dx}$=xy

(OR)

15.(ii)(a) Find the internal and external centers of similitude of the circles $x^2+y^2=9$ and $x^2+y^2-16x+2y+49=0$.

15.(ii)(b) Find the variance and standard deviation of the following frequency distribution

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

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VOCATIONAL BRIDGE COURSE
MATHEMATICS – Second Year (2004)

March 2020

Time: 3 Hours Max. Marks: 75

Section – A 10x3=30

Note: i) Answer all the questions

ii) Each question carries 2 marks.

1. Express $(1-i)^3(1+i)$ in the form of a+ib

2. Obtain the quadratic equations whose roots are $\frac{m}{n}, \frac{n}{m}$

3. If -1, 2 , α are the roots of equation $2x^3+x^2$-7x-6 = 0, then find α.

4. Evaluate $\int\frac{1+\cos 2x}{1-\cos 2x}$ dx

5. Evaluate $\int x^2e^x\,dx$

6. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$

7. Find the equation of the circle having centre (1, 4) and radius r=5

8. Find the eccentricity and latus rectum of the hyperbola $x^2 - 4y^2 = 4$

9. Find the mean deviation about the mean for the data: 3, 6,10,4,9,10

10. If A , B are two independent events such that P(A)=0.6, P(B)=0.7. Then find

(i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(A/B)$

Section – B 3x15=45

Note: i) Answer any 3 questions

ii) Each question carries 15 marks.

11(i)(a) Resolve into partial fractions $\frac{3x^2+2x}{(x-3)(x^2+2)}$.

11(i)(b) Show that the points in Argand plane represented by the complex numbers 2 + i, 4 +3i, 2 +5i, 3i forms a square.

(OR)

11(ii)(a) If α,β are the roots of the equation ax^2+b x + c=0, then find the following

(i) $\alpha^2+\beta^2$ (ii) $\alpha^3+\beta^3$ (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$.

11(ii)(b) If $(n+1)P_5 : nP_5 = 3:2$ then find the value of n.

12(i)(a) Evaluate $\int\frac{2x+5}{\sqrt{x^2-2x+10}}dx$

12(i)(b) Evaluate $\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}dx$

(OR)

12(ii)(a) Evaluate $\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}$

12(ii)(b) Solve $\frac{dy}{dx}=\frac{xy+y}{xy+x}$

13(i)(a) Show that the line 5x+12y+4=0 touches the circle x^2+y^2-6x +4y -12=0 and find its point of contact.

13(i)(b) Find the equation of the parabola whose focus is (3,5) and vertex is (1,3)

(OR)

13(ii)(a) Find the condition that the line lx + my + n =0 is to be a tangent with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1

13(ii)(b) Obtain the equation of Parabola y^2 = 4ax

14(i) (a) Find the variance and standard deviation of the following frequency distribution

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	18	4	3

14. (i) (b) State and prove addition theorem on Probability.

(OR)

14(ii)(a) If A and B are two events with $P(A\cup B)$=0.65 $P(A\cap B)$=0.15. Then find the value of $P(A^C)+P(B^C)$

14(ii)(b) Find the variance and standard deviation of the following data 5,12,3,18,6,8,2,10.

15(i)(a) Resolve into partial fractions $\frac{x^2+5x+7}{(x-3)^3}$.

15(i)(b) Solve the differential equation $\frac{dy}{dx}=\frac{y^2-2xy}{x^2-xy}$

(OR)

15(ii)(a) Find the internal and external centers of the circles $x^2+y^2=9$ and x^2+y^2 -16x + 2y + 49 = 0.

15(ii)(b) Find the mean deviation about median of the following data.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

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VOCATIONAL BRIDGE COURSE

MATHEMATICS – Second Year (2004)

September 2021

Time: 3 Hours Max. Marks: 75

Section – A 10x3=30

Note: i) Answer all the questions

ii) Each question carries 2 marks.

1. Express $(1-i)^3(1+i)$ in the form of a + ib.

2. Find the quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

3. If 1, -2 , 3are the roots of the equation $x^3-2x^2+ax-6=0$, then find a.

4. Evaluate $\int \sqrt{1+\sin 2x}\,dx$

5. Evaluate $\int \frac{1}{6-2x^2}dx$

6. Evaluate $\int_0^{\frac{\pi}{2}} \cos^8 x\,dx$

7. Find the centre and the radius for the circle $2x^2+2y^2-3x+2y-1=0$.

8. Find the centre, eccentricity and length of the latus rectum of the hyperbola $16x^2-9y^2=144$.

9. Find the mean deviation from the median of the discrete data 6,7,10,12,13,4,12,16.

10. Find the probability that a non leap year contains 52 Sundays only.

Section – B 3x15=45

Note: i) Answer any 3 questions

ii) Each question carries 15 marks.

11(i)(a) Resolve $\frac{x+4}{(x^2-4)(x+1)}$ into partial fractions.

(i)(b) Show that $Z_1=\frac{2+11i}{25}$, $Z_2=\frac{-2+i}{(1-2i)^2}$ are conjugate to each other.

OR

11(ii)(a) Find the relation between the roots and coefficients of the equation $x^4-2x^3+4x^2+6x-21=0$

(ii)(b) Find the sum of all 4-digit numbers that can be formed using the digit 1,2,4,5,6.

12(i)(a) Evaluate $\int \frac{(1+\log x)^5}{x}dx$

(i)(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{5+4\cos x}dx$

OR

12(ii)(a) Evaluate $\int_0^1 \frac{3x^2}{x^6+1}dx$

(ii)(b) Solve $\frac{dy}{dx}=\sqrt{\frac{1+y^2}{1+x^2}}$

13(i)(a) Find the equation of the circle passing through the points (2,1),(5,5),(-6,7)

(i)(b) Find the angle between the pair of circles $x^2+y^2+4x-14y+28=0$ and $x^2+y^2+4x-5=0$

OR

13(ii)(a) Obtain the equation of Parabola in standard form $y^2=4ax$.

(ii)(b) Find the equation of ellipse whose focus is (2,1), eccentricity $\frac{3}{4}$ and directrix 2x – y +3 = 0.

14(i)(a) Find the mean deviation about the mean for the following data

x_i	2	5	7	8	10	35
f_i	6	8	10	6	8	2

(i)(b) Let A and B be independent events with P(A) = 0.2; P(B) = 0.5 Find

(i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A\cup B)$

OR

14(ii)(a) A problem in calculus is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of the problem being solved if both of then try independently.

(ii)(b) Calculate the variance and standard deviation for a discrete frequency distribution

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

15(i)(a) Resolve $\frac{x}{(x+1)(2x+1)}$ into partial fractions.

(i)(b) Solve $\frac{dy}{dx}=(3x+y+4)^2$

OR

(ii)(a) Find the equations of the common tangent of the pair of circles $x^2+y^2-8x-2y+8=0$ and $x^2+y^2-2x+6y+6=0$

(ii)(b) Find the mean deviation about median for the following continuous distribution

marks obtained	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of boys	6	8	14	16	4	2

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VOCATIONAL BRIDGE COURSE
MATHEMATICS – Second Year(2004)
May 2022

Time: 3 Hours Max. Marks: 75

Section – A 10x3=30

Note: i) Answer all the questions
ii) Each question carries 2 marks.

1. Find the multiplicative inverse of -5 + 12i.
2. Find the roots of the equation $x^2 - 7x + 12 = 0$.
3. If 1, -2 , 3are the roots of the equation $x^3 - 2x^2 + ax - 6 = 0$, then find a.
4. Evaluate $\int \sqrt{1+\cos 2x}\,dx$.
5. Evaluate $\int \frac{\cos(\log x)}{x}\,dx$.
6. Evaluate $\int \frac{3x^2}{x^6+1}\,dx$.
7. Find the equation of the circle having centre (-1, 2) and radius r=4.
8. Find the centre, eccentricity and length of the latus rectum of the hyperbola $9x^2-25y^2=225$.
9. Find the mean deviation from the median of the following data 6,7,10,12,13,4,12,16.
10. Find the probability of drawing an Ace from a well shuffled pack of 52 playing cards.

Section – B 3x15=45

Note: i) Answer any 3 questions
ii) Each question carries 15 marks.

11(i)(a) Resolve $\frac{5x+1}{(x-1)(x+2)}$ into partial fractions.

(i)(b) If $(a+ib)^2 = x + iy$ find $x^2 + y^2$

OR

(ii)(a) Find the relation between the roots and coefficients of the equation $3x^3 - 10x^2 + 7x + 10 = 0$.

(ii)(b) Find the sum of all 4-digit numbers that can be formed using the digit 1, 3, 5, 7, 9.

12(i)(a) Evaluate $\int \frac{1}{\cosh x+\sinh x}\,dx$.

(i)(b) Evaluate $\int_2^3 \frac{2x}{1+x^2}\,dx$.

OR

12(ii)(a) Evaluate $\int_0^2 |1-x|\,dx$.

(ii)(b) Find the order and degree of the following differential equation $[\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3]^{\frac{6}{5}} = 6y$.

13(i)(a) Find the equation of the circle passing through the points (5,7),(8,1),(1,3).

(i)(b) Find the angle between the pair of circles $x^2 + y^2 - 6x - 8y + 12 = 0$ and $x^2 + y^2 - 4x + 6y - 24 = 0$.

OR

13(ii)(a) Obtain the equation of Parabola in standard form $y^2 = 4ax$.

(ii)(b) Find the equation of the ellipse whose focus is (1,-1), eccentricity $\frac{2}{3}$ and directrix is x + y + z = 0.

14(i)(a) Find the mean deviation about the mean for the following distribution

x_i	10	30	50	70	90
f_i	4	24	28	16	8

(i)(b) Suppose A and B are independent events with P(A)=0.6, P(B)=0.7. Then compute (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A/B)$ (iv) $P(A^C \cap B^C)$.

OR

14(ii)(a) A, B, C are 3 news papers from a city. 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% both A and C, 4% both B and C and 2% all the three. Find the percentage of the population who read at least one news paper.

(ii)(b) Find the variance and standard deviation of the following frequency distribution

x_i	4	8	11	1	20	24	32
f_i	3	5	9	5	4	3	1

15(i)(a)Resolve $\frac{13x+43}{(2x+5)(x+6)}$ into partial fractions.

(i)(b) Solve $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$.

OR

15(ii)(a) Find the equations of common tangents of the following pair of circles $x^2+y^2+10x-2y+22=0$ and $x^2+y^2+2x-8y+8=0$.

(ii)(b) Find the mean deviation about the mean for the following continuous distribution

marks obtained	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	8	8	15	16	6

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MATHEMATICS (BRIDGE COURSE) -II
Model Paper

Answers

Section – A

1. Express $\frac{2+5i}{3-2i}+\frac{2-5i}{3+2i}$ in the form of a+ib

Sol: Given complex number $\frac{2+5i}{3-2i}+\frac{2-5i}{3+2i}$

$\frac{2+5i}{3-2i}+\frac{2-5i}{3+2i}=\frac{2+5i}{3-2i}\times\frac{3+2i}{3+2i}+\frac{2-5i}{3+2i}\times\frac{3-2i}{3-2i}$

$=\frac{6+4i+15i-10}{(9+4)}+\frac{6-4i-15i-10}{(9+4)}=\frac{-4+19i}{13}+\frac{-4-19i}{13}$

$=\frac{-8}{13}=\frac{-8}{13}+i(0)$

2. Obtain the quadratic equations whose roots are $\frac{m}{n}$, $\frac{-n}{m}$ (m≠0, n≠0)

(i) Sol: Roots $\alpha=\frac{m}{n}$ $\beta=-\frac{n}{m}$

Required equation is $x^2+(\alpha+\beta)+\alpha\beta=0$

$x^2+(\frac{m}{n}-\frac{n}{m})+\left(\frac{m}{n}\right)(-\frac{n}{m})=0$

$x^2+\frac{(m^2-n^2)}{mn}x-1=0$

$mnx^2+(m^2-n^2)x-mn=0$

3. Find the coefficient of x^{11} in the expansion $(2x^2+\frac{3}{x^3})^{13}$

Sol: Given expression is $(2x^2+\frac{3}{x^3})^{13}$

Let x $=2x^2$; y $=\frac{3}{x^3}$; n =13

$T_{r+1}={}^nC_r\, x^{n-r}y^r={}^{13}C_r\,(2x^2)^{13-r}(\frac{3}{x^3})^r$

$={}^{13}C_r\,(2)^{13-r}(3)^r(x^2)^{13-r}x^{-3r}$

$={}^{13}C_r\,(\mathbf{2})^{13-r}(3)^r(x)^{26-2r-3r}$

$={}^{13}C_r\,(\mathbf{2})^{13-r}(3)^r(x)^{26-5r}$

If we want x^{11} coefficient 26-5r =11 ⇒r = 3

x^{11} Coefficient = ${}^{13}C_3\,(\mathbf{2})^{13-3}(3)^3$

$={}^{13}C_3\,(\mathbf{2})^{10}(3)^3=286\times2^{10}\times3^3$

4. Evaluate $\int\frac{1+\cos 2x}{1-\cos 2x}dx$

Sol: Let I$=\int\frac{1+\cos 2x}{1-\cos 2x}dx=\int\frac{1+2\cos^2 x-1}{1-(1-2\sin^2 x)}\,dx$

$=\int\frac{2\cos^2 x}{2\sin^2 x}dx=\int\frac{\cos^2 x}{\sin^2 x}dx=\int\cot^2 x\,dx$

$=\int(\operatorname{cosec}^2 x-1)\,dx=\int\operatorname{cosec}^2 x\,dx-\int 1\,dx$

$=-\cot x-x+C$

5. Evaluate $\int x^2e^x\,dx$

Sol: I$=\int x^2e^x\,dx$

$=x^2\int e^x\,dx-\int[\frac{d}{dx}(x^2)\int e^x\,dx]dx$

$=x^2e^x-\int 2x\,e^x\,dx$

$=x^2e^x-2[x\int e^x\,dx-\frac{d}{dx}\int e^x\,dx]dx$

$=x^2e^x-2xe^x-2e^x+C$

$=e^x(x^2-2x-2)+C$

6. Evaluate $\int_0^1\frac{1}{1+x^2}dx$

Sol: I$=\int_0^1\frac{1}{1+x^2}dx=[\tan^{-1}x]_0^1$

I$=\tan^{-1}1-\tan^{-1}0=\frac{\pi}{4}-0=\frac{\pi}{4}$

7. Find the equations of the circles whose extremities of a diameter are (7,-3)and (3,5)

Sol: Let P = (x_1,y_1) = (7,-3)

Q = (x_2,y_2) = (3,5)

Equation of the required circle is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$(x-7)(x-3)+(y+3)(y-5)=0$

$x^2-3x-7x+21+y^2-5y+3y-15=0$

$\therefore x^2+y^2-10x-2y+6=0$

8. Find the eccentricity and latus rectum of the hyperbola x^2-$4y^2$=4

Sol: Equation of the hyperbola is $x^2-4y^2=4$

$\frac{x^2}{4}-\frac{y^2}{1}=1\Rightarrow a^2=4;\ b^2=1$

Eccentricity e$=\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{4+1}{4}}=\frac{\sqrt5}{2}$

Length of the latus rectum $=2\frac{b^2}{a}=2\frac{1}{2}=1$

9. Find the mean deviation about the mean for the data: 3,6,10,4,10,9.

Sol: Mean $\bar{x}=\frac{3+6+10+4+10+9}{6}$

Mean $\bar{x}=\frac{42}{6}=7$

The absolute values of mean deviation are

$|x_i-\bar{x}|$ = 4,1,3,3,3,2

∴Mean deviation about the mean$=\frac{\sum_{i=1}^{10}(x_i-\bar{x})}{n}$

$=\frac{4+1+3+3+3+2}{6}=\frac{16}{6}=2.67$

10. Two dice are rolled. Find the probability that none of the dice shows the number '2'.

Sol: Random experiment is rolling 2 dice

$n(S)=6^2=36$

Let E be the event of not getting 2

n(E)=5X5=25

$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{25}{36}$

Section – B

11.(i)(a) If the coefficient of 10^{10} in the expansion of $(ax^2+\frac{1}{bx})^{11}$ is equal to the coefficient of 10^{-10} in the expansion of $(ax-\frac{1}{bx^2})^{11}$. Find the relation between *a* and *b* where *a* and *b* are real numbers.

Sol: We have T_{r+1} is the general term in the expansion of $(\mathbf{ax^2}+\frac{\mathbf{1}}{\mathbf{bx}})^{\mathbf{11}}$

i.e. $T_{r+1}=11_{cr}(ax^2)^{11-r}(\frac{1}{bx})^r$

$=11_{cr}\,a^{11-r}x^{22-2r}\frac{1}{b^rx^r}$

$=11_{cr}\,\frac{a^{11-r}}{b^r}x^{22-3r}$

To find the coefficient of x^{10} in the expansion, we should consider 22 - 3r = 10 ⟹r = 4.

Hence the coefficient of 10^{10} in the expansion of $(ax^2+\frac{1}{bx})^{11}$ is $11_{c4}\,\frac{a^7}{b^4}$ ……(1)

We have T_{r+1} is the general term in the expansion of $(ax-\frac{1}{bx^2})^{11}$

i.e. $T_{r+1}=11_{cr}(ax)^{11-r}(\frac{-1}{bx^2})^r$

$=(-1)^r11_{cr}\,a^{11-r}x^{11-r}\frac{1}{b^rx^{2r}}$

$=(-1)^r\ 11_{cr}\ \frac{a^{11-r}}{b^r}x^{11-3r}$

To find the coefficient of x^{-10} in the expansion, we should consider 11 - 3r = -10 ⟹r = 7.

Hence the coefficient of x^{-10} in the expansion of $(ax-\frac{1}{bx^2})^{11}$ is $-11_{c7}\ \frac{a^4}{b^7}$ ……(2)

Hence from equations (1) and (2), we get

$11_{c4}\ \frac{a^7}{b^4} = -11_{c7}\ \frac{a^4}{b^7}$

i.e. $\frac{a^7}{b^4} = \frac{a^4}{b^7}$ [$\because 11_{c4} = -11_{c7}$]

i.e. $a^3 = -\frac{1}{b^4}$

11.(i)(b) Resolve $\frac{5x+1}{(x-1)(x+2)}$ into partial fractions.

Sol: Let $\frac{5x+1}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$ where A and B are real numbers

$\frac{5x+1}{(x-1)(x+2)} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)}$

$\Rightarrow A(x+2)+B(x-1)=5x+1$

If x=1 then A(1+2) +B(1-1) =5(1)+1

3A = 6⇒A =2

If x=-2 then A(-2+2) +B(-2-1) = 5(-2)+1

-3B = -9⇒B=3

$\therefore \frac{5x+1}{(x-1)(x+2)} = \frac{2}{x-1}+\frac{3}{x+2}$

(OR)

11.(ii)(a) If α,β are the roots of the equation ax^2+b x + c=0, then find the following (i)$\alpha^3+\beta^3$ (ii) $\alpha^2+\beta^2$ (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$.

Sol: (i)$\alpha^2+\beta^2$

Given equation is ax^2+b x + c

Given roots are α, β

Sum of the roots α+β= - -$\frac{b}{a}$

Product of the roots αβ=$\frac{c}{a}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-\frac{b}{a})^2-2(\frac{c}{a})=\frac{b^2-2ac}{a^2}$

Sol :(ii) $\alpha^3+\beta^3$

Given equation is ax^2+b x + c

Given roots are α, β

Sum of the roots α+β = - -$\frac{b}{a}$

Product of the roots αβ = $\frac{c}{a}$

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$=(-\frac{b}{a})^3-3(\frac{c}{a})(--\frac{b}{a}) = -\frac{b^3}{a^3}+3\frac{bc}{a^2}=\frac{-b^3-3abc}{a^3}$

Sol: (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Given equation is ax^2+b x + c

Given roots are α, β

Sum of the roots α+β = - $\frac{b}{a}$

Product of the roots αβ = $\frac{c}{a}$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$=\frac{(-\frac{b}{a})^2-2(\frac{c}{a})}{(\frac{c}{a})^2}=\frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\frac{c^2}{a^2}}=\frac{\frac{b^2-2ac}{a^2}}{\frac{c^2}{a^2}}=\frac{b^2-2ac}{c^2}$

11.(ii)(b) If (n+1)P_5: nP_5=3:2 then find the value of n.

Sol: We know that $(n+1)_{P_5}$=(n+1)n(n-1)(n-2)(n-3) and n_{P_5}=n(n-1)(n-2)(n-3)(n-4)

Given $(n+1)_{P_5}:n_{P_5}$= 3:2

$\Rightarrow \frac{(n+1)_{P_5}}{n_{P_5}}=\frac{3}{2}$

$\Rightarrow \frac{(n+1)n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)(n-4)}=\frac{3}{2}$

$\Rightarrow \frac{(n+1)}{(n-4)}=\frac{3}{2}$

⇒2n+2=3n-12

⇒n=14

12.(i)(a)Evaluate $\int\frac{\sec^2x}{(1+\tan x)^2}dx$

Sol: I=$\int\frac{sec^2x}{(1+\tan x)^2}$dx

Let t=1+tan x

dt=sec^2x dx

$\therefore \int\frac{sec^2x}{(1+\tan x)^2}dx=\int\frac{dt}{t^3}=\int t^{-3}\ dt=\frac{t^{-3+1}}{-3+1}+C=\frac{t^{-2}}{-2}+C$

$=\frac{1}{-2t^2}+C=\frac{1}{-2(1+\tan x)^2}+C$

12.(i)(b) Evaluate $\int_0^1\frac{1}{\sqrt{3-2x}}dx$

Sol: I=$\int_0^1\frac{1}{\sqrt{3-2x}}dx=\int_0^1(3-2x)^{-\frac{1}{2}}\ dx$

I=$[\frac{(3-2x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}]_0^1=[\frac{(3-2x)^{\frac{1}{2}}}{\frac{1}{2}}]_0^1$

I=$[2\sqrt{3-2x}]_0^1$=2-2$\sqrt{3}$=2(1-$\sqrt{3}$)

(OR)

12.(ii)(a).Evaluate $\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}$

Sol: I=$\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}$

Let tan$\frac{x}{2}$=t cos $x=\frac{1-t^2}{1+t^2}$

⇒dx=$\frac{2\,dt}{1+t^2}$ ⇒x=0⇒t=0; x=$\frac{\pi}{2}$ ⇒t=1

$\therefore$ I=$\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}=\int_0^1\frac{\frac{2\,dt}{1+t^2}}{4+5\frac{1-t^2}{1+t^2}}=\int_0^1\frac{\frac{2\,dt}{1+t^2}}{4+5\frac{1-t^2}{1+t^2}}$

I=$2\int_0^1\frac{1}{9-t^2}dt=2\frac{1}{2.3}[\log\frac{3+t}{3-t}]_0^1=\frac{1}{3}\log\frac{4}{2}=\frac{1}{3}\log 2$

12.(ii)(b).Solve $\frac{dy}{dx}=\frac{xy+y}{xy+x}$

Sol: Given equation is $\frac{dy}{dx}=\frac{xy+y}{xy+x}=\frac{y(x+1)}{x(y+1)}=\frac{y(1+x)}{x(1+y)}$

$\Longrightarrow \frac{(1+x)}{x}dx=\frac{(1+y)}{y}dy$

$(1+\frac{1}{x})dx-(1+\frac{1}{y})dy=0$

Now taking integration on both sides we get

$\int(1+\frac{1}{x})dx-\int(1+\frac{1}{y})dy=\int 0$

⇒ x+ log x –(y+ log y) = log c

⇒ x- y + log$\frac{x}{y}$ = log c

13.(i)(a) Show that the line x+y+1=0 touches towards the circle $x^2+y^2-3x+7y+14=0$ and find its point of contact.

Sol: $x^2+y^2-3x+7y+14=0$

$x^2+y^2+2gx+2fy+C=0$

$g=-\frac{3}{2}$; $f=-\frac{7}{2}$; c=14

Centre(-g,-f) = $(\frac{3}{2},\frac{-7}{2})$

Radius of the circle $r=\sqrt{g^2+f^2-c}$

$=\sqrt{(-\frac{3}{2})^2+(\frac{7}{2})^2-14}$

$=\sqrt{\frac{9}{4}+\frac{49}{4}-14}$

$\sqrt{\frac{9+49-56}{4}}=\sqrt{\frac{2}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$

D= perpendicular distance from centre $C(\frac{3}{2},\frac{-7}{2})$ to the line x+y+1=0

$=\left|\frac{\frac{3}{2},\frac{-7}{2}+1}{\sqrt{1^2+1^2}}\right|=\left|\frac{\frac{3-7+2}{2}}{\sqrt{2}}\right|=\left|\frac{\frac{-2}{2}}{\sqrt{2}}\right|=\frac{1}{\sqrt{2}}$

Clearly r= d i.e., the given circle touches the given line.

Point of contact: The point of contact is the foot of the perpendicular P(h, k) from $C(\frac{3}{2},\frac{-7}{2})$ on the line x+y+1= 0

$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$

$\frac{h-\frac{3}{2}}{1}=\frac{k+\frac{7}{2}}{1}=\frac{-(\frac{3}{2}-\frac{7}{2}+1)}{1^2+1^2}$

$h-\frac{3}{2}=k+\frac{7}{2}=\frac{-(\frac{3-7+2}{2})}{2}=\frac{1}{2}$

$h-\frac{3}{2}=\frac{1}{2}\Rightarrow h=2 \quad k+\frac{7}{2}=\frac{1}{2}\Rightarrow k=-3$

∴Point of contact P(h,k) + (2,-3)

13.(i)(b) Find the equation of the parabola whose focus is (3,5) and vertex is (1,3).

Sol: vertex A(1,3) and focus S(3,5)

Let Z(x,y) be the projection of S on directrix.

The A is the midpoint of S

$\Rightarrow(1,3)=(\frac{3+x}{2},\frac{5+y}{2})\Rightarrow x=-1$; y=1 Z=(-1,1)

Slope of directrix$=\frac{-1}{\text{Slope of SA}}=\frac{-1}{\frac{5-3}{3-1}}=-1$

Equation of directrix is y-1=-1(x+1)

x+y=0 ……(1)

Let P(x,y) be any point on the parabola,

Then SP=PM ⇒ $SP^2=PM^2$

Where PM is the perpendicular from P to the directrix,

$\Rightarrow(x-3)^2+(y-5)^2=\frac{(x+y)2}{1+1}$

$2[(x-3)^2+(y-5)^2]=(x+y)^2$

$2[x^2-6x+9+y^2-10y+25]=x^2+2xy+y^2$

$2x^2+2y^2-12x-20y+68]=x^2+2xy+y^2$

$x^2-2xy+y^2-12x-20y+68=0.$

(OR)

13.(ii)(a) Find the equation of the ellipse whose focus is (1,-1), eccentricity $\frac{2}{3}$ and directrix is x+y+2=0.

Sol: Let $P(x_1,y_1)$ be any point on the ellipse.

Equation of the directrix L=x+y+2=0

By definition of ellipse SP=e.PM

$SP^2=e^2.PM^2$

$(x_1-1)^2+(y_1+1)^2=(\frac{2}{3})^2[\frac{x1+y1+2}{\sqrt{1+1}}]^2=\frac{4}{9}[\frac{x1+y1+2}{\sqrt{2}}]^2$

$9[(x_1-1)^2+(y_1+1)^2]=2(x_1+y_1+2)^2$

$9[x_1^2-2x_1+1+y_1^2+2y_1+1]=2[x_1^2+y_1^2+4+2x_1y_1+4x_1+4y_1$

$9x_1^2+9y_1^2-18x_1+18y_1+18]=$

$2x_1^2+2y_1^2+4x_1y_1+8x_1+8y_1+8$

$7x_1^2+7y_1^2-4x_1y_1-26x_1+10y_1+10=0$

$7x_1^2-4x_1y_1+7y_1^2-26x_1+10y_1+10=0$

Locus of $P(x_1,y_1)$ is $7x^2-4xy+7y^2-26x+10y+10=0$

13.(ii)(b) Find the equation of the hyperbola whose asymptotes are the straight lines x+2y+3=0, 3x+4y+5=0 and which passes through the point (1,-1).

Sol: Equation of the hyperbola having the given lines as asymptotes is (x+2y+3)(3x+4y+5)=k….(1)

This passes through (1,-1)

So, eq(1) ⇒(1+2(-1)+3)(3+4(-1)+5)=k

⇒k=2(4)=8

∴The equation of required hyperbola is

(x+2y+3)(3x+4y+5)=8

$3x^2+4xy+5x+6xy+8y^2+10y+9x+12y+15=8$

$3x^2+10xy+8y^2+14x+22y+7=0$

14. (i) (a) Find the probability that a non leap year contains i) 53 Sundays ii) 52 Sundays only.

Sol: A non leap year contains 365 days=52 weeks and 1day more.

i)We get 53 days when the remaining day is Sunday

Number of days in week=7

∴n(S)=7

Number of ways getting 53 Sundays

n(E)=1 ∴$P(E)=\frac{n(E)}{n(S)}=\frac{1}{7}$

∴ probability of getting 53 Sundays$=\frac{1}{7}$

ii) probability of getting 52 Sundays

$P(E)=1-p(E)=1-\frac{1}{7}=\frac{6}{7}$

14. (i) (b) Find the mean deviation about median of the following frequency distribution.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Sol: Writing the observations in the ascending order

x_i	f_i	Cumulative Frequency CF	$\lvert x_i-\bar{x}\rvert$	$f_i \lvert x_i-\bar{x}\rvert$
5	8	8	2	16
7→ M	6	$14>\frac{N}{2}$	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
	N =26			$\sum f_i \lvert x_i-M\rvert$ =84

Median = $\frac{26}{2}$ th observation=13th observation=7

Mean deviation about Median = $\frac{\sum_{i=1}^{6} f_i |x_i - M|}{N}$

$=\frac{84}{26}=4$

(or)

14.(ii)(a) If A and B are two events with $P(A\cup B)=0.65, P(A\cap B)=0.15$. Then find the value of $P(A^C)+P(B^C)$

Sol: By addition theorem on probability

$P(A\cup B)=P(A)+P(B) - P(A\cap B)$

$P(A)+P(B) = P(A\cup B)+P(A\cap B)=0.65+0.15=0.8$

$P(A^C)+P(B^C) = 1-P(A)+1-P(B)$

$=2-[P(A)+P(B)]=2-[0.8]=1.2$

14.(ii)(b). Find the variance and standard deviation of the data 5,12,3,18,6,8,2,10

Sol: Mean $(\bar{x})=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})$

$=\frac{5+12+3+18+6+8+2+10}{8}=\frac{64}{8}=8$

To find the variance, we construct the following table

x_i	5	12	3	18	6	8	2	10
$x_i-\bar{x}$	-3	4	-5	10	-2	0	-6	2
$(x_i-\bar{x})^2$	9	16	25	100	4	0	36	4

Here $\sum(x_i-\bar{x})^2=194$

∴Variance$(\sigma^2)=\frac{1}{n}\sum_{i=1}^{8}(x_i-\bar{x})^2=\frac{1}{8}X194=24.25$

Standard deviation $\sigma=\sqrt{24.25}=4.95$

15.(i)(a)Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions.

Sol: $\frac{2x^2+3x+4}{(x-1)(x^2+2)}=\frac{A}{(x-1)}+\frac{Bx+C}{(x^2+2)}$

$\frac{2x^2+3x+4}{(x-1)(x^2+2)}=\frac{A(x^2+2)+(Bx+C)(x-1)}{(x-1)(x^2+2)}$

$2x^2+3x+4=A(x^2+2)+(Bx+C)(x-1)$

$2x^2+3x+4=A(x^2+2)+B(x^2-x)+C(x-1)$

$2x^2+3x+4=(A+B)x^2+(-B+C)x+2A-C$

Equating coefficients

A+B=2….(1)

-B+C=3…..(2)

2A-C=4

Add eq(1) and eq(2)

A+B =2

-B+C=3

A +C=5 …..(4)

Add eq(3) and eq(4)

2A-C=4

A +C=5

3A = 9⇒A =3

A+B =2

3+B =2⇒B = -1

A+C = 5

3+C = 5⇒C =2

∴ $\frac{2x^2+3x+4}{(x-1)(x^2+2)}=\frac{3}{(x+1)}+\frac{2-x}{(x^2+2)}$

15.(i)(b) Solve$(x^2-y^2)\frac{dy}{dx}$=xy

Sol: Given $(x^2-y^2)\frac{dy}{dx}$=xy

$\frac{dy}{dx}=\frac{xy}{x^2-y^2}$

Let y=Vx

$\frac{dy}{dx}=V+x\frac{dV}{dx}$

∴ $\frac{dy}{dx}=\frac{xy}{x^2-y^2}$

$V+x\frac{dV}{dx}=\frac{x(Vx)}{x^2-V^2x^2}=\frac{2V}{1-V^2}$

$x\frac{dV}{dx}=\frac{V}{1-V^2}-V=\frac{V-V-V^3}{1-V^2}=\frac{V^3}{1-V^2}$

$\frac{1-V^2}{V^3}dv=\frac{dx}{x}$

$[\frac{1}{V^3}-\frac{1}{V}]dv=\frac{dx}{x}$

Integrating on both sides

$\int[\frac{1}{V^3}-\frac{1}{V}]dv=\int\frac{dx}{x}$

$-\frac{1}{2V^2}-\log V=\log x+c$

$-\frac{x^2}{2y^2}=\log V+\log x+c=\log Vx+c=\log y+c$

$-\frac{x^2}{2y^2}=\log y+c$

$-x^2=2y^2(\log y+c)$

$x^2+2y^2(\log y+c)=0$

(or)

15.(ii)(a) Find the internal and external centers of similitude of the circles $x^2+y^2=9$ and $x^2+y^2-16x+2y+49=0$.

Sol: Let the given equations of circles be

$S = x^2+y^2=9$ …..(1)

$S' = x^2+y^2-16x+2y+49=0$ ….(2)

Let C_1, C_2 be the centers and r_1, r_2 be radii of circles (1) and (2) respectively.

$S = x^2+y^2=9$

$=(x+0)^2+(y+0)^2=9=3^2$

We have $C_1(h_1,k_1)=C_1(0,0)$ and $r_1=3$

$S'=x^2+y^2-16x+2y+49=0$

$=x^2-16x+64+y^2+2y+1-64-1+49=0$

$=(x-8)^2+(y+1)^2=16=4^2$

We have $C_2(h_2,k_2)=C_2(8,-1)$ and $r_2=4$

15.(i)(b) Find the variance and standard deviation of the following frequency distribution

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Sol:

x_i	f_i	$f_i x_i$	$x_i-\bar{x}$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	216	-1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	N= 40	760			1736

Mean of the given data$\bar{x}=\frac{1}{N}\sum_{i=1}^{7}f_i x_i=\frac{760}{40}=19$

Variance of the given data (σ^2)

$=\frac{1}{N}\sum_{i=1}^{7}(x_i-\bar{x})^2=\frac{1}{40}X1736=43.4$

Standard deviation $\sigma=\sqrt{43.4}=6.59$

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MATHEMATICS (BRIDGE COURSE) -II
March 2020

Answers

Section – A

1. Express $(1-i)^3$(1+i) in the form of a+bi

Sol: Let $(1-i)^3(1+i) = (1-i)^3(1-i)(1+i)$

$= (1+i^2-2i)(1^2-i^2)$

$= (1-1-2i)(1+1) = (-2i)(2) = -4i$

$(1-i)^3(1+i) = 0+i(-4)$

2. Obtain the quadratic equations whose roots are $\frac{m}{n}, \frac{n}{m}$

(i) Sol: Roots $\alpha=\frac{m}{n}$ $\beta=\frac{n}{m}$

Required equation is $x^2 +(\alpha+\beta) +\alpha\beta= 0$

$$x^2 +\left(\frac{m}{n}+\frac{n}{m}\right)+\left(\frac{m}{n}\right)\left(\frac{n}{m}\right) = 0$$

$$x^2 + \frac{(m^2+n^2)}{mn}\, x +1 = 0$$

$$mnx^2 + (m^2+ n^2)\, x +mn =0$$

3. If 1,-2 and α are the roots of equation $2x^3+x^2$-7x-6=0, then find α.

Sol: 1,-2 and α are the roots of

$2x^3+x^2-7x-6=0$,

$ax^3+bx^2+c\,x+d=0$

$\Rightarrow a=2; b=1; c=-7; d=-6$

Sum=1-2 + $\alpha=-\frac{b}{a} = -\frac{1}{2}$

$\alpha= -\frac{1}{2}+1=\frac{1}{2}$

4. Evaluate $\int \frac{1+\cos 2x}{1-\cos 2x} dx$

Sol: Let I=$\int \frac{1+\cos 2x}{1-\cos 2x} dx=\int \frac{1+2\cos^2 x-1}{1-(1-2\sin^2 x)}\ dx$

$=\int \frac{2\cos^2 x}{2\sin^2 x} dx=\int \frac{\cos^2 x}{\sin^2 x} dx=\int \cot^2 x\, dx$

$=\int (\mathrm{cosec}^2 x-1)\, dx=\int \mathrm{cosec}^2 x\, dx-\int 1\, dx$

$= -\cot x-x+C$

5. Evaluate $\int x^2 e^x dx$

Sol: I=$\int x^2 e^x\, dx$

$=x^2\int e^x dx-\int[\frac{d}{dx}(x^2)\int e^x dx]dx$

$=x^2e^x-\int 2x\, e^x\, dx$

$=x^2e^x-2[x\int e^x dx-\frac{d}{dx}\int e^x dx]dx$

$=x^2e^x-2xe^x-2e^x+C$

$=e^x(x^2-2x-2)$+C

6. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$

Sol: I=$\int_0^1 \frac{1}{1+x^2} dx=[\tan^{-1}x]_0^1$

I=$\tan^{-1}1-\tan^{-1}0=\frac{\pi}{4}-0=\frac{\pi}{4}$

7. Find the equation of the circle having Centre C=(1,4) and radius r = 5

Sol: Here (h,k)=(1,4) and r=5

The equation of the circle with centre at C(h,k) and radius r is

$(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y-4)^2=5^2$

$x^2-2x+1+ y^2-8y+16=25$

$\therefore$ Equation of circle is $x^2+ y^2 -2x -8y-8=0$

8. Find the eccentricity and latus rectum of the hyperbola $x^2-4y^2=4$.

Sol: Equation of the hyperbola is $x^2-4y^2=4$

$\frac{x^2}{4}-\frac{y^2}{1}=1\Rightarrow a^2=4; b^2=1$

Eccentricity e=$\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{4+1}{4}}=\frac{\sqrt{5}}{2}$

Length of the latus rectum = $2\frac{b^2}{a}=2\frac{1}{2}=1$

9. Find the mean deviation about the mean for the data 3,6,10,4,10,9.

Sol: Mean $\bar{x}=\frac{3+6+10+4+10+9}{6}$

Mean $\bar{x}=\frac{42}{6}=7$

The absolute values of mean deviation are

$|xi-\bar{x}|$ = 4,1,3,3,3,2

$\therefore$Mean deviation about the mean=$\frac{\sum_{i=1}^{10}(x_i-\bar{x})}{n}$

$=\frac{4+1+3+3+3+2}{6}=\frac{16}{6}=2.67$

10. If A and B are two independent events such that P(A)=0.6, P(B)=0.7. Then find

(i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(A/B)$

Sol: Given that A and B are two independent events such that P(A)=0.6, P(B)=0.7

(i) $P(A\cap B)$ =P(A).P(B)= 0.6X0.7=0.42

(ii) $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

=0.6 + 0.7 - 0.42=0.88

(iii) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{0.42}{0.7}=0.6$

Section – B

11(i)(a). Resolve $\frac{3x^2+2x}{(x-3)(x^2+2)}$ into partial fractions.

Sol: $\frac{3x^2+2x}{(x-3)(x^2+2)}=\frac{A}{(x-3)}+\frac{Bx+C}{(x^2+2)}$

$\frac{3x^2+2x}{(x^2+2)(x-3)}=\frac{A(x^2+2)+(Bx+C)(x-3)}{(x-3)(x^2+2)}$

$3x^2+2x= A(x^2+2)+(Bx+C)(x-3)$

$3x^2+2x= Ax^2+2A+Bx^2-3Bx+Cx-3C$

$3x^2+2x= (A+B)x^2+(C-3B)x+2A-3C$

Equating coefficients

A+B=3 $\Rightarrow$A=3-B ... (1)

C-2B=2 ... (2)

2A-3C=0 ... (3)

Put A=3-B in eq(3) 2A-3C=0

2(3-B)-3C=0 $\Rightarrow$6-2B-3C=0$\Rightarrow$2B+3C=6 ...(4)

2B+3C=6

-2B+ C=2

$\Rightarrow$4C=8 $\Rightarrow$C=2

2-2B=2$\Rightarrow$B=0

A=3-B=3-0=3

$\therefore \frac{3x^2+2x}{(x-3)(x^2+2)}=\frac{3}{(x-3)}+\frac{(0)x+2}{(x^2+2)}=\frac{3}{(x-3)}+\frac{2}{(x^2+2)}$

11.(i)(b). Show that the points in Argand plane represented by the complex numbers 2+i, 4+3i, 2+5i, 3i forms a square.

Sol: Let the four points represented by the given complex numbers be A,B,C,D Then A=(2,1), B=(4,3), C=(2,5), D=(0,3)

AB=$\sqrt{(4-2)^2+(3-1)^2}$ =$\sqrt{4+4}$ =2$\sqrt{2}$
BC=$\sqrt{(4-2)^2+(3-5)^2}$ =$\sqrt{4+4}$ =2$\sqrt{2}$
AC=$\sqrt{(2-2)^2+(5-1)^2}$ =$\sqrt{0+16}$ = 4
BD=$\sqrt{(4-0)^2+(3-3)^2}$ =$\sqrt{16+0}$ = 4
AB=BC and AC=BD
Given complex numbers are vertices of a square.

(OR)

11.(ii)(a). If α,β are the roots of the ax^2+b x + c=0, then find (i)$\alpha^2+\beta^2$ (ii) $\alpha^3+\beta^3$ (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Sol: (i)$\alpha^2+\beta^2$
Given equation is ax^2+b x + c
Given roots are α, β
Sum of the roots α+β= - -$\frac{b}{a}$
Product of the roots αβ= $\frac{c}{a}$
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$=(-\frac{b}{a})^2-2(\frac{c}{a})=\frac{b^2-2ac}{a^2}$

Sol :(ii) $\alpha^3+\beta^3$
Given equation is ax^2+b x + c
Given roots are α, β
Sum of the roots α+β = - -$\frac{b}{a}$
Product of the roots αβ = $\frac{c}{a}$
$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$
$=(-\frac{b}{a})^3-3(\frac{c}{a})(-\frac{b}{a}) = -\frac{b^3}{a^3}+3\frac{bc}{a^2}=\frac{-b^3-3abc}{a^3}$

Sol: (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$
Given equation is ax^2+b x + c
Given roots are α, β
Sum of the roots α+β = - $\frac{b}{a}$
Product of the roots αβ = $\frac{c}{a}$
$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$
$=\frac{(-\frac{b}{a})^2-2(\frac{c}{a})}{(\frac{c}{a})^2}=\frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\frac{c^2}{a^2}}=\frac{\frac{b^2-2ac}{a^2}}{\frac{c^2}{a^2}}=\frac{b^2-2ac}{c^2}$

11.(ii)(b).If $(n+1)_{P_5}: n_{P_5}$= 3:2, then find the value of 'n'.

Sol: We know that $(n+1)_{P_5}$=(n+1)n(n-1)(n-2)(n-3)
and n_{P_5}=n(n-1)(n-2)(n-3)(n-4)
Given $(n+1)_{P_5}: n_{P_5}$= 3:2
$\Rightarrow \frac{(n+1)_{P_5}}{n_{P_5}}=\frac{3}{2}$
$\Rightarrow \frac{(n+1)n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)(n-4)}=\frac{3}{2}$

$\Rightarrow \frac{(n+1)}{(n-4)}=\frac{3}{2}$
⇒2n+2=3n-12
⇒n=14

12.(i)(a).Evaluate $\int\frac{2x+5}{\sqrt{x^2-2x+10}}dx$

Sol: Given problem $\int\frac{2x+5}{\sqrt{x^2-2x+10}}dx$
px + q write and evaluate
A$\frac{d}{dx}(ax^2$+bx+c) + B
2x+5 = A$\frac{d}{dx}(x^2$-2x+10) + B
2x+5 = A(2x -2) +B .....(1)
Comparing x co-efficient OBS
2A =2 ⟹ A = 1
Comparing constants OBS
-2A + B = 5
-2(1) + B = 5
B = 5 +2 =7
$\int\frac{2x+5}{\sqrt{x^2-2x+10}}dx=\int\frac{2x-2}{\sqrt{x^2-2x+10}}dx+7\int\frac{1}{\sqrt{x^2-2x+10}}dx+c$

$=2\sqrt{x^2-2x+10}+\int\frac{1}{\sqrt{(x-1)^2+3^2}}dx+c$

$=2\sqrt{x^2-2x+10}+\sinh^{-1}(\frac{x-1}{3})+c$

12.(i)(b).Evaluate $\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}dx$

Sol: Let I=$\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}dx$
We have $\int_0^a f(x)dx=\int_0^a f(a-x)dx$
$\therefore \int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}dx=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{5}{2}}(\frac{\pi}{2}-x)}{\sin^{\frac{5}{2}}(\frac{\pi}{2}-x)+\cos^{\frac{5}{2}}(\frac{\pi}{2}-x)}dx$
$=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{5}{2}}x}{\cos^{\frac{5}{2}}x+\sin^{\frac{5}{2}}x}dx$
Now I+I=$\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}dx+\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{5}{2}}x}{\cos^{\frac{5}{2}}x+\sin^{\frac{5}{2}}x}dx$
$\Rightarrow$ 2I=$\int_0^{\frac{\pi}{2}}\{\frac{\cos^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}+\frac{\sin^{\frac{5}{2}}x}{\cos^{\frac{5}{2}}x+\sin^{\frac{5}{2}}x}\}dx$
2I=$\int_0^{\frac{\pi}{2}}\{\frac{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}{\sin^{\frac{5}{2}}x+\cos^{\frac{5}{2}}x}\}dx=\int_0^{\frac{\pi}{2}}\mathbf{1}\,dx=[x]_0^{\frac{\pi}{2}}=\frac{\pi}{2}-0=\frac{\pi}{2}$

∴I= $\frac{\pi}{4}$

12.(ii)(a).Evaluate $\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}$

Sol: I=$\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}$
Let $\tan\frac{x}{2}$=t $\cos x=\frac{1-t^2}{1+t^2}$
⇒dx=$\frac{2\,dt}{1+t^2}$ ⇒x=0⇒t=0; x=$\frac{\pi}{2}$ ⇒t=1
∴ I=$\int_0^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}=\int_0^1\frac{\frac{2\,dt}{1+t^2}}{4+5\frac{1-t^2}{1+t^2}}=\int_0^1\frac{\frac{2\,dt}{1+t^2}}{4+5\frac{1-t^2}{1+t^2}}$
I=$2\int_0^1\frac{1}{9-t^2}dt=2\frac{1}{2.3}[\log\frac{3+t}{3-t}]_0^1=\frac{1}{3}\log\frac{4}{2}=\frac{1}{3}\log 2$

12.(ii)(b).Solve $\frac{dy}{dx}=\frac{xy+y}{xy+x}$

Sol: Given equation is $\frac{dy}{dx}=\frac{xy+y}{xy+x}=\frac{y(x+1)}{x(y+1)}=\frac{y(1+x)}{x(1+y)}$
$\Longrightarrow \frac{(1+x)}{x}dx=\frac{(1+y)}{y}dy$
$(1+\frac{1}{x})dx-(1+\frac{1}{y})dy=0$
Now taking integration on both sides we get
$\int(1+\frac{1}{x})dx-\int(1+\frac{1}{y})dy=\int 0$
⟹ x+ log x –(y+ log y) = log c

$\Rightarrow$ x- y + log$\frac{x}{y}$ = log c

13.(i)(a).Show that the line 5x+12y-4=0 touches the circle $x^2+y^2-6x+4y-12=0$ and find its point of contract.

Sol: Let (x_1,y_1) be the point of contact

$x^2+y^2+2gx+2fy+C=0$

g = -3; f =2 ; C = -12

$(x_1+g)x + (y_1+f)y + (gx_1+fy_1+C) = 0$

$\frac{x_1-3}{4} = \frac{y_1+2}{-3} = \left[\frac{-3x_1+2y_1-12}{7}\right]$ ....(1)

$(x_1-3)x + (y_1+2)y + (-3x_1+2y_1-12) = 0$

From first and second equalities of (1), we get

$3x_1 + 4y_1 = 1$ ....(2)

First and third equalities of (1) ,we get

$19x_1 - 8y_1 = -27$ ....(3)

$6x_1 + 8y_1 = 2$

$19x_1 - 8y_1 = -27$

$25x_1 = -25$

$\Rightarrow x_1 = -1$

$3(-1) + 4y_1 = 1 \Rightarrow y_1 = 1$

Hence the point of contact is (-1,1)

13.(i)(b).Find the equation of the parabola whose focus is (3,5),and the vertex is at the point (1,3).

Sol: vertex A(1,3) and focus S(3,5)

Let Z(x,y) be the projection of S on directrix.

The A is the midpoint of S

$\Rightarrow(1,3) = (\frac{3+x}{2}, \frac{5+y}{2}) \Rightarrow$x=-1; y=1

Z=(-1,1)

Slope of directrix=$\frac{-1}{\text{Slope of SA}} = \frac{-1}{\frac{5-3}{3-1}}$ = -1

Equation of directrix is y-1=-1(x+1)

x+y=0 ......(1)

Let P(x,y) be any point on the parabola,

Then SP=PM $\Rightarrow SP^2=PM^2$

Where PM is the perpendicular from P to the directrix,

$\Rightarrow(x-3)^2+(y-5)^2 = \frac{(x+y)2}{1+1}$

$2[(x-3)^2+(y-5)^2]=(x+y)^2$

$2[x^2-6x+9+y^2-10y+25]= x^2+2xy+y^2$

$2x^2+2y^2-12x-20y+68]= x^2+2xy+y^2$

$x^2-2xy+y^2-12x-20y+68= 0.$

13.(ii)(a).Find the condition that the line lx + my-n=0 is to be a tangent with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ =1.

Sol: Equation of line**,** lx+my-n=0

$\Rightarrow$my =-lx+n

y =$-\frac{l}{m}$x+n

Here m=$-\frac{l}{m}$ and c=$\frac{n}{m}$

Equation of ellipse , $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ =1

By condition of tangency,

C=$\sqrt{a^2m^2+b^2}$

$\Rightarrow \frac{n}{m} = \sqrt{a^2(-\frac{l}{m})^2+b^2}$

$(\frac{n}{m})^2 = a^2\frac{l^2}{m^2}+b^2$

$\frac{n^2}{m^2} = a^2\frac{l^2}{m^2}+b^2$

$\frac{n^2}{m^2} = a^2\frac{l^2}{m^2}+b^2\frac{m^2}{m^2}$

$n^2 = a^2l^2+b^2m^2$

Thus $a^2l^2+b^2m^2 = n^2$

This is the required condition.

13.(ii)(b) Obtain the equation of parabola in standard form $y^2 = 4ax$.

Let S be the focus, *l* be the directrix. Let Zbe the projection of S on *l* and A be the midpoint of SZ. A lies on the parabola because SA = AZ, A is called the vertex of the parabola. Let YAY′ be the straight line through A and parallel to the directrix. Now take ZX as the X- axis and YY′ as the Y- axis.

Then A is the origin (0,0). Let S = $(a,0), a > 0$. Then Z = $(-a,0)$ and the equation of the directrix is $x + a = 0$.

If P(x, y) is a point on the parabola and PM is the perpendicular distance from P to the directrix *l*, then $\frac{SP}{PM} = e =$ M

$\therefore SP^2 = PM^2$

$\Longrightarrow(x-a)^2 + y^2 = (x+a)^2$

$\therefore y^2 = 4ax.$

Conversely if P(x, y) is any point such that $y^2 = 4ax$ then

SP = $\sqrt{(x-a)^2+y^2} = \sqrt{x^2-2ax+a^2+4ax}$

$= \sqrt{x^2+2ax+a^2} = \sqrt{(x+a)^2} = |x+a| =$ PM

Hence P(x, y) is on the locus. In other words, a necessary and sufficient condition for the P(x, y) point to be on the parabola is that $y^2 = 4ax$.

Thus the equation of the parabola is $y^2 = 4ax$.

14.(i)(a).Find the variance and standard deviation of the data

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	18	4	3

x_i	f_i	$f_i x_i$	$x_i-\bar{x}$	$(X_i-\bar{x})^2$	$f_i(X_i-\bar{x})^2$
6	2	12	-14	196	392
10	4	40	-10	100	400
14	7	98	-6	36	252
18	12	216	-2	4	48
24	18	432	4	16	288
28	4	112	8	64	256
30	3	90	10	100	300
	N= 50	1000			1936

Mean of the given data$\bar{x}=\frac{1}{N}\sum_{i=1}^{7} f_i x_i = \frac{1000}{50} = 20$

Variance of the given data (σ^2)

$= \frac{1}{N}\sum_{i=1}^{7}(X_i-\bar{x})^2 = \frac{1}{50}$X1936=38.72

Standard deviation $\sigma=\sqrt{38.72}$=6.22

14.(i)(b). State and prove addition theorem on probability.

Sol: The addition rule is a result used to determine the probability that event A or event B occurs or both occur.

The result is often written as follows, using set notation:
P(A U B) = P(A) + P(B) - P(P int B)
where:
P(A) = probability that event A occurs
P(B) = probability that event B occurs
P(A U B) = probability that event A or event B occurs
P(A int B) = probability that event A and event B both occur
Proof:
For mutually exclusive events, that is events which cannot occur together: P(A int B) = 0
The addition rule therefore reduces to
P(A U B)= P(A) + P(B)
For independent events, that is events which have no influence on each other:
P(A int B) = P(A).P(B)
The addition rule therefore reduces to
P(AUB)=P(A)+P(B)-P(A).P(B)
In both cases the rules stands true.

14.(ii)(a). If A and B are two events with $P(A\cup B)=0.65$ $P(A\cap B)=0.15$. Then find the value of $P(A^C)+P(B^C)$

Sol: By addition theorem on probability
$P(A\cup B)=P(A)+P(B) - P(A\cap B)$
$P(A)+P(B) = P(A\cup B)+ P(A\cap B)=0.65+0.15=0.8$
$P(A^C)+P(B^C) = 1-P(A)+1-P(B)$
$=2-[P(A)+P(B)]=2-[0.8]=1.2$

14.(ii)(b). Find the variance and standard deviation of the data 5,12,3,18,6,8,2,10

Sol: Mean $(\bar{x})=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})$
$=\frac{5+12+3+18+6+8+2+10}{8}=\frac{64}{8}=8$

To find the variance, we construct the following table

x_i	5	12	3	18	6	8	2	10
$x_i-\bar{x}$	-3	4	-5	10	-2	0	-6	2
$(x_i-\bar{x})^2$	9	16	25	100	4	0	36	4

Here $\sum(x_i-\bar{x})^2=194$
∴Variance$(\sigma^2)=\frac{1}{n}\sum_{i=1}^{8}(x_i-\bar{x})^2=\frac{1}{8}X194=24.25$
Standard deviation $\sigma=\sqrt{24.25}=4.95$

15.(i)(a).Resolve into partial fractions $\frac{x^2+5x+7}{(x-3)^3}$

Sol: Let put x-3=y then x=y+3

$$\frac{x^2+5x+7}{(x-3)^3}=\frac{(y+3)^2+5(y+3)+7}{y^3}=\frac{y^2+6y+9+5y+15+7}{y^3}$$
$$=\frac{y^2+11y+31}{y^3}=\frac{y^2}{y^3}+\frac{11y}{y^3}+\frac{31}{y^3}=\frac{1}{y}+\frac{11}{y^2}+\frac{31}{y^3}$$
$$\therefore \frac{x^2+5x+7}{(x-3)^3}=\frac{1}{x-3}+\frac{11}{(x-3)^2}+\frac{31}{(x-3)^3}$$

15.(i)(b).Solve the differential equation $\frac{dy}{dx}=\frac{y^2-2xy}{x^2-xy}$

Sol: Given $\frac{dy}{dx}=\frac{y^2-2xy}{x^2-xy}$
Let y=Vx
$$\frac{dy}{dx}=V+x\frac{dV}{dx}$$
$$\therefore \frac{dy}{dx}=\frac{y^2-2xy}{x^2-xy}$$
$$V+x\frac{dV}{dx}=\frac{V^2x^2-2xVx}{x^2-xVx}=\frac{V^2-2V}{1-V}$$
$$x\frac{dV}{dx}=\frac{V^2-2V}{1-V}-V=\frac{V^2-2V-V+V^2}{1-V}=\frac{2V^2-3V}{1-V}$$
$$\frac{1-V}{2V^2-3V}dv=\frac{dx}{x}$$
Integrating on both sides
$$\int\frac{1-V}{2V^2-3V}dv=\int\frac{dx}{x}$$
$$-\frac{1}{3}\int(\frac{1}{V}+\frac{1}{2V-3})dv=\int\frac{dx}{x}$$
$$-\frac{1}{3}[\log V+\frac{1}{2}\log(2V-3)]=\log x-\log C$$
$$-\frac{1}{3}\log(V\sqrt{2V-3})=\log x-\log C$$
$$\log(V\sqrt{2V-3})=-3\log x+3\log C=-\log x^3+\log C^3$$
$$\log(V\sqrt{2V-3})+\log x^3=\log C^3$$
$$\log(x^3V\sqrt{2V-3})=\log C^3$$
$$x^3V\sqrt{2V-3}=C^3$$
$$x^3\frac{y}{x}\sqrt{2\frac{y}{x}-3}=C^3$$
$$x^2y\sqrt{2\frac{y}{x}-3}=C^3$$
$$xy\sqrt{2xy-3x^2}=C^3$$

15.(ii)(a).Find the internal and external centers of the circles $x^2+y^2=9$ and $x^2+y^2-16x+2y+49=0$

Sol: Let the given equations of circles be
$S = x^2+y^2=9$ …..(1)
$S' = x^2+y^2-16x+2y+49=0$ ….(2)
Let C_1, C_2 be the centers and r_1, r_2 be radii of circles (1) and (2) respectively.
$S = x^2+y^2=9$
$= (x+0)^2 +(y+0)^2=9=3^2$
We have $C_1(h_1,k_1)=C_1(0,0)$ and $r_1=3$
$S' = x^2+y^2-16x+2y+49=0$
$= x^2-16x +64+y^2+2y+1-64-1+49=0$
$= (x-8)^2 +(y+1)^2=16=4^2$
We have $C_2(h_2,k_2)=C_2(8,-1)$ and $r_2=4$

15.(ii)(b).Find the mean deviation about the median for the following data.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Sol: Writing the observations in the ascending order

x_i	f_i	Cumulative Frequency CF	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
5	8	8	2	16
7→ M	6	$14>\frac{N}{2}$	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
	N =26			$\sum f_i\lvert x_i-M\rvert$ =84

Median = $\frac{26}{2}$ th observation=13th observation=7

Mean deviation about Median $=\frac{\sum_{i=1}^{6}f_i\lvert x_i-M\rvert}{N}$
$=\frac{84}{26}=4$

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MATHEMATICS (BRIDGE COURSE) -II
Sept 2021

Answers

Section – A

1. Express $(1-i)^3$(1+i) in the form of a+bi

Sol: Let $(1-i)^3(1+i) = (1-i)^3(1-i)(1+i)$

$= (1+i^2-2i)(1^2-i^2)$

$= (1-1-2i)(1+1) = (-2i)(2) = -4i$

$(1-i)^3(1+i) = 0+i(-4)$

2. Find the quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

Sol: Let $\alpha = 2+\sqrt{3}$ and $\beta = 2-\sqrt{3}$

Now, $\alpha+\beta = 2+\sqrt{3} + 2-\sqrt{3} = 4$

$\alpha.\beta = (2+\sqrt{3})(2-\sqrt{3}) = 4+2\sqrt{3}-2\sqrt{3} - 3 = 1$

Since a quadratic equation having roots are α and β is of the form $x^2 - (\alpha+\beta)x + \alpha.\beta = 0$

i.e. $x^2 - 4x + 1 = 0$

This is a quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

3.If 1,-2,3 are the roots of the equation $x^3-2x^2+ax+6=0$, then find a.

Sol: 1, -2 and 3 are roots of $x^3-2x^2+ax+6=0$

$\Rightarrow \alpha=1, \beta=-2, \gamma=3$

$\alpha\beta+\beta\gamma+\gamma\alpha = a$

(1)(-2)+(-2)(3)+(3)(1) =a

-2-6+3=a

⇒ a = -5

4. Evaluate $\int \sqrt{1+\sin 2x}\, dx$

Sol: $I = \int \sqrt{1+\sin 2x}\, dx$

$= \int \sqrt{\cos^2 x + \sin^2 x + 2\sin x \cos x}\, dx$

$= \int \sqrt{(\cos x + \sin x)^2}\, dx$

$= \int (\cos x + \sin x) dx$

$= \int \cos x\, dx + \int \sin x\, dx = \sin x - \cos x + C$

5. Evaluate $\int \frac{1}{6-2x^2} dx$

Sol: $I = \int \frac{1}{6-2x^2} dx = \frac{1}{2}\int \frac{1}{3-x^2} dx = \frac{1}{2}\int \frac{1}{(\sqrt{3})^2-x^2} dx$

$= \frac{1}{2}\left(\frac{1}{2\sqrt{3}}\log\left(\frac{\sqrt{3}+x}{\sqrt{3}-x}\right)\right)+c \quad \left[\because \int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\frac{a+x}{a-x}+c\right]$

$= \frac{1}{4\sqrt{3}}\log\left(\frac{\sqrt{3}+x}{\sqrt{3}-x}\right)+c$

6. Evaluate $\int_0^{\frac{\pi}{2}} \cos^8 x\, dx$

Sol: Let $I_8 = \int_0^{\frac{\pi}{2}} \cos^8 x\, dx$

We have $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\, dx = \begin{cases} \frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}..\frac{1}{2}\frac{\pi}{2} \text{ if n is even} \\ \frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}..\frac{2}{3}.1 \text{ if n is odd} \end{cases}$

Since $n = 8$ which is even

$I_8 = \int_0^{\frac{\pi}{2}} \cos^8 x\, dx = \frac{8-1}{8}.\frac{8-3}{8-2}.\frac{8-5}{8-4}.\frac{8-7}{8-6}\frac{\pi}{2} = \frac{7}{8}.\frac{5}{6}.\frac{3}{4}.\frac{1}{2}\frac{\pi}{2} = \frac{35\pi}{256}$

7.Find the centre and the radius of the circle $2x^2+2y^2-3x+2y-1=0$

Sol: Given equation of circle is

$2x^2+2y^2-3x+2y-1=0$ written as

$x^2+y^2-\frac{3}{2}x+y-\frac{1}{2}=0$

$x^2+y^2+2gx+2fy+C=0$

Here $2g = -\frac{3}{2}$; $2f = 1$; $C = -\frac{1}{2}$

$\Rightarrow g = -\frac{3}{4}$; $f = \frac{1}{2}$; $C = -\frac{1}{2}$

∴ Centre $(-g, -f) = (-(-\frac{3}{4}), -(-\frac{1}{2})) = (\frac{3}{4}, \frac{1}{2})$

Radius $= \sqrt{g^2+f^2-c}$

$= \sqrt{(-\frac{3}{4})^2+(\frac{1}{2})^2-(-\frac{1}{2})}$

$= \sqrt{\frac{9}{16}+\frac{1}{4}+\frac{1}{2}} = \sqrt{\frac{9+4+8}{16}} = \sqrt{\frac{21}{16}} = \frac{\sqrt{21}}{4}$

∴ Radius r is $\frac{\sqrt{21}}{4}$

8. Find the centre, eccentricity, length of latus rectum of the hyperbola $16x^2-9y^2=144$

Sol: Given equation of hyperbola

$16x^2-9y^2=-144$

$\frac{x^2}{9} - \frac{y^2}{16} = -1$ it is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$\Rightarrow a^2=9$; $b^2=16 \Rightarrow a=3$; $b=4$ $a<b$

Centre C(0,0)

Eccentricity $e = \sqrt{\frac{a^2+b^2}{b^2}} = \sqrt{\frac{9+16}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

Length of the latus rectum$= \frac{2a^2}{b} = \frac{2(9)}{4} = \frac{9}{2}$

9. Find the mean deviation about the mean for the data 6,7,10,12,13,4,12,16

Sol: Mean $\bar{x} = \frac{6+7+10+12+13+4+12+16}{8}$

Mean $\bar{x} = \frac{80}{8} = 10$

The absolute values of mean deviation are

$|xi - \bar{x}| = 4,3,0,2,3,6,2,6$

∴Mean deviation about the mean$= \frac{\sum_{i=1}^{10}(x_i-\bar{x})}{n}$

$= \frac{4+3+0+2+3+6+2+6}{8} = \frac{26}{8} = 3.25$

10. Find the probability that a non leap year contains 52 Sundays only.

Sol: A non leap year contains 365 days=52 weeks and 1day more.

Number of days in week=7

∴n(S)=7

Let A be the event that a non-leap year contains 52 Sundays. Then A $\{M, T, W, Th, F, Sa\}$ so that $n(A) = 6$. The probability that a non-leap year contains 52 Sundays is $P(A) = \frac{n(A)}{n(S)} = \frac{6}{7}$

Section – B

11.(i)(a).Resolve $\frac{x+4}{(x^2-4)(x+1)}$ into partial fractions.

Sol: $\frac{x+4}{(x^2-4)(x+1)} = \frac{A}{(x+2)} + \frac{B}{(x-2)} + \frac{C}{(x+1)}$

$\frac{x+4}{(x^2-4)(x+1)} = \frac{A(x-2)(x+1)+B(x+2)(x+1)+C(x+2)(x-2)}{(x+2)(x-2)(x+1)}$

$A(x-2)(x+1)+B(x+2)(x+1)+C(x+2)(x-2) = x+4$

Put x=2 then

$A(2-2)(2+1)+B(2+2)(2+1)+C(2+2)(2-2)$=2+4

12B=6 ⇒B=$\frac{1}{2}$

Put x=-2 then

$A(-2-2)(-2+1)+B(-2+2)(-2+1)+C(-2+2)(-2-2)$=-2+4

4A=2 ⇒A= $\frac{1}{2}$

Put x=-1 then

$A(-1-2)(-1+1)+B(-1+2)(-1+1)+C(-1+2)(-1-2)$=-1+4

-3C=3 ⇒C= -1

$\therefore \frac{x+4}{(x^2-4)(x+1)} = \frac{1}{2(x+2)} + \frac{1}{2(x-2)} - \frac{1}{(x+1)}$

11.(i)(b).Show that $Z_1 = \frac{2+11i}{25}$, $Z_2 = \frac{-2+i}{(1-2i)^2}$ are conjugate to each other.

Sol: $\frac{-2+i}{(1-2i)^2} = \frac{-2+i}{1-4-4i} = \frac{-2+i}{-3-4i} = \frac{2-i}{3+4i}$

$= \frac{(2-i)(3-4i)}{(3+4i)(3-4i)} = \frac{(6-4)+i(-8-3)}{3^2+4^2} = \frac{2-11i}{25}$

Since this complex number is the conjugate of $\frac{2+11i}{25}$.

The given complex numbers Z_1,Z_2 are conjugate each other.

(OR)

11.(ii)(a).Find the relation between the roots and the coefficients of the equation

$x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$

Sol: Given equation is

$x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$…(1)

On comparing eq(1) with

$x^4 + P_1 x^3 + P_2 x^2 + P_3 x + P_4 = 0$

We have $P_1 = -2$; $P_2 = 4$; $P_3 = 6$; $P_4 = -21$

Let $\alpha, \beta, \gamma, \delta$ be its roots

Then $S_1 = \sum \alpha = -P_1 = 2$

$S_2 = \sum \alpha\beta = P_2 = 4$

$S_3 = \sum \alpha\beta\gamma = -P_3 = 6$

$S_4 = \sum \alpha\beta\gamma\delta = P_4 = -21$

11. (ii)(b) Find the sum of all 4-digit numbers that can be formed using the digits 1,2,4,5,6.

Sol: Number of digits = 5

$(n-1)_{P_{(r-1)}}$

${}^{(n-1)}P_{(r-1)}$(Sum of all digits) $*$(111.............. r times)

Here, $n = 5$ and $r = 4$

$n - 1 = 4$ and $r - 1 = 3$

Then,

${}^{(n-1)}P_{(r-1)}$(Sum of all digits) $*$(11.............. r times)

$= {}^4P_3 \times (1+2+4+5+6) \times (1111)$

$= 24 * 18 * 1111$

$= 432 * 1111$

$= 479952$

Therefore, the sum of 1111 is 479952.

12.(i)(a).Evaluate $\int \frac{(1+\log x)^5}{x} dx$

Sol: Let I = $\int \frac{(1+\log x)^5}{x} dx$

Put 1+log x=t ⇒ $\frac{1}{x}$dx=dt

I = $\int t^5\, dt = \frac{t^{5+1}}{5+1} + c = \frac{t^6}{6} + c = \frac{(1+\log x)^6}{6} + c$

12.(i)(b).Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\cos x}$

Sol: I=$\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\cos x}$

Let $\tan\frac{x}{2}$=t $\cos x = \frac{1-t^2}{1+t^2}$

⇒dx=$\frac{2\,dt}{1+t^2}$ ⇒x=0⇒t=0; x=$\frac{\pi}{2}$ ⇒t=1

∴ I=$\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\cos x} = \int_0^1 \frac{\frac{2\,dt}{1+t^2}}{4+5\frac{1-t^2}{1+t^2}} = \int_0^1 \frac{\frac{2\,dt}{1+t^2}}{4+5\frac{1-t^2}{1+t^2}}$

I=$2\int_0^1 \frac{1}{9-t^2}\, dt = 2\frac{1}{2.3}[\log\frac{3+t}{3-t}]_0^1 = \frac{1}{3}\log\frac{4}{2} = \frac{1}{3}\log 2$

(OR)

12.(ii)(a).Evaluate $\int_0^1 \frac{3x^2}{x^6+1} dx$

Sol: I=$\int_0^1 \frac{3x^2}{x^6+1} dx = \int_0^1 \frac{3x^2}{(x^3)^2+1} dx$

Put x^3=t

$3x^2$ dx= dt

Upper limit x=1 ⇒t=1^3=1

Lower limit x=0 ⇒t=0^3=0

∴I=$\int_0^1 \frac{3x^2}{(x^3)^2+1} dx = \int_0^1 \frac{1}{1+t^2} dt$

I=$[\tan^{-1} t]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

12.(ii)(b).Solve $\frac{dy}{dx} = \sqrt{\frac{1+y^2}{1+x^2}}$

Sol: Given equation is $\frac{dy}{dx} = \sqrt{\frac{1+y^2}{1+x^2}}$ ⇒ $\frac{dy}{\sqrt{1+y^2}} = \frac{dx}{\sqrt{1+x^2}}$

⇒ $\frac{dy}{\sqrt{1+y^2}} - \frac{dx}{\sqrt{1+x^2}} = 0$

Integrating on both sides

$\int \frac{dy}{\sqrt{1+y^2}} - \int \frac{dx}{\sqrt{1+x^2}} = \int 0$

$\sinh^{-1} y + \sinh^{-1} x$=C

13.(i)(a) Find the equation of the circle passing through the points (2,1), (5,5), (-6,7)

Sol: Let the equation of the circle passing through three points be $x^2 + y^2 + 2gx + 2fy + c = 0$

At point (2,1) is $2^2 + 1^2 + 2g(2) + 2f(1) + c = 0$

4+ 1+4g +2f+c=0

5+4g +2f+c=0 …..(1)

At point (5,5) is $(5)^2 + (5)^2 + 2g(5) + 2f(5) + c = 0$

25+ 25 + 10g + 10f+c=0

50 + 10g +10f+c=0 …..(2)

At point (-6,7) is $(-6)^2 + (7)^2 + 2g(-6) + 2f(7) + c = 0$

36+ 49 - 12g +14f+c=0

85 -12g +14f+c = 0 …..(3)

25+ 20g +10f+5c=0…..(1)X5

50 +10g +10f+ c =0 …..(2)

-25 + 10g +4c = 0 ……(4)

35 + 28g +14f+7c =0 …..(1)X7

85 - 12g +14f+ c = 0 …..(3)

-50 +40g +6c = 0 …..(5)

$-100 + 40g + 16c = 0$ .....(4)X4
$-\ \ 50 + 40g + 6c = 0$ .....(5)
$-50 \qquad + 10c = 0$

$\Longrightarrow c = \frac{50}{10} = 5$

Substitute c = 5 in equation (5)

$-50 + 40g + 6c = 0$

$-50 + 40g + 6(5) = 0 \Longrightarrow g = \frac{1}{2}$

Substitute c=5 and $g = \frac{1}{2}$ in equation (1)

$5 + 4g + 2f + c = 0$

$5 + 4(\frac{1}{2}) + 2f + 5 = 0 \Longrightarrow f = -6$

∴ Equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 + 2(\frac{1}{2})x + 2(-6)y + 5 = 0$

$x^2 + y^2 - x - \mathbf{12}y + 5 = 0$

13.(i)(b) Find the angle between the pair of circles $x^2+y^2+4x-14y+28=0$ and $x^2+y^2+4x-5=0$

Sol: Given circles

$x^2+y^2+4x-14y+28=0$ and $x^2+y^2+4x-5=0$

$x^2 + y^2 + 2gx + 2fy + C = 0 \qquad x^2 + y^2 + 2g'x + 2f'y + C' = 0$

$\Rightarrow g=2;\ f=-7;\ C=28 \qquad \Rightarrow g'=2;\ f'=0;\ C'=-5$

Let θ be the angle between the circles(1) and (2)

$$\cos\theta = \frac{C + C' - 2gg' - 2ff'}{2\sqrt{g^2+f^2-C}\sqrt{g'^2+f'^2-C'}}$$

$$= \frac{28-5-2(2)(2)-2(-7)(0)}{2\sqrt{2^2+(-7)^2-28}\sqrt{(2)^2+(0)^2-(-5)}}$$

$$= \frac{28-5-8-0}{2\sqrt{4+49-28}\sqrt{4+0+5}}$$

$$= \frac{15}{2\sqrt{25}\sqrt{9}} = \frac{15}{2(5)(3)} = \frac{1}{2}$$

$$\therefore \theta = \cos^{-1}\frac{1}{2} = 60°$$

Hence the angle between the circles is 60°.

13.(ii)(a) Obtain the equation of parabola in standard form $y^2 = 4ax$.

Let S be the focus, *l* be the directrix. Let Zbe the projection of S on *l* and A be the midpoint of SZ. A lies on the parabola because SA = AZ, A is called the vertex of the parabola. Let YAY′ be the straight line through A and parallel to the directrix. Now take ZX as the X- axis and YY′ as the Y- axis.

Then A is the origin (0,0). Let S = $(a,0), a > 0$. Then Z = $(-a,0)$ and the equation of the directrix is $x + a = 0$.

If P(x, y) is a point on the parabola and PM is the perpendicular distance from P to the directrix *l*, then $\frac{SP}{PM} = e = M$

$\therefore SP^2 = PM^2$

$\Longrightarrow (x-a)^2 + y^2 = (x+a)^2$

$\therefore y^2 = 4ax$.

Conversely if P(x, y) is any point such that $y^2 = 4ax$ then

$SP = \sqrt{(x-a)^2 + y^2} = \sqrt{x^2 - 2ax + a^2 + 4ax}$

$= \sqrt{x^2 + 2ax + a^2} = \sqrt{(x+a)^2} = |x + a| = PM$

Hence P(x, y) is on the locus. In other words, a necessary and sufficient condition for the P(x, y) point to be on the parabola is that $y^2 = 4ax$.

Thus the equation of the parabola is $y^2 = 4ax$.

13.(ii)(b) Find the equation of the ellipse whose focus is (2,1), eccentricity $\frac{3}{4}$ and directrix is 2x-y+3=0.

Sol: Let P(x_1, y_1) be any point on the ellipse.

Focus S=(2,1), eccentricity $e = \frac{3}{4}$

Equation of the directrix M=2x-y+3=0

Equation of ellipse having focus(x_1, y_1) and directrix ax+by+c=0 is

$(a^2 + b^2)[(x-x_1)^2 + (y-y_1)^2] = e^2(ax+by+c)^2$

Equation of ellipse having focus(2,1), $e = \frac{3}{4}$

M=2x-y+3=0 is

$(2^2 + (-1)^2)[(x-2)^2 + (y-1)^2] = (\frac{3}{4})^2(2x-y+3)^2$

$\Rightarrow 80[x^2-4x+4+y^2-2y+1] = 9[4x^2+y^2+9-4xy-6y+12x]$

$80x^2-320x+80y^2-160y+400 =$

$36x^2+9y^2+81-36xy+108x-54y$

$44x^2+71y^2+36xy-428x-106y+319=0$

Which is required equation of ellipse.

14.(i)(a) Find the mean deviation about the mean for the following data.

x_i	2	5	7	8	10	35
f_i	6	8	10	6	8	2

Sol: We can form the following table from the given data

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
2	6	12	6	36
5	8	40	3	24
7	10	70	1	10
8	6	480	0	0
10	8	80	2	16
35	2	70	27	54
	N =40	$\sum f_i x_i = 320$		$\sum f_i \lvert x_i - \bar{x}\rvert = 140$

$\therefore Mean(\bar{x}) = \frac{\sum f_i x_i}{N} = \frac{320}{40} = 8$

Mean deviation about the mean$= \frac{\sum_{i=1}^{6} f_i |x_i - \bar{x}|}{N}$

$= \frac{140}{40} = 3.5$

14.(i)(b) Let A and B be independent events with P(A)=0.2; P(B) = 0.5
Find (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A\cup B)$

Sol: Given that A,B are two independent events
P(A)=0.2; P(B) = 0.5
$P(A\cap B)=P(A)P(B)=0.2X0.5=0.1$
(i) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{0.1}{0.5}=\frac{1}{5}$
(ii) $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{0.1}{0.2}=\frac{1}{2}$
(iii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.2+0.5-0.1=0.6$

(OR)

14.(ii)(a) A problem in calculus is given to solving it are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of the problem being solved if both of then try independently.

Sol: Let E_1 and E_2 denote the events that the problem is solved by two students A and B respectively. We are given that $P(E_1)=\frac{1}{3}$ and
$P(E_2)=\frac{1}{4}$
Since E_1 and E_2 are two independent events. So that
$P(E_1\cap E_2) = P(E_1).P(E_2)=\frac{1}{3}.\frac{1}{4}=\frac{1}{12}$
The probability that the problem being solved is
$P(E_1\cup E_2) = P(E_1)+P(E_2) - P(E_1\cap E_2)$
$=\frac{1}{3}+\frac{1}{4}-\frac{1}{12}=\frac{4+3-1}{12}=\frac{6}{12}=\frac{1}{2}$

14.(ii)(b) Calculate the variance and standard deviation for a discrete frequency distribution

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

x_i	f_i	$f_i x_i$	$x_i-\bar{x}$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	N= 30	420			1374

Variance of the given data (σ^2)
$=\frac{1}{N}\sum_{i=1}^{7}(x_i-\bar{x})^2=\frac{1}{30}X1374=45.8$
Standard deviation $\sigma=\sqrt{45.8}=6.77$

15.(i)(a) Resolve $\frac{x}{(x+1)(2x+1)}$ into partial fractions.

Sol: Let $\frac{x}{(x+1)(2x+1)}=\frac{A}{x+1}+\frac{B}{2x+1}$ where A and B are real numbers
$\frac{x}{(x+1)(2x+1)}=\frac{A(2x+1)+B(x+1)}{(x+1)(2x+1)}$
$\Rightarrow A(2x+1)+B(x+1)=x$
If $x=-\frac{1}{2}$ then
$A\left(2(-\frac{1}{2})+1\right)+B(-\frac{1}{2}+1)=(-\frac{1}{2})$
$\frac{1}{2}B=-\frac{1}{2}\Rightarrow B=-1$
If x=-1 then
$A(2(-1)+1)+B(-1+1)=(-1)$
$-A=-1\Rightarrow A=1$
$\therefore \frac{x}{(x+1)(2x+1)}=\frac{1}{(x+1)}-\frac{1}{(2x+1)}$

15.(i)(b) Solve $\frac{dy}{dx}=(3x+y+4)^2$

Sol: Given equation is $\frac{dy}{dx}=(3x+y+4)^2$
Put 3x+y+4=z
$3+\frac{dy}{dz}=\frac{dz}{dx}\Rightarrow\frac{dy}{dz}=\frac{dz}{dx}-3$
$\frac{dy}{dx}=(3x+y+4)^2=\frac{dz}{dx}-3=z^2$
$\frac{dz}{dx}=z^2+3$
$\frac{dz}{z^2+3}=dx\Rightarrow\frac{dz}{z^2+3}-dx=0$
Integrating on both sides
$\int\frac{dz}{z^2+3}-\int dx=\int 0$
$\frac{1}{\sqrt{3}}\tan^{-1}(\frac{z}{\sqrt{3}})-x=C$
$\therefore\frac{1}{\sqrt{3}}\tan^{-1}(\frac{3x+y+4}{\sqrt{3}})-x=C$

(OR)

15.(ii)(a) Find the equations of common tangents of the pair of circles $x^2+y^2-8x-2y+8=0$ and $x^2+y^2-2x+6y+6=0$

Sol: Given circles
$S=x^2+y^2-8x-2y+8=0$
$S'=x^2+y^2-2x+6y+6=0$
Equation of common chords is L=S -S' = 0
$=(x^2+y^2-8x-2y+8)-(x^2+y^2-2x+6y+6)$
$=-6x-8y+2=0\Rightarrow 3x+4y+1=0$

15.(ii)(b) Find the mean deviation about the median for the following continuous distribution

marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of boys	6	8	14	16	4	2

Marks obtained	No. of boys	CF	Mid point	(X_i-M)	$f_i(X_i-M)$
0-10	6	6	5	22.86	187.16
10-20	8	14	15	12.86	102.88
20-30	14	28	25	2.86	40.04
30-40	16	44	35	7.14	114.24
40-50	4	48	45	17.14	68.56
50-60	2	50	55	27.14	54.28
					517.16

Median class: class containing $\frac{50}{2}$th item i.e. 20-30
Here L=20; C=10; f=14; m =14; N =50
Median $=L+\frac{\frac{N}{2}-M}{f}x C=20+\frac{\frac{50}{2}-14}{14}x10$
=20+7.89 =27.86
Mean deviation through median
$=\frac{1}{N}\sum_{i=1}^{n}fi(x_i-\bar{x})=\frac{517.16}{50}=10.34$

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MATHEMATICS (BRIDGE COURSE) -II
May 2022

Answers

Section – A

1. Find the multiplicative inverse of -5+12i

Sol: Since (x+iy)[$\frac{x-iy}{x^2+y^2}$) =1 it follows that the multiplicative inverse of x+iy is $\frac{x-iy}{x^2+y^2}$

Multiplicative inverse of -5+12i is

$\frac{-5-12i}{(-5)^2+12^2} = \frac{-5-12i}{25+144} = \frac{-5-12i}{169}$

2. Find the roots of the equation $x^2 - 7x + 12 = 0$

Sol: On comparing the given equation with $a x^2 + bx + c = 0$, we have $a = 1, b = -7$ and $c = 12$.

∴ Roots of the quadratic equation are

$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-7)\pm\sqrt{(-7)^2-4(1)(12)}}{2(1)}$

$= \frac{7\pm\sqrt{49-48}}{2} = \frac{7\pm\sqrt{1}}{2}$

$= \frac{7\pm1}{2} = \frac{7+1}{2}, \frac{7-1}{2} = \frac{8}{2}, \frac{6}{2} = 4,3$

Hence the roots of the equation $x^2 - 7x + 12 = 0$ are 4 and 3.

3.If 1,-2,3 are the roots of the equation $x^3-2x^2+ax+6=0$, then find a.

Sol: 1, -2 and 3 are roots of $x^3-2x^2+ax+6=0$

$\Rightarrow \alpha=1, \beta=-2, \gamma=3$

$\alpha\beta + \beta\gamma+\gamma\alpha = a$

(1)(-2)+(-2)(3)+(3)(1) =a

-2-6+3=a

⇒ a = -5

4. Evaluate $\int \sqrt{1+\cos 2x}\, dx$

Sol: Let I=$\int \sqrt{1+\cos 2x}\, dx$

=$\int \sqrt{1+2\cos^2 x-1}\, dx$=$\int \sqrt{2\cos^2 x}\, dx$

=$\sqrt{2}\int \cos x\, dx$=$\sqrt{2}\sin x$+C

5. Evaluate $\int \frac{\cos(\log x)}{x} dx$

Sol: Let t=$\log x \Rightarrow$dt= $\frac{1}{x}$ dx

∴ $\int \frac{\cos(\log x)}{x}dx=\int \cos t\, dt=\sin t+C$

$=\sin(\log x))+C$

6. Evaluate $\int \frac{3x^2}{x^6+1} dx$

Sol: I=$\int \frac{3x^2}{x^6+1} dx$

Put x^3=t⇒ $3x^2\, dx$ = dt

∴I=$\int \frac{3x^2}{x^6+1}dx=\int \frac{1}{t^2+1}dt=\tan^{-1} t+C=\tan^{-1}(x^3)+C$

7. Find the equation of the circle whose Centre C=(-1,2) and radius r=4

Sol: Here (h,k)=(-1,2) and r=4

The equation of the circle with centre at C(h,k) and radius r is

$(x-h)^2+(y-k)^2=r^2$

$(x+1)^2+(y-2)^2=4^2$

$x^2+2x+1+ y^2-4y+4=16$

∴ Equation of circle is $x^2+ y^2 +2x -4y-11=0$

8. Find the centre, eccentricity, length of latus rectum of the hyperbola $9x^2-25y^2=225$

Sol: Given equation of hyperbola $9x^2-25y^2=225$

$\frac{x^2}{25} - \frac{y^2}{9} = 1$ it is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$\Rightarrow a^2=25; b^2=9 \Rightarrow a=5; b=3$

Centre C(0,0)

Eccentricity e =$\sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{25+9}{25}} = \sqrt{\frac{34}{25}} = \frac{\sqrt{34}}{5}$

Length of the latus rectum = $\frac{2b^2}{a} = \frac{2(9)}{5} = \frac{18}{5}$

9. Find the mean deviation about the mean for the data 6,7,10,12,13,4,12,16

Sol: Mean $\bar{x}$= $\frac{6+7+10+12+13+4+12+16}{8}$

Mean $\bar{x} = \frac{80}{8}$ =10

The absolute values of mean deviation are

$|xi - \bar{x}|$ = 4,3,0,2,3,6,2,6

∴Mean deviation about the mean=$\frac{\sum_{i=1}^{10}(x_i-\bar{x})}{n}$

$= \frac{4+3+0+2+3+6+2+6}{8} = \frac{26}{8}$ =3.25

10. Find the probability of drawing an Ace from a well shuffled pack of 52 playing cards.

Sol: A pack of cards means of pack containing 52 cards, 26 of them are red and 26 of them are black coloured. These 52 cards are divided into 4 sets namely Hearts, Spades, Diamonds and Clubs. Each set contains of 13 cards names A, 2, 3, 4, 5, 6, 7, 8, 9, 10, K, Q,J.

P(A) is the probability of drawing Ace= $\frac{4}{52} = \frac{1}{13}$

Section – B

11(i)(a) Resolve $\frac{5x+1}{(x-1)(x+2)}$ into partial fractions.

Sol: Let $\frac{5x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$ where A and B are real numbers

$\frac{5x+1}{(x-1)(x+2)} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)}$

$\Rightarrow A(x+2) + B(x-1) = 5x+1$

If x=1 then A(1+2) +B(1-1) =5(1)+1

3A = 6⇒A =2

If x=-2 then A(-2+2) +B(-2-1) = 5(-2)+1

-3B = -9⇒B=3

∴ $\frac{5x+1}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{3}{x+2}$

11(i)(b) If $(a+ib)^2$= x+iy find x^2+y^2

Sol: Given $(a+ib)^2$ = x+ib

a^2-b^2+2abi = x+iy

Here x = a^2-b^2; y = 2ab

$\therefore x^2 + y^2 = (a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$

(OR)

11(ii)(a) Find the relation between the roots and the coefficients of the equation $3x^3 - 10x^2 + 7x + 10 = 0$

Sol: Given equation $3x^3 - 10x^2 + 7x + 10 = 0$

Comparing with $ax^3 + bx^2 + cx + d = 0$

$\Rightarrow a = 3;\ b = -10;\ c = 7;\ d = 10$

$\sum \alpha$ = sum of roots = $S_1 = -\frac{b}{a} = -\frac{-10}{3} = \frac{10}{3}$

$\sum \alpha\beta \quad = S_2 = \frac{b}{a} = \frac{7}{3}$

$\alpha\beta\gamma \quad = S_3 = -\frac{d}{a} = -\frac{10}{3}$

11(ii)(b) Find the sum of all 4-digit numbers that can be formed using the digits 1,3,5,7,9.

Sol: The number of 4 digit numbers that can be formed using the given 5 digits is $5P_4 = 120$

No. of distinct digits n=5

The sum of the r digital numbers that can be formed using the given n distinct digits ($1 \le n \le 9$) is $(n-1)P_{r-1}$X sum of the digitsx111….1(r times) Hence n=5, r=4

The sum of all 4 digited numbers that can be formed using the digits {1,3,5,7,9} without repetitions is $4\text{-}1P_{4\text{-}1}$x(1+3+5+7+9)X1111

$= 4P_3$X25x1111=24x25x1111=666600

12(i)(a). Evaluate $\int \frac{1}{\cosh x + \sinh x} dx$

Sol: $I = \int \frac{1}{\cosh x + \sinh x} \times \frac{\cosh x - \sinh x}{\cosh x - \sinh x} dx$

$= \int \frac{\cosh x - \sinh x}{\cosh^2 x - \sinh^2 x} dx = \int \frac{\cosh x - \sinh x}{1} dx$

$= \int \cosh x\, dx - \int \sinh x\, dx = \sinh x - \cosh x + C$

12(i)(b) Evaluate $\int_2^3 \frac{2x}{1+x^2} dx$

Sol: Let $\int_2^3 \frac{2x}{1+x^2} dx \quad \because \int \frac{f'(x)}{f(x)} dx = \log x + C$

$\int_2^3 \frac{2x}{1+x^2} dx = [\log(1+x^2)]_2^3$

$= \log(1+3^2) - \log(1+2^2)$

$= \log 10 - \log 5 = \log \frac{10}{5} = \log 2$

(OR)

12(ii)(a)Evaluate $\int_0^2 |1-x|\ dx$

Sol: $\int_0^2 |1-x|\, dx = \int_0^1 |1-x|\, dx + \int_1^2 |x-1|\, dx$

$= [x - \frac{x^2}{2}]_0^1 + [\frac{x^2}{2} - x]_1^2$

$= 1 - \frac{1}{2} + \frac{4}{2} - 2 - (\frac{1}{2} - 1) = \frac{1}{2} + 0 + \frac{1}{2} = 1$

12(ii)(b) Find the order and degree of the following differential equations $[\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3]^{\frac{6}{5}} = 6y$

Sol: Given equation is $[\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3]^{\frac{6}{5}} = 6y$

i.e. $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = [6y]^{\frac{5}{6}}$

Order = 2, degree = 1

13(i)(a) Find the equation of the circle passing through the points (5,7), (8,1), (1,3)

Sol: Let the equation of the circle passing through three points be $x^2 + y^2 + 2gx + 2fy + c = 0$

At point (5,7) is $5^2 + 7^2 + 2g(5) + 2f(7) + c = 0$

$25 + 49 + 10g + 14f + c = 0$

$74 + 10g + 14f + c = 0$ …..(1)

At point (8,1) is $(8)^2 + (1)^2 + 2g(8) + 2f(1) + c = 0$

$64 + 1 + 16g + 2f + c = 0$

$65 + 16g + 2f + c = 0$ …..(2)

At point (1,3) is $(1)^2 + (3)^2 + 2g(1) + 2f(3) + c = 0$

$1 + 9 + 2g + 6f + c = 0$

$10 + 2g + 6f + c = 0$ …..(3)

$74 + 10g + 14f + c = 0$ …..(1)

$\underline{455 + 112g + 14f + 7c = 0}$ …..(2)X7

$\underline{-381 - 102g \quad - 6c = 0}$

$381 + 102g \quad + 6c = 0$ ……(4)

$195 + 48g + 6f + 3c = 0$ …..(2)X3

$\underline{10 + 2g + 6f + c = 0}$ …..(3)

$185 + 46g \quad + 2c = 0$ …..(5)

$381 + 102g + 6c = 0$ ……(4)

$\underline{555 + 138g + 6c = 0}$ …..(5)

$\underline{-174 - 36g \quad = 0}$

$\Rightarrow g = -\frac{174}{36} = -\frac{29}{6}$

Substitute $g = -\frac{29}{6}$ in equation (5)

$555 + 138g + 6c = 0$

$555 + 138(-\frac{29}{6}) + 6c = 0$

$555 - 23(29) + 6c = 0$

$555 - 667 + 6c = 0 \quad \Rightarrow c = \frac{112}{6} = \frac{56}{3}$

Substitute $c = \frac{56}{3}$ and $g = -\frac{29}{6}$ in equation (3)

$10 + 2g + 6f + c = 0$ …..(3)

$10 + 2(-\frac{29}{6}) + 6f + \frac{56}{3} = 0$

$10 - \frac{29}{3} + 6f + \frac{56}{3} = 0$

$\Rightarrow f = -\frac{19}{6}$

$\therefore$ Equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 + 2(-\frac{29}{6})x + 2(-\frac{19}{6})y + \frac{56}{3} = 0$

$x^2 + y^2 - \frac{29}{3}x - \frac{19}{3}y + \frac{56}{3} = 0$

13(i)(b) Find the angle between the circles $x^2+y^2-6x-8y+12=0$ and $x^2+y^2-4x+6y-24=0$

Sol: Given circles

$x^2+y^2-6x-8y+12=0$ and $x^2+y^2-4x+6y-24=0$

$x^2+y^2+2gx+2fy+C=0 \qquad x^2+y^2+2g'x+2f'y+C'=0$

$\Rightarrow g=-3;\ f=-4;\ C=12 \qquad \Rightarrow g'=-2;\ f'=3;\ C'=-24$

Let θ be the angle between the circles(1) and (2)

$\cos\theta = \frac{C+C'-2gg'-2ff'}{2\sqrt{g^2+f^2-C}\sqrt{g'^2+f'^2-C'}}$

$= \frac{12-24-2(-3)(-2)-2(-4)(3)}{2\sqrt{(-3)^2+(-4)^2-12}\sqrt{(-2)^2+(3)^2-(-24)}}$

$= \frac{12-24-12+24}{2\sqrt{9+16-12}\sqrt{4+9+24}} = 0$

$\therefore \theta=\cos^{-1} 0 = 90°$

Hence the angle between the circles is 90°

(OR)

13(ii)(a) Obtain the equation of parabola in standard form $y^2 = 4ax$.

Let S be the focus, l be the directrix. Let Zbe the projection of S on l and A be the midpoint of SZ. A lies on the parabola because SA = AZ, A is called the vertex of the parabola. Let YAY′ be the straight line through A and parallel to the directrix. Now take ZX as the X- axis and YY′ as the Y- axis.

Then A is the origin (0,0). Let S = $(a,0), a > 0$. Then Z = $(-a,0)$ and the equation of the directrix is $x + a = 0$.

If P(x, y) is a point on the parabola and PM is the perpendicular distance from P to the directrix l, then $\frac{SP}{PM} = e =$ M

$\therefore SP^2 = PM^2$

$\Longrightarrow (x-a)^2 + y^2 = (x+a)^2$

$\therefore y^2 = 4ax$.

Conversely if P(x, y) is any point such that $y^2 = 4ax$ then

$SP = \sqrt{(x-a)^2 + y^2} = \sqrt{x^2 - 2ax + a^2 + 4ax}$

$= \sqrt{x^2 + 2ax + a^2} = \sqrt{(x+a)^2} = |x + a| = PM$

Hence P(x, y) is on the locus. In other words, a necessary and sufficient condition for the P(x, y) point to be on the parabola is that $y^2 = 4ax$.

Thus the equation of the parabola is $y^2 = 4ax$.

13(ii)(b) Find the equation of the ellipse whose focus is (1,-1), eccentricity $\frac{2}{3}$ and directrix is x+y+z=0.

Sol: wrong question no solution x+y+2 =0 instead of x+y+Z

Sol: Let P(x_1,y_1) be any point on the ellipse.

Equation of the directrix L=x+y+2=0

By definition of ellipse SP=e.PM

$SP^2=e^2.PM^2$

$(x_1-1)^2+(y_1+1)^2=(\frac{2}{3})^2[\frac{x1+y1+2}{\sqrt{1+1}}]^2 = \frac{4}{9}[\frac{x1+y1+2}{\sqrt{2}}]^2$

$9[(x_1-1)^2+(y_1+1)^2]=2(x_1+y_1+2)^2$

$9[x_1^2-2x_1+1+y_1^2+2y_1+1]=2[x_1^2+y_1^2+4+2x_1y_1+4x_1+4y_1$

$9x_1^2+9y_1^2-18x_1+18y_1+18]=$

$2x_1^2+2y_1^2+4x_1y_1+8x_1+8y_1+8$

$7x_1^2+7y_1^2-4x_1y_1-26x_1+10y_1+10=0$

$7x_1^2-4x_1y_1+7y_1^2-26x_1+10y_1+10=0$

Locus of P(x_1,y_1) is $7x^2-4xy+7y^2-26x+10y+10=0$

14(i) (a) Find the mean deviation about the mean for the following distribution

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Sol: We can form the following table from the given data

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	N=80	$\sum f_i x_i$ = 4000		$\sum f_i \lvert x_i - \bar{x}\rvert$ =1280

$\therefore$ Mean$(\bar{x}) = \frac{\sum f_i x_i}{N} = \frac{4000}{80} = 50$

Mean deviation about the mean$=\frac{\sum_{i=1}^{5} f_i |x_i-\bar{x}|}{N}$

$=\frac{1280}{80} = 16$

14(i)(b) Suppose A and B are independent events with P(A)=0.6, P(B)=0.7. Then compute

(i) $P(A \cap B)$ (ii) $P(A\cup B)$ (iii) $P(A/B)$

(iv) $P(A^c \cap B^c)$

Sol: Given that A and B are two independent events such that P(A)=0.6, P(B)=0.7

(i) $P(A \cap B) = P(A).P(B) = 0.6X0.7 = 0.42$

(ii) $P(A\cup B) = P(A) + P(B) - P(A \cap B)$

$=0.6 + 0.7 - 0.42 = 0.88$

(iii) $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.42}{0.7} = 0.6$

(iv) $P(A^c \cap B^c) = P(A\cup B)^c = 1 - P(A\cup B) = 1 - 0.88 = 0.12$

(OR)

14(ii)(a) A,B,C are 3 news papers from a city. 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% both A and C, 4% both B and C and 2% all the three. Find the percentage of the population who read at least one news paper.

Sol: Given $P(A)= \frac{20}{100} = 0.2$

$P(B) = \frac{16}{100} = 0.16$

$P(C) = \frac{14}{100} = 0.14$

$P(A\cap B) = \frac{8}{100} = 0.08$

$P(B\cap C) = \frac{4}{100} = 0.04$

$P(A\cap C) = \frac{5}{100} = 0.05$

$P(A\cap B\cap C) = \frac{2}{100} = 0.02$

$P(A\cup B \cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

$=0.2+0.16+0.14-0.08-0.04-0.05+0.02$

$=0.52-0.17=0.35$

Percentage population who read at least one news paper=0.35X100=35%

14(ii)(b) Find the variance and standard deviation of the following frequency distribution

x_i	4	8	11	1	20	24	32
f_i	3	5	9	5	4	3	1

Sol: No solution data not correct

Actual problem is

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

x_i	f_i	$f_i x_i$	$x_i-\bar{x}$	$(X_i-\bar{x})^2$	$f_i(X_i-\bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	N= 30	420			1374

Variance of the given data (σ^2)

$=\frac{1}{N}\sum_{i=1}^{7}(x_i-\bar{x})^2=\frac{1}{30}$X1374=45.8

Standard deviation $\sigma=\sqrt{45.8}$=6.77

15(i)(a) Resolve $\frac{13x+43}{(2x+5)(x+6)}$ into partial fractions.

Sol: Let $\frac{13x+43}{(2x+5)(x+6)}=\frac{A}{2x+5}+\frac{B}{x+6}$ where A and B are real numbers

$\frac{13x+43}{(2x+5)(x+6)} = \frac{A(x+6)+B(2x+5)}{(2x+5)(x+6)}$

$\Rightarrow A(x+6)+B(2x+5)= \mathbf{13x+43}$

If x= - 6 then

$A(-6+6)+B(2(-6)+5)= \mathbf{13(-6)+43}$

$-7B$ = -35⇒B = 5

If x= $-\frac{5}{2}$ then

$A\left((-\frac{5}{2})+6\right)+B(2(-\frac{5}{2})+5)= \mathbf{13(-\frac{5}{2})+43}$

$\frac{7}{2}A =-\frac{65}{2}$ +43

7A =− 65 +86

7A =21⇒A=3

$\therefore \frac{13x+43}{(2x+5)(x+6)} = \frac{3}{2x+5}+\frac{5}{x+6}$

15(i)(b) Solve $\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$

The given equation is $\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$

Which is a first order and first degree differential equation. It can be solved by the method of variables separable.

$\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$

$\Rightarrow (1+y^2)dy = (1+x^2)dx$

$\Rightarrow (1+x^2)dx - (1+y^2)dy = 0$

Now taking integration on both sides we get

$\Rightarrow \int(1+x^2)dx - \int(1+y^2)dy = \int 0$

$\Rightarrow (x+\frac{x^3}{3}) - (y+\frac{y^3}{3}) = c$

$\Rightarrow x^3 - y^3+3x-3y=c$

Hence the required solution is $x^3 - y^3+3x-3y=c$

(OR)

15(ii)(a) Find the equations of common tangents of the pair of circles $x^2+y^2+10x-2y+22=0$ and $x^2+y^2+2x-8y+8=0$

Sol: Given circles

$S= x^2+y^2+10x-2y+22=0$

$S'= x^2+y^2+2x-8y+8=0$

Equation of common chords is L=S -S' = 0

$=(x^2+y^2+10x-2y+22)-(x^2+y^2+2x-8y+8)$

=8x+6y+14=0 ⇒ 4x+3y+7=0

15(ii)(b) Find the mean deviation about the mean for the following continuous distribution

marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	15	16	6

Median class: class containing $\frac{100}{2}$th item i.e.

Marks obtained	No. of students	Midpoint x_i	di	$f_i d_i$	$\lvert X_i-\bar{x}\rvert$	$f_i\lvert X_i-\bar{x}\rvert$
0-10 10-20 20-30 30-40 40-50	5 8 15 16 6	5 15 25 35 45	- 2 - 1 0 1 2	- 10 -8 0 16 12	22 12 2 8 18	110 96 30 128 108
	50			10		472

Let us assumed mean be A=25, here C=10

Mean $\bar{x}$= A+$\frac{1}{N}\sum_{i=1}^{5}f_i d_i$ XC =25 +$\frac{10(10)}{50}$ = 25 +2 =27

Mean deviation about mean is $\frac{1}{N}\sum_{i=1}^{n}f(x_i-\bar{x})=\frac{472}{50}$

=9.44

*^%